Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol. 12, No. 3, Sep. 2013, pp 317-336

ON 2- BANACH SPACE VALUED PARANORMED SEQUENCE SPACE $C_0 (X, M , ||. , .||, \bar{\lambda} , \bar{p})$ DEFINED BY ORLICZ FUNCTION

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Received : Aug. 30, 2013

Abstract : The aim of this paper is to introduce and study a new class $c_0 (X, M , ||. , .||, \bar{\lambda} , \bar{p})$ of 2-Banach space valued sequences using Orlicz function as a generalization of sequence space $c_0 (X , M, \bar{\lambda} , \bar{p})$ studied in [20], which is the generalization of the familiar sequence space c_0. Besides the investigation of conditions pertaining to the containment relation of the class $c_0 (X, M , ||. , . ||, \bar{\lambda} , \bar{p})$ in terms of different $\bar{\lambda}$ and $\bar{p}$, our primarily interest is to explore the linear topological structures of the class $c_0 (X, M , ||. , .||, \bar{\lambda} , \bar{p})$ when topologized it with suitable natural paranorm.

Keywords : 2-normed space, paranormed space, Orlicz sequence space, normal space.

2010 Mathematics Subject Classification: Primary-46A45; Secondary-46B 15

1. Introduction and Preliminaries

The notion of 2–nor $\bar{\lambda}$, $\bar{p}$ med space was initially introduced by S. GÄahler [4] as an interesting linear generalization of a normed linear space, which was subsequently studied in [3], [19] and many others. Recently a lot of activities have been started by many researchers to study this concept in different directions, for instances,(see, [1], [6], [14] ,[18]).

Definition 1.1: Let X be a vector space of dimension greater than 1 over K, the field of real or complex numbers. A 2 - *norm* on X is a real valued function $\|.\,,.\|$ on $X \times X$ satisfying the following conditions:

(i) $\| x, y \| \geq 0$ and $\| x, y \| = 0$ if and only if x and y are linearly dependent;

(ii) $\| x, y \| = \| y, x \|$, for all $x, y \in X$;

(iii) $\| \alpha x, y \| = |\alpha| \, \|x, y \|$, where $\alpha \in K$ and $x, y \in X$;

(iv) $\| x_1 + x_2, y \| \leq \| x_1, y \| + \| x_2, y \|$ for all x_1, x_2 and $y \in X$.

The pair $(X, \|.\,,.\|)$ is called a 2–normed space.Recall that $(X, \|.\,,.\|)$ is a 2-Banach space if every Cauchy sequence in X is convergent to some x_0 in X.

Geometrically, a 2-norm function generalizes the concept of area function of parallelogram spanned by the two associated vectors, see [6].

Definition 1.2: A *paranormed space* (X, G) is a linear space X with zero element θ together with a function $G : X \rightarrow \mathbf{R}_+$ (called a paranorm on X) which satisfies the following axioms:

$PN1$: $G(\theta) = 0$;

$PN2$: $G(x) = G(-x)$ for all $x \in X$;

$PN3$: $G(x + y) \leq G(x) + G(y)$ for all $x, y \in X$; and

$PN4$: Scalar multiplication is continuous.

Note that the continuity of scalar multiplication is equivalent to (i) if $G(x_n) \rightarrow 0$ and $\alpha_n \rightarrow \alpha$ as $n \rightarrow \infty$, then $G(\alpha_n x_n) \rightarrow 0$ as $n \rightarrow \infty$ and (ii) if $\alpha_n \rightarrow 0$ as $n \rightarrow \infty$ and x be any element in X, then $G(\alpha_n x) \rightarrow 0$, (see, Wilansky [20]). A paranorm is called total if $G(x) = 0$ implies $x = \theta$.

The concept of paranorm is closely related to linear metric space, (see, Wilansky [20]) and its studies on sequence spaces were initiated by Maddox [11] and many others.In particular, various types of paranormed sequence spaces were further investigated by several workers,(see [2], [7], [12]).

Definition 1.3: A function $M : [0, \infty) \to [0, \infty)$ is called an *Orlicz function*, if it is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. An Orlicz function M can be represented in the following integral form

$$M(x) = \int_0^x q(t)\, dt$$

where q, known as the kernel of M, is right-differentiable for $t \geq 0$, $q(0) = 0$, $q(t) > 0$ for $t > 0$, q is non decreasing, and $q(t) \to \infty$ as $t \to \infty$, (see, Krasnosel'skiî and Rutickiî [9]).

Definition 1.4: An Orlicz function M is said to satisfy Δ_2 - condition for all values of t, if there exists a constant $K > 0$ such that

$$M(2t) \leq KM(t), \text{ for all } t \geq 0.$$

The Δ_2-condition is equivalent to the satisfaction of inequality $M(Lt) \leq KL\, M(t)$ for all values of t for which $L > 1$, (see, Krasnosel'skiî and Rutickiî [9]).

