Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol. 12, No. 3, Sep. 2013, pp 337-344

BIANCHI TYPE V STRING DUST COSMOLOGICAL MODEL IN GENERAL RELATIVITY

DEO KARAN SINGH REWAR
Department of Mathematics, Government Khetan Polytechnic College, Jaipur-302004 (Rajasthan)
E-mail: devkaranrewar@gmail.com

Received : Sep. 25, 2013

Abstract : A model formed by string dust in the context of general relativity is studied. This model is used as a source of Bianchi Type V space-time. The model represents big bang model in which expansion decreases with time. The model in general represents anisotropic space-time. However, the model isotropizes for large values of time. The model allows particle horizon and represents Point Type singularity. For large values of T, the string tension vanishes. Thus anisotropy is maintained due to the presence of string. As soon as string disappears, anisotropy also disappears.

Keywords: Bianchi V, String dust, Cosmological.

2010 Mathematics Subject Classification: 83C15

1. Introduction

It has not yet convincingly been decided whether the physical universe can better be explained by a closed or an open model. But many theoretical considerations have pointed towards the possibility of an open universe. Bianchi Type V models represent the open generalized Friedmann-Robertson-Walker (FRW) cosmological model with $K = -1$ where K is the curvature of three-dimensional space at any time t. Nayak and Sahoo [13] have investigated Bianchi Type V models with a matter distribution admitting anisotropic pressure and heat flow. Ram et al.[14] investigated Bianchi Type V cosmological models with perfect fluid and heat conduction in Lyra geometry.

It seems that after the big-bang, the universe might have experienced a number of phase transitions. The phase transitions produce vacuum domain structure such as domain walls, strings and monopoles (Kibble [10], Everett [9]). Cosmic strings have excited considerable interest as they act as gravitational lenses and gives rise to density perturbations leading to the formation of galaxies. (Turok [19]). Letelier [11] has done a pioneering work in the formulation of energy-momentum tensor for classical massive strings. Stachel [17] considered massless (geometric string) models to develop a realistic treatment of strings. Banerjee et al. [8] have investigated some cosmological solutions for massive strings in Bianchi Type I space-time in presence and absence of magnetic field. Tikekar and Patel [18] have obtained some exact solutions of string cosmology in Bianchi Type III space-time. Roy and Banerjee [16] have investigated some LRS Bianchi Type II string cosmological models for cloud of geometrical and massive strings using the condition $\varepsilon = \lambda$ and (σ/θ) constant respectively. Bali and Dave [1] have investigated some special strings solutions for Bianchi Type IX space-time using the condition $\rho = \lambda$ and $a = e^{\alpha t}$ where a is metric potential. Many authors viz. Bali et al. [1,2,3,4,5,6,7], Reddy and Rao [15], Wang [21] have investigated string cosmological models in different contexts.

2. The Metric and Field Equations

We consider Bianchi Type V metric in the form

$$dt^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{2x} dz^2 \qquad \ldots(1)$$

where A, B, C are functions of t-alone.

The energy-momentum tensor for string dust is given by Letelier [9]

$$T_i^j = \varepsilon v_i v^j - \lambda x_i x^j \qquad \ldots(2)$$

together with

$$v^i v_i = -x_i x^i = -1 \qquad \ldots(3)$$

$$v_1 = 0 = v_2 = v_3,\ v^4 = 1 \qquad \ldots(4)$$

$$x_1 \neq 0,\ x_2 = 0 = x_3 = x_4 \qquad \ldots(5)$$

where ε is the rest energy density of the system of strings, λ the string tension density, v^i the flow vector, x^i the direction of string.

The Einstein field equation

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\,\pi\, T_i^j \text{(using the units in which c = G = 1)} \qquad \ldots(6)$$

for the line-element (2.1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = 8\pi\lambda \qquad \ldots(7)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = 0 \qquad \ldots(8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = 0 \qquad \ldots(9)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{3}{A^2} = 8\pi\varepsilon \qquad \ldots(10)$$

$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \qquad \ldots(11)$$

3. Solutions of Field Equations

Equation (11) leads to

$$A^2 = \ell^2 BC \qquad \ldots(12)$$

where ℓ is constant of integration. From equations (8) and (9), we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = -\frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) \qquad \ldots(13)$$

which after using (11) leads to

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = -\frac{1}{2}\frac{(BC)_4}{BC} \qquad ...(14)$$