Definition 1.5: Let X be a normed space over C, the field of complex numbers. Let $\omega(X)$ denotes the linear space of all sequences $\bar{x} = (\xi_k)$ with $\xi_k \in X$, $k \geq 1$ with usual coordinate wise operations i.e.,

$$\bar{x} + \bar{y} = (\xi_k + \zeta_k) \text{ and } \alpha \bar{x} = (\alpha\xi_k), \text{ for each } \bar{x}, \bar{y} \in \omega(X) \text{ and } \alpha \in C.$$

We shall denote $\omega(C)$ by ω. Further, $\bar{\lambda} = (\lambda_k) \in \omega$ and $\bar{x} \in \omega(X)$ we shall write $\bar{\lambda}\bar{x} = (\lambda_k \xi_k)$. Further by a vector valued sequence space we mean a linear subspace of $\omega(X)$.

Definition 1.6: Lindenstrauss and Tzafriri [10] used the idea of Orlicz function to construct the sequence space ℓ_M of scalars (ξ_k) such that

$$\ell_M = \left\{ \bar{x} = (\xi_k) \in \omega : \sum_{k=1}^{\infty} . M\left(\frac{|\xi_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}.$$

The space ℓ_M with the norm

$$\| \bar{x} \|_M = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} . M\left(\frac{|\xi_k|}{\rho} \right) \leq 1 \right\}$$

becomes a Banach space which is called an *Orlicz sequence space*. The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with

$$M(x) = x^p : 1 \leq p < \infty.$$

Subsequently, various algebraic and topological properties of sequence spaces using Orlicz function have been studied in [2], [5], [7], [8], [12], [13], [15], [16], [17], [18], as the generalization of various well known sequence spaces.

Definition 1.7: A sequence space S is said to be *normal* if $\bar{x} = (\xi_k) \in S$ and $\bar{\alpha} = (\alpha_k)$ a sequence of scalars with $|\alpha_k| \leq 1$, for all $k \geq 1$, then $\bar{\alpha}\bar{x} = (\alpha_k \xi_k) \in S$.

2. The Class $c_0 (X, M, \|. , .\|, \bar{\lambda}, \bar{p})$

Let $\bar{p} = (p_k)$ and $\bar{q} = (q_k)$ be any sequences of strictly positive real numbers and $\bar{\lambda} = (\lambda_k)$ and $\bar{\mu} = (\mu_k)$ be sequences of non zero complex numbers. Let $(X, \|.,.\|)$ be the 2- Banach space over the field C of complex numbers and θ denotes the zero element of X. We now introduce the following class of 2-Banach space X–valued sequences using Orlicz function M.

$c_0 (X, M, \|. , .\|, \bar{\lambda}, \bar{p}) = \{\bar{x} = (x_k) \in \omega(X)$: there exists $\rho > 0$ satisfying

$$M\left(\frac{1}{\rho} \| \lambda_k x_k, z \|^{p_k} \right) \to 0 \text{ as } k \to \infty, \text{ for each } z \in X \}. \qquad \ldots(1)$$

Further, when $\lambda_k = 1$ for all k, then $c_0 (X, M, \|. , .\|, \bar{\lambda}, \bar{p})$ will be denoted by $c_0 (X, M, \|. , .\|, \bar{p})$ and when $p_k = 1$ for all k, then $c_0 (X, M, \|. , .\|, \bar{\lambda}, \bar{p})$ will be denoted by $c_0\,(X, M, ||.\,,.||,\ \bar{\lambda})$ .If $p_k = \lambda_k = 1$ for all k , then the class $c_0\,(X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p})$ will be denoted by $c_0\,(X, M, ||.\,,.||)$. Further, when $X = \mathbf{C}$ we simply write $c_0\,(X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p})$ as $c_0\,(M, ||.\,,.||,\ \bar{\lambda}, \bar{p})$.

If in the definition of $c_0\,(X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p})$, the phrase 'for some $\rho > 0$' is replaced by 'for every $\rho > 0$' then we denote this subclass by $\bar{c}_0\,(X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p})$. Thus

$$\bar{c}_0\ (X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p}) = \{\bar{x} = (x_k) \in \omega(X) : M\left(\frac{1}{\rho}||\lambda_k x_k\,, z||^{p_k}\right) \to 0 \text{ as } k \to \infty,$$

$$\text{for every } \rho > 0 \text{ and for each } z \in X\}; \qquad \ldots(2)$$

3. Containment Relations

In this section, we investigate some inclusion relations between the the classes $c_0(X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p})$ arising in terms of different $\bar{p}$ and $\bar{\lambda}$. Throughout, we shall denote $t_k = \left|\frac{\lambda_k}{\mu_k}\right|^{p_k}$, $\sup p_k = L$ for all $k \geq 1$ and for scalar α, $A\,[\alpha] = \max\,(1, |\alpha|)$.

But when the sequences p_k and q_k occur, then to distinguish L we use the notations $L(p)$ and $L(q)$ respectively.

Lemma 3.1: $c_0\,(X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p}) \subset c_0\ (X, M, ||.\,,.||,\ \bar{\mu}, \bar{p})$ *if and only if*

$$\lim\inf_k t_k > 0.$$

Proof: For the sufficiency, assume that $\lim\inf_k t_k > 0$.Then there exists $m > 0$ such that $m\,|\mu_k|^{p_k} < |\lambda_k|^{p_k}$ for all sufficiently large values of k. Let $\bar{x} = (x_k) \in c_0\,(X, M, ||.\,,.||,\ \bar{\lambda}, \bar{p})$. Then for some $\rho > 0$,

$$M\left(\frac{1}{\rho}||\ \lambda_k x_k,\ z||^{p_k}\right) \to 0 \text{ as } k \to \infty\,, \text{ for each } z \in X.$$

Now we choose $\rho_1 > 0$ such that $m\rho_1 \geq \rho$. Since M is non–decreasing, we have

$$M\left(\frac{1}{\rho_1}\|\mu_k x_k, z\|^{p_k}\right) \le M\left(\frac{|\lambda_k|^{p_k}}{m\rho_1}\|x_k, z\|^{p_k}\right)$$

$$\le M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ as } k \to \infty \text{, for each } z \in X.$$

This clearly implies that $\bar{x} \in c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p})$ and hence

$$c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p}).$$

For the necessity, assume that $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p})$ but $\liminf_k t_k = 0$. Then we can find a sequence $(k(n))$ of integers such that $1 \le k(n) < k(n+1)$, $n \ge 1$, for which

$$n^2 |\lambda_{k(n)}|^{p_{k(n)}} < |\mu_{k(n)}|^{p_{k(n)}}, \text{ for all } n \ge 1. \qquad \text{...(3)}$$

Now, corresponding to $y \in X$ and $y \neq \theta$, we define the sequence $\bar{x} = (x_k)$ by

$$x_k = \begin{cases} \lambda_{k(n)}^{-1}\, n^{-2/p_{k(n)}}\, y, \text{ if } k = k(n),\ n \ge 1 \text{ and} \\ \theta, \text{ otherwise.} \end{cases} \qquad \text{...(4)}$$

Let $\rho > 0$. Then for $k = k(n)$, $n \ge 1$, using convexity of M, we have

$$M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) = M\left(\frac{1}{\rho}\|n^{-2/p_{k(n)}} y, z\|^{p_{k(n)}}\right)$$

$$\le \frac{1}{n^2} M\left(\frac{1}{\rho}\|y, z\|^{p_{k(n)}}\right)$$

$$\le \frac{1}{n^2} M\left(\frac{A\,[\,\|y, z\|^{L(p)}\,]}{\rho}\right) \to 0 \text{ as } n \to \infty,$$

where $\|y, z\|^{p_{k(n)}} \le A\,[\,\|y, z\|^{L(p)}]$ for each $n \ge 1$ is used and $M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) = 0$, for $k \neq k(n)$, $n \ge 1$.

Which shows that $\bar{x} \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$. On the other hand, let us choose $z \in X$ such that $\|y, z\| = 1$. Then for any $\rho > 0$ and $k = k(n)$, $n \ge 1$, and in view of (3) and (4), we have

$$M\left(\frac{1}{\rho}\|\mu_k x_k, z\|^{p_k}\right) = M\left(\frac{1}{\rho}\left|\frac{\mu_{k(n)}}{\lambda_{k(n)}}\right|^{p_{k(n)}}\frac{1}{n^2}\|y,z\|^{p_{k(n)}}\right) \geq M\left(\frac{1}{\rho}\right),$$

showing that $\bar{x} \notin c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p})$, a contradiction. This completes the proof.

Lemma 3.2: $c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$

if and only if $\lim \sup_k t_k < \infty$.

Proof: For the sufficiency, assume that lim $\sup_k t_k < \infty$. Then we can find a positive number d such that $d\,|\mu_k|^{p_k} > |\lambda_k|^{p_k}$ for all sufficiently large values of k. Then analogous to the Lemma 3.1, the result follows.

For the necessity, suppose that $c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ but lim $\sup_k t_k = \infty$. Then there exists a sequence $(k(n))$ of positive integers with

$1 \leq k(n) < k(n+1)$, $n \geq 1$ satisfying

$$|\lambda_{k(n)}|^{p_{k(n)}} > n^2\,|\mu_{k(n)}|^{p_{k(n)}} \text{ ,for each } n \geq 1. \qquad ...(5)$$

Now, as written in Lemma 3.1,corresponding to $y \in X$ and $y \neq \theta$, we define a sequence $\bar{x} = (x_k)$ by

$$x_k = \begin{cases} \mu_{k(n)}^{-1}\, n^{-2/p_{k(n)}} y, & \text{if } k = k(n),\ n \geq 1 \text{ and} \\ \theta, & \text{otherwise.} \end{cases} \qquad ...(6)$$

Then we can show that $\bar{x} \in c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p})$,but $\bar{x} \notin c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$,a contradiction. The proof is now complete.

On combining the Lemmas 3.1 and 3.2, we get:

Theorem 3.3: $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) = c_0(X, M, \|.,.\|, \bar{\mu}, \bar{p})$

if and only if $0 < \lim \inf_k t_k \leq \lim \sup_k t_k < \infty$.