After integration, it leads to

$$C^2\left(\frac{B}{C}\right)_4 = \frac{L}{\sqrt{BC}} \qquad ...(15)$$

L being constant of integration. To get deterministic model of the universe, we apply string dust condition $\in = \lambda$. Thus equations (7) and (10) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{2}{A^2} = 0 \qquad ...(16)$$

Using (12) in (16), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{1}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{2}{\ell^2 BC} = 0 \qquad ...(17)$$

To solve equations (15) and (17), we assume

$$BC = \mu, \frac{B}{C} = \nu \qquad ...(18)$$

Using (18) in equations (15) and (17), we have

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^{3/2}} \qquad ...(19)$$

and $$\frac{\mu_{44}}{\mu} - \frac{\mu_4^2}{\mu^2} + \frac{1}{2}\frac{\nu_4^2}{\nu^2} + \frac{2}{\ell^2\mu} = 0$$

which after using (19) leads to

$$\frac{\mu_{44}}{\mu} - \frac{\mu_4^2}{\mu^2} = -\frac{L^2}{2\mu^3} - \frac{2}{\ell^2\mu} \qquad ...(20)$$

which leads to

$$2\mu_{44} - \frac{2}{\mu}\mu_4^2 = -\frac{L^2}{\mu^2} - \frac{4}{\ell^2} \qquad ...(21)$$

To find the solution of (21), we assume that

$$\mu_4 = f(\mu)$$

Thus $\mu_{44} = f f^1, f^1 = df/d\mu.$

Now equation (21) leads to

$$\frac{d}{d\mu}(f^2) - \frac{2}{\mu} f^2 = -\frac{L^2}{\mu^2} - \frac{4}{\ell^2} \qquad \ldots(22)$$

The solution of (22) is given by

$$f^2 = \frac{a^2 + b^2\mu^2 + M\mu^3}{\mu} \qquad \ldots(23)$$

where $a^2 = \frac{L^2}{3}, b^2 = \frac{4}{\ell^2}$ ...(24)

Equation (23) leads to

$$\left(\frac{d\mu}{dt}\right)^2 = \frac{a^2 + b^2\mu^2 + M\mu^3}{\mu} \qquad \ldots(25)$$

The value of μ in terms of cosmic time t is not solvable, therefore, after suitable transformation of coordinates, the metric (1) leads to

$$ds^2 = -\frac{T\, dT^2}{a^2 + b^2T^2 + MT^3} + T\, dX^2 + T\, \nu e^{2x} dY^2 + T\, \nu^{-1} e^{2x} dz^2 \ldots(26)$$

where $\log \nu = \int \frac{L\, dT}{T\sqrt{a^2 + b^2T^2 + MT^3}}$ ...(27)

and $\mu = T, \ell x = X, y = Y, z = Z.$

4. Some Physical and Geometrical Aspects

The energy density ($\in$), the string tension density (λ), the expansion (θ), the shear (σ), the spatial volume (R^3), the deceleration parameter (q) for the model (26) are given by

$$8\pi\in=\frac{3}{4T}(a^2+b^2T^2+MT^3)-\frac{L^2}{4T^3}-\frac{3}{\ell^2T}=\lambda \qquad ...(28)$$

$$\theta=\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}=\frac{3}{2T}\sqrt{\frac{a^2}{T}+b^2T+MT^2} \qquad ...(29)$$

$$\sigma=\frac{L}{2}\left(\frac{B_4}{B}-\frac{C_4}{C}\right)=\frac{L}{2T^{3/2}} \qquad ...(30)$$

$$R^3=ABC=\sqrt{\ell}\,T^{3/2} \qquad ...(31)$$

$$q=-\frac{\ddot{R}/R}{\dot{R}^2/R^2}=2-\frac{2b^2T^2+3T^3}{a^2+b^2T^2+MT^3} \qquad ...(32)$$

Particle Horizon. Since

$$\int_{T_0}^{T}\frac{dT}{R^3(T)}=\int_{T_0}^{T}\frac{dT}{\sqrt{\ell}\,T^{3/2}}=\text{finite}$$

This is convergent integral, so the particle horizon exists.