Corollary 3.4:

(i) $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{p})$ *if and only if* $\lim \inf_k |\lambda_k|^{p_k} > 0$;

(ii) $c_0(X, M, \|.,.\|, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ *if and only if*

$$\limsup_k |\lambda_k|^{p_k} < \infty; \text{and}$$

(iii) $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) = c_0(X, M, \|.,.\|, \bar{p})$ *if and only if*

$$0 < \liminf_k |\lambda_k|^{p_k} \le \limsup_k |\lambda_k|^{p_k} < \infty.$$

Proof: By taking $\mu_k = 1$ for all k, in Lemmas 3.1, 3.2 and in Theorem 3.3,the assertions (i),(ii) and (iii) follow.

Lemma 3.5: $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{q})$

if and only if $\liminf_k (q_k/p_k) > 0$.

Proof: For the sufficiency of the condition, suppose that $\liminf_k (q_k/p_k) > 0$. Then there exists a $m > 0$ such that $q_k > m\, p_k$ for all sufficiently large values of k.

Let $\bar{x} = (x_k) \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$. Then for some $\rho > 0$, and for each $z \in X$

$$M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ as } k \to \infty.$$

Hence for a given $\varepsilon > 0$, if we choose $0 < \eta < 1$ satisfying $\eta^m (M/\rho) < \varepsilon$, then we have

$$M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \le M\left(\frac{\eta}{\rho}\right),$$

and since M is non decreasing, therefore

$$\|\lambda_k x_k, z\|^{p_k} < \eta < 1$$

for each $z \in X$ and for all sufficiently large values of k.Thus using the convexity of M, we have

$$M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{q_k}\right) \le M\left(\frac{1}{\rho}[\|\lambda_k x_k, z\|^{p_k}]^m\right)$$

$$\le M\left(\frac{\eta^m}{\rho}\right) \le \eta^m M\left(\frac{1}{\rho}\right) < \varepsilon,$$

for each $z \in X$ and for all sufficiently large values of k and consequently

$\bar{x} \in c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{q})$. Hence

$$c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p}) \subset c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{q}).$$

For the necessity, suppose that the inclusion holds but $\liminf_k\,(q_k/p_k) = 0$. Then there exists a sequence $(k(n))$ of positive integers such that $1 \leq k(n) < k(n+1)$, $n \geq 1$, for which

$$n\, q_{k(n)} < p_{k(n)}\ , \text{ for each } n \geq 1. \qquad \ldots(7)$$

Let $y \in X$ and $y \neq \theta$. We define a sequence $\bar{x} = (x_k)$ by

$$x_k = \begin{cases} \lambda_{k(n)}^{-1}\, n^{-1/p_{k(n)}}\, y, & \text{for } k = k(n)\ ,\ n \geq 1 \text{ and} \\ \theta, & \text{otherwise.} \end{cases} \qquad \ldots(8)$$

Let $\rho > 0$. Then for each $z \in X$, $k = k(n)$, $n \geq 1$, we have

$$\begin{aligned} M\left(\frac{1}{\rho}\|\lambda_k x_k\,, z\|^{p_k}\right) &= M\left(\frac{1}{\rho}\|n^{-1/p_{k(n)}} y, z\|^{p_{k(n)}}\right) \\ &= M\left(\frac{1}{n\rho}\|y, z\|^{p_{k(n)}}\right) \\ &\leq \frac{1}{n} M\left(\frac{A\,[\,\|y, z\,\|^{L(p)}\,]}{\rho}\right) \to 0 \text{ as } n \to \infty \end{aligned}$$

and $M\left(\frac{1}{\rho}\|\lambda_k x_k\,, z\|^{p_k}\right) = 0$, for $k \neq k(n)$, $n \geq 1$.

This shows that $\bar{x} \in c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$. But on the other hand, let us choose $z \in X$ such that $\|\,y, z\,\| = 1$. Then for $k = k(n)$, $n \geq 1$, in view of (7) and (8) ,we have

$$\begin{aligned} M\left(\frac{1}{\rho}\|\lambda_k x_k\,, z\|^{q_k}\right) &= M\left(\frac{1}{\rho}\|\,n^{-1/p_{k(n)}} y, z\|^{q_{k(n)}}\right) \\ &\geq M\left(\frac{1}{\rho n^{1/n}}\|y, z\|^{q_{k(n)}}\right) \geq M\left(\frac{1}{\rho\sqrt{e}}\right). \end{aligned}$$

This shows that $\bar{x} \notin c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{q})$, which contradicts our assumption. This completes the proof of the theorem.

Lemma 3.6: $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{q}) \subset c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$

if and only if $\limsup_k (q_k/p_k) < \infty$.

Proof: For the sufficiency, assume that $\limsup_k (q_k/p_k) < \infty$. Hence there exists $L > 0$ such that $q_k < L\, p_k$ for all sufficiently large values of k. Then analogous to the Lemma 3.5 we can easily show that $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{q}) \subset c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$.

For the necessity, suppose that the inclusion holds but $\limsup_k (q_k/p_k) = \infty$. Then there exists a sequence $(k(n))$ of positive integers such that $1 \le k(n) < k(n+1)$, $n \ge 1$, for which

$$q_{k(n)} > n\, p_{k(n)}, \text{for all } n \ge 1. \qquad \text{...(9)}$$

Corresponding to $y \in X$ and $y \neq \theta$, we define a sequence $\bar{x} = (x_k)$ by

$$x_k = \begin{cases} \lambda_{k(n)}^{-1}\, n^{-1/q_{k(n)}} y, & \text{for } k = k(n),\ n \ge 1 \text{ and} \\ \theta, & \text{otherwise.} \end{cases} \qquad \text{...(10)}$$

Then analogous to the Lemma 3.5 we can immediately $\bar{x} \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{q})$ and $\bar{x} \notin c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$, a contradiction. This completes the proof.

On combining the Lemmas 3.5 and 3.6, one obtain

Theorem 3.7 : $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) = c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{q})$ *if and only if*

$$0 < \liminf_k (q_k/p_k) \le \limsup_k (q_k/p_k) < \infty.$$

Corollary 3.8:

(i) $c_0(X, M, \|.,.\|, \bar{\lambda}) \subset c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ *if and only if* $\liminf_k p_k > 0$;

(ii) $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\lambda})$ *if and only if*

$\limsup_k p_k < \infty$; *and*

(iii) $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) = c_0(X, M, \|.,.\|, \bar{\lambda})$ *if and only if*

$$0 < \lim\inf_k p_k \leq \lim\sup_k p_k < \infty.$$

Proof: Proof follows by taking $p_k = 1$ for all k and replacing $\bar{q}$ by $\bar{p}$ in Lemmas 3.5 and 3.6 and in Theorem 3.7.

Theorem 3.9: $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) \subset c_0(X, M, \|.,.\|, \bar{\mu}, \bar{q})$ *if and only if*

(i) $\lim\inf_k t_k > 0$; *and* (ii) $\lim\inf_k (q_k/p_k) > 0$.

Proof: Proof of the theorem follows immediately from the Lemmas 3.1 and 3.5.

In the following example, $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ may strictly be contained in $c_0(X, M, \|.,.\|, \bar{\mu}, \bar{q})$ inspite of the satisfaction of the conditions (i) and (ii) of Theorem 3.9 .

Example 3.10:

Let $(X, \|.,.\|)$ be a 2- normed space and consider a sequence $\bar{x} = (x_k)$ defined by $x_k = k^{-k} y$, if $k = 1, 2, 3, \ldots$,where $y \in X$ and $y \neq 0$.Further, let $p_k = k^{-1}$, if k is odd integer, $p_k = k^{-2}$, if k is even integer, $q_k = k^{-1}$ for all values of k, $\lambda_k = 3^k$, $\mu_k = 2^k$ for all values of k. Then

$$t_k = \left|\frac{\lambda_k}{\mu_k}\right|^{p_k} = \frac{3}{2} \text{ or } \left(\frac{3}{2}\right)^{1/k} \text{according as } k \text{ is odd or even integer}$$

and hence $\lim\inf_k t_k > 0$. Further, $(q_k/p_k) = 1$, if k is odd integer, $(q_k/p_k) = k$, if k is even integer. Therefore $\lim\inf_k (q_k/p_k) > 0$. Hence the conditions (i) and (ii) of Theorem 3.9 are satisfied.

Let $\rho > 0$. Then for each $z \in X$, we have

$$M\left(\frac{1}{\rho}\|\mu_k x_k , z\|^{q_k}\right) = M\left(\frac{1}{\rho}\|2^k k^{-k} y , z\|^{1/k}\right) \leq \frac{1}{k} M\left(\frac{2}{\rho}\|y , z\|^{1/k}\right)$$

$$\leq \frac{1}{k} M\left(\frac{2A\,[\,\|y, z\|\,]}{\rho}\right) \to 0 \text{ as } k \to \infty,$$

showing that $\bar{x} \in c_0(X, M, \|.,.\|, \bar{\mu}, \bar{q})$. But on the other hand, let us choose $z \in X$ such that $\|y, z\| = 1$. Then for even integer k,

$$M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) = M\left(\frac{1}{\rho}\|3^k k^{-k} y, z\|^{1/k^2}\right)$$

$$= M\left(\frac{(3/k)^{1/k}}{\rho}\|y, z\|^{1/k^2}\right) > M\left(\frac{1}{2\rho}\right).$$

This implies that $\bar{x} \notin c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$. Thus the containment of $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ in $c_0(X, M, \|.,.\|, \bar{\mu}, \bar{q})$ is strict inspite of the satisfaction of the conditions (i) and (ii) of the Theorem 3.9.

4. Linear Topological Structure of $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$

In this section, we shall investigate some results that characterize the linear topological structure of the class $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ by endowing it with suitable natural paranorm.

We shall use frequently

$$|a + b|^{p_k} \le D\{|a|^{p_k} + |b|^{p_k}\},$$

where $a, b \in \mathbf{C}$, $0 < p_k \le \sup_k p_k = L$ and $D = \max(1, 2^{L-1})$.

Theorem 4.1 : $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ *forms a linear space over* C.