5. Discussion and Conclusion

The reality condition $\rho > 0$ is satisfied when

$$3\ell^2a^2T^3+3\ell^2b^2T^5+3M\ell^2T^6-12T^2>L^2\ell^2$$

The model (26) starts with a big bang at T = 0 and the expansion decreases with time. The model in general represents anisotropic space-time. However, for large values of T, it isotropizes. The deceleration parameter q > 0, hence the model decelerates. The model allows particle horizon. The model also represents Point Type singularity at t = 0 (MacCallum [12]). For large values of T, string tension (λ) vanishes. Thus anisotropy is maintained due to the presence of string. As soon as string disappears, anisotropy also disappears.

Acknowledgement

The author is extremely thankful to Professor Raj Bali, CSIR Emeritus Scientist, Department of Mathematics, University of Rajasthan, Jaipur for his kind suggestion and helpful discussion.

References

[1] Bali, R. and Dave, S. (2001). Bianchi Type IX string cosmological model in General Relativity, *Pramana – J. Phys.*, **56**, 513-517.

[2] Bali, R. and Upadhaya, R.D. (2003). An LRS Bianchi Type I Bulk viscous fluid string cosmological model in General Relativity, *Astrophys. and Space-Science,* **288**, 287-292.

[3] Bali, R. and Singh, D.K. (2005). Bianchi Type V bulk viscous fluid string dust cosmological model in General Relativity, *Astrophys. and Space-Science*, **300**, 387-394.

[4] Bali, R. and Anjali (2006). Bianchi Type I magnetized string cosmological model in General Relativity, *Astrophys. and Space-Science*, **302**, 201-205.

[5] Bali, R., Pareek, U.K. and Pradhan, A. (2007). Bianchi Type I massive string magnetized barotropic perfect fluid cosmological model in General Relativity, *Chin. Phys. Lett.* , **24**, 2455-2458.

[6] Bali, R. and Jain, S. (2007). Bianchi Type V magnetized string dust cosmological model in General Relativity, *Int. J. Mod. Phys,.* **D16**, 11-17.

[7] Bali, R., Chandnani, N.K. and Gupta, L.K. (2010). Bianchi Type I string dust magnetized cosmological models in Lyra geometry, *Comm. Theor. Phys,.* **54**, 197-202.

[8] Banerjee, A., Sanyal, A.K. and Chakraborty, S.(1990). String cosmology in Bianchi I space-time, *Pramana – J. Phys.* **34**, 1-11.

[9] Everett, A.E. (1988). Cosmic strings in unified gauge theories, *Phys. Rev.* **D24**, 858-868.

[10] Kibble, T.W.R. (1976). Topology of cosmic domains and strings. *J. Phys.* **A9**, 1387-1398.

[11] Letelier, P.S. (1983). String Cosmology, *Phys. Rev*,. **D 28**, 2414-2419.

[12] MacCallum, M.A.H. (1971). Homogeneous cosmologies – III, *Comm. Math. Phys.* **20**, 57-70.

[13] Nayak, B.K. and Sahoo, B.K. (1989). Bianchi Type V models with matter distribution admitting anisotropic pressure and heat flow, *Gen. Relativ. Gravit.* **21**, 211-225.

[14] Ram, S., Zeyauddin, M. and Singh, C.P. (2008). Bianchi Type V cosmological models with perfect fluid and heat conduction in Lyra geometry, *Int. J. Mod. Phys.* , **A 23**, 4991-5005.

[15] Reddy, D.R.K. and Rao, M.V.S. (2006). Axially symmetric string cosmological model in Brans-Dicke Theory of Gravitation, *Astrophys and Space-Science*, **305**, 183-186.

[16] Roy, S.R. and Banjerjee, S.K. (1995). Bianchi type II string cosmological models in general relativity, *Class. Quant. Gravity*, **12**, 1943-1948.

[17] Stachel, J. (1980). Thickening the string I. *The string perfect dust, Phys. Rev.* **D 21**, 2171-2181.

[18] Tikekar, R. and Patel, L.K. (1992). Some exact solutions of string cosmology in Bianchi III space-time, *Gen. Relativ. Gravit.* 24, 397-404.

[19] Turok, N. (1989). Global texture as the origin of cosmic structure, *Phys. Rev. Lett.* 63, 2625-2628.

[20] Vilenkin, A. (1981). Cosmic strings. *Phys. Rev.* **D 24**, 2082-2089.

[21] Wang, X.X. (2006). Bianchi Type III string cosmological model with bulk viscosity and magnetic field, *Chin. Phys. Lett.* **23**, 1702-1705.