Proof: Let $\bar{x}, \bar{y} \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ and $\alpha, \beta \in C$. Then there exist $\rho_1 > 0$ and $\rho_2 > 0$ such that for each $z \in X$,

$$M\left(\frac{1}{\rho_1}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ and } M\left(\frac{1}{\rho_2}\|\lambda_k y_k, z\|^{p_k}\right) \to 0, \text{ as } k \to \infty.$$

We now choose $\rho > 0$ such that

$2D\rho_1 A[|\alpha|^L] \le \rho$ and $2D\rho_2 A[|\beta|^L] \le \rho$, where $D = A[2^{L-1}]$.

For such ρ, using non decreasing and convex properties of M, we have

$$M\left(\frac{1}{\rho}\|\lambda_k(\alpha x_k+\beta y_k), z\|^{pk}\right) \le M\left[\frac{1}{\rho}\left(D\|\alpha\lambda_k x_k, z\|^{pk}+D\|\beta\lambda_k y_k, z\|^{pk}\right)\right]$$

$$= M\left[\frac{D}{\rho}|\alpha|^{pk}\|\lambda_k x_k, z\|^{pk}+\frac{D}{\rho}|\beta|^{pk}\|\lambda_k y_k, z\|^{pk}\right]$$

$$= M\left[\frac{D}{\rho}A\,[\,|\alpha|^{L}]\,\|\lambda_k x_k, z\|^{pk}+\frac{D}{\rho}A\,[\,|\beta|^{L}]\,\|\lambda_k y_k, z\|^{pk}\right]$$

$$\le M\left[\frac{1}{2\rho_1}\|\lambda_k x_k, z\|^{pk}+\frac{1}{2\rho_2}\|\lambda_k y_k, z\|^{pk}\right]$$

$$\le \frac{1}{2}M\left(\frac{1}{\rho_1}\|\lambda_k x_k, z\|^{pk}\right)+\frac{1}{2}M\left(\frac{1}{\rho_2}\|\lambda_k y_k, z\|^{pk}\right)$$

which tends to 0 as $k\to\infty$, for each $z\in X$ and hence $\alpha\bar{x}+\beta\bar{y}\in c_0\,(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$. This implies that $c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$ forms a linear space over C.

Theorem 4.2 : *If* $inf_k\, p_k = l > 0$, *then* $c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$ *forms a paranormed space with respect to*

$$G(\bar{x}) = inf\left\{\rho>0 : \sup_k M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k/L}\right)\le 1, for\ each\ z\in X\right\}. \qquad \ldots(11)$$

Proof: Obviously $G(\bar{\theta}) = 0$ and $G(-\bar{x}) = G(\bar{x})$ easily follow, so PN_1 and PN_2 are obvious.

To proceed the further proof, for $\bar{x}\in c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$, let us denote

$$Q\,(\bar{x}) = \left\{\rho>0 : \sup_k M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k/L}\right)\le 1, \text{ for each } z\in X\right\}. \qquad \ldots(12)$$

Now for $\bar{x}, \bar{y}\in c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$,consider $\rho_1\in Q\,(\bar{x})$ and $\rho_2\in Q(\bar{y})$. Then clearly by the convexity of M we have

$$\sup_k M\left(\frac{1}{\rho_1+\rho_2}\|\lambda_k(x_k+y_k), z\|^{p_k/L}\right)$$

$$\le \sup_k M\left[\frac{1}{\rho_1}\|\lambda_k x_k, z\|^{p_k/L}\times\frac{\rho_1}{\rho_1+\rho_2}+\frac{1}{\rho_2}\|\lambda_k y_k, z\|^{p_k/L}\times\frac{\rho_2}{\rho_1+\rho_2}\right]$$

$$\leq \frac{\rho_1}{\rho_1+\rho_2} \sup_k M\left(\frac{1}{\rho_1}\|\lambda_k x_k , z\|^{p_k/L}\right) + \frac{\rho_2}{\rho_1+\rho_2} \sup_k M\left(\frac{1}{\rho_2}\|\lambda_k x_k , z\|^{p_k/L}\right)$$

$\leq$ 1, for each $z \in X$.

This shows that $\rho_1 + \rho_2 \in Q\,(\bar{x} + \bar{y})$. Thus $G(\bar{x} + \bar{y}) \leq \rho_1 + \rho_2$ for each $\rho_1 \in Q\,(\bar{x})$ and $\rho_2 \in Q\,(\bar{y})$ implies that

$$G(\bar{x} + \bar{y}) \leq G(\bar{x}) + G(\bar{y}) \text{ i.e., } PN_3 \text{ holds.}$$

Finally we show PN_4 i.e., the continuity of scalar multiplication.

(i) Let $\bar{x}^{(n)} = (x_k^{(n)})$ be a sequence in $c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$ such that $G(\bar{x}^{(n)}) \to 0$ as $n \to \infty$ and (α_n) a sequence of scalars such that $\alpha_n \to \alpha$. Then we have

$$G\,(\alpha_n \bar{x}^{(n)}) = \inf\left\{\rho : \sup_k M\left(\frac{1}{\rho}\|\lambda_k \alpha_n\ x_k^{(n)}, z\|^{p_k/L}\right) \leq 1 \text{ , for each } z \in X\right\}$$

$$= \inf\left\{\rho : \sup_k M\left(\frac{|\alpha_n|^{p_k/L}}{\rho}\|\lambda_k x_k^{(n)} , z\|^{p_k/L}\right) \leq 1\text{, for each } z \in X\right\}$$

$$\leq \inf\left\{\rho : \sup_k M\left(\frac{s^{p_k/L}}{\rho}\|\lambda_k x_k^{(n)} , z\|^{p_k/L}\right) \leq 1\text{, for each } z \in X\right\}$$

where $s = \sup_n|\alpha_n|$. Clearly $s^{p_k/L} \leq A[s]$ for each $k \geq 1$ and using $\rho = r\,A[s]$ implies that

$$G(\alpha_n \bar{x}^{(n)}) \leq \inf\left\{\rho : \sup_k M\left(\frac{A[s]}{\rho}\|\lambda_k x_k^{(n)} , z\|^{p_k/L}\right) \leq 1 \text{ , for each } z \in X\right\}$$

$$= \inf\left\{ r\,A[s] : \sup_k M\left(\frac{1}{r}\|\lambda_k x_k^{(n)} , z\|^{p_k/L}\right) \leq 1\text{, for each } z \in X\right\}$$

$$= A[s]\ G\,(\bar{x}^{(n)})$$

implies that $G\,(\alpha_n \bar{x}^{(n)}) \to 0$, as $G\,(\bar{x}^{(n)}) \to 0$ as $n \to \infty$.

(ii) Let $\alpha_n \to 0$ as $n \to \infty$ and $\bar{x}$ be any element in $c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$. We show that $G\,(\alpha_n\,\bar{x}) \to 0$. Now for $0 < \varepsilon < 1$, we can find a positive integer N such that $|\alpha_n| \le \varepsilon$ for all $n \ge N$. In view of $\inf_k p_k = l > 0$, we get

$$|\alpha_n|^{p_k/L} \le \varepsilon^{l/L}.$$

Thus for each $n \ge N$, we have

$$M\left(\frac{1}{\rho}\|\,\alpha_n\,\lambda_k x_k, z\|^{p_k/L}\right) \le M\left(\frac{|\alpha_n|^{p_k/L}}{\rho}\|\,\lambda_k\,x_k, z\|^{p_k/L}\right)$$
$$\le M\left(\frac{\varepsilon^{l/L}}{\rho}\|\,\lambda_k\,x_k,\,z\|^{p_k/L}\right),$$

for each $z \in X$ and if $\rho \in Q\,(\varepsilon^{l/L}\,\bar{x})$, then $\rho \in Q\,(\alpha_n\,\bar{x})$ and consequently

$$Q\,(\varepsilon^{l/L}\,\bar{x}) \subseteq Q\,(\alpha_n\,\bar{x}).$$

Now taking infimum over such ρ's,we get

$$\inf\{\,\rho : \rho \in Q\,(\alpha_n\,\bar{x})\,\} \le \inf\{\,\rho : \rho \in Q\,(\varepsilon^{l/L}\,\bar{x})\,\} = \varepsilon^{l/L} \inf\{\,\rho : \rho \in Q\,(\bar{x})\,\}$$

which shows that $G\,(\alpha_n\,\bar{x}) \le \varepsilon^{l/L}\,G\,(\bar{x})$ for all $n \ge N$, i.e., $G(\alpha_n\,\bar{x}) \to 0$ as $n \to \infty$. Hence $c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$ forms a paranormed space. The proof is now complete.

Theorem 4.3 : *Paranormed space* $(c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p}), G)$ *is complete.*

Proof: Let $(\bar{x}^{(i)})$ be a Cauchy sequence in $c_0\,(X, M, \|.\,,.\|, \bar{\lambda}, \bar{p})$ and $0 < \varepsilon < 1$. Let r be a fixed positive real number such that $M\,(r) \ge 1$. Then for $\frac{\varepsilon}{r}$, there exists an integer $N \ge 1$ such that

$$G(\bar{x}^{(i)} - \bar{x}^{(j)}) < \frac{\varepsilon}{r},\ \text{for all } i, j \ge N. \qquad ...(13)$$

Using definition of paranorm G, we see that

$$\sup_k M\left(\frac{\|\lambda_k x_k^{(i)} - \lambda_k x_k^{(j)}, z\|^{p_k/L}}{G(\bar{x}^{(i)} - \bar{x}^{(j)})}\right) \leq 1, \text{ for all } i, j \geq N \text{ and each } z \in X. \qquad \ldots (14)$$

Thus for all $i, j \geq N$ and $k \geq 1$ and each $z \in X$, we have

$$M\left(\frac{\|\lambda_k x_k^{(i)} - \lambda_k x_k^{(j)}, z\|^{p_k/L}}{G(\bar{x}^{(i)} - \bar{x}^{(j)})}\right) \leq 1 \leq M(r).$$

But M is non decreasing, therefore

$$\frac{\|\lambda_k (x_k^{(i)} - x_k^{(j)}), z\|^{p_k/L}}{G(\bar{x}^{(i)} - \bar{x}^{(j)})} < r.$$

Hence in view of (13), we have for each $k \geq 1$ and for each $z \in X$

$$\|\lambda_k (x_k^{(i)} - x_k^{(j)}), z\|^{p_k/L} < \varepsilon, \text{ for all } i, j \geq N \qquad \ldots (15)$$

This shows that $(x_k^{(i)})$ is a Cauchy sequence in X for each $k \geq 1$. But X is complete, therefore for each $k \geq 1$ there exists x_k (say) in X such that $x_k^{(i)} \to x_k$ as $i \to \infty$. We show that $\bar{x} = (x_k) \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$.

Now we choose $\rho > 0$ such that

$$G(\bar{x}^{(i)} - \bar{x}^{(j)}) < \rho < \varepsilon, \text{ for all } i, j \geq N. \qquad \ldots (16)$$

Since M is non decreasing, therefore by (16) we have for each $k \geq 1$

$$\sup_k M\left(\frac{1}{\rho}\|\lambda_k (x_k^{(i)} - x_k^{(j)}), z\|^{p_k/L}\right) \leq \sup_k M\left(\frac{\|\lambda_k (x_k^{(i)} - x_k^{(j)}), z\|^{p_k/L}}{G(\bar{x}^{(i)} - \bar{x}^{(j)})}\right) \leq 1$$

for all $i, j \geq N$ and for each $z \in X$.

Since M is continuous, taking limit as $j \to \infty$, we see that for each $k \geq 1$

$$\sup_k M\left(\frac{1}{\rho}\|\lambda_k (x_k^{(i)} - x_k), z\|^{p_k/L}\right) \leq 1, \text{for all } i \geq N \text{ and for each } z \in X.$$

Hence $G(\bar{x}^{(i)} - \bar{x}) = \inf\left\{\rho : \sup_k M\left(\frac{1}{\rho}\|\lambda_k (x_k^{(i)} - x_k), z\|^{p_k/L}\right) \leq 1 \text{ for all } i \geq N \text{ and for each } z \in X\right\}$

$\leq \rho < \varepsilon.$

This shows that $G(\bar{x}^{(i)} - \bar{x}) < \varepsilon$, for all $i \geq N$ and consequently $\bar{x}^{(i)} \to \bar{x}$ as $i \to \infty$. Clearly $\bar{x}^{(i)} - \bar{x} \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$,for all $i \geq N$. Since, $\bar{x}^{(N)}$ and $\bar{x}^{(N)} - \bar{x} \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$,therefore it follows that $\bar{x} = \bar{x}^{(N)} - (\bar{x}^{(N)} - \bar{x}) \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$. This completes the proof.

Theorem 4.4: *The space* $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ *is normal.*

Proof: Let $\bar{x} = (x_k) \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$. So that

$$M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ as } k \to \infty, \text{ for some } \rho > 0 \text{ and for each } z \in X.$$

Let (ξ_k) be a sequence of scalars satisfying $|\xi_k| \leq 1$ for all $k \geq 1$.Using non decreasing property of M, we have

$$\begin{aligned} M\left(\frac{1}{\rho}\|\lambda_k \xi_k x_k, z\|^{p_k}\right) &= M\left(\frac{1}{\rho}|\xi_k|^{p_k}\|\lambda_k x_k, z\|^{p_k}\right) \\ &\leq M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ as } k \to \infty \end{aligned}$$

for each $z \in X$. This shows that $(\xi_k x_k) \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ and hence $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ is normal.

Theorem 4.5: *If* M *satisfies the* Δ_2 *-condition, then*

$$c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p}) = \bar{c_0}(X, M, \|.,.\|, \bar{\lambda}, \bar{p}).$$

Proof: To prove the equality, it suffices to show that $c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ is a subset of $\bar{c}_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$ since the reverse inclusion is always true. Let $\bar{x} \in c_0(X, M, \|.,.\|, \bar{\lambda}, \bar{p})$. Then for some $\rho > 0$ and for each $z \in X$,

$$M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ as } k \to \infty.$$

Let us consider an arbitrary $\rho_1 > 0$.

Case I: If $\rho \leq \rho_1$, then obviously, we have

$$M\left(\frac{1}{\rho_1}\|\lambda_k x_k, z\|^{p_k}\right) \leq M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ as } k \to \infty$$

for each $z \in X$. Hence we get $\bar{x} \in \bar{c}_0\ (X, M, \|.,.\|, \bar{\lambda}, \bar{p})$.

Case II: If $\rho > \rho_1$, $(\rho/\rho_1) > 1$. In this case using Δ_2 -condition of M, we get

$$M\left(\frac{1}{\rho_1}\|\lambda_k x_k, z\|^{p_k}\right) = M\left(\frac{\rho}{\rho_1}.\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right)$$

$$\leq K.\frac{\rho}{\rho_1}\ M\left(\frac{1}{\rho}\|\lambda_k x_k, z\|^{p_k}\right) \to 0 \text{ as } k \to \infty$$

for each $z \in X$, where K is the number involved in Δ_2- condition. This proves that

$$\bar{x} \in \bar{c}_0\ (X, M, \|.,.\|, \bar{\lambda}, \bar{p}).$$

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