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CYLINDRICAL SHOCK WAVES IN A LOW CONDUCTING NON-IDEAL GAS IN PRESENCE OF AXIAL AND AZIMUTHAL COMPONENTS OF MAGNETIC INDUCTION

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Abstract : The propagation of diverging cylindrical shock waves in a low conducting non-ideal gas under the influence of spatially variable axial and azimuthal components of magnetic induction, is investigated by using method of self-similarity. The initial density of the medium is assumed to be constant. Also both the components of the initial magnetic induction are taken to vary as some power of the distance from the axis of symmetry. The effects of variation of non-idealness parameter δ and the ratio of specific heats of the gas, on the propagation of the shock and the flow-field behind it are investigated.

Keywords : Shock waves, self-similar flow, non-ideal gas, variable initial magnetic field, low electrical conductivity.

2010 Mathematics Subject Classification : 76L05, 35L67

1. Introduction

Lin [10] has extended the Taylor's analysis [19] of the intense spherical explosion to the cylindrical case. The law of variation of the radius of a strong cylindrical shock wave produced by a sudden release of a finite amount of energy was obtained. Since at high temperatures that prevail in the problems associated with shock waves a gas is ionized, electromagnetic effects may also be significant. A complete analysis of such a problem should therefore consist of the study of the gas dynamic flow and the electromagnetic field simultaneously. The study of the propagation of cylindrical shock waves in a conducting gas in presence of an axial or azimuthal magnetic induction is relevant to the experiments on pinch effect, exploding wires and so forth. This problem both in the uniform or non-uniform medium was undertaken by many investigators, for example, Pai [13], Cole and Greifinger [5], Sakurai [18], Bhutani [3], Christer and Helliwell [4], Deb Ray [6], Vishwakarma and Yadav [26]. One of the basis assumptions of these works is that the shock wave is propagated in a medium which is an ideal gas.

The assumption that the gas is ideal is no more valid when the flow takes place at extreme conditions. In recent years, several studies have been performed concerning the problem of strong shock waves in non-ideal gases, in particular by Anisimov and Spiner [1], Rangarao and Purohit [14], Wu and Roberts [27], Madhumita and Sharma [11], Arora and Sharma [2] and Vishwakarma and Nath [23, 24] among others. Anisimov and Spiner [1] have taken an equation of state for a low density non-ideal gas in a simplified form, and studied the effects of non-idealness of the gas on the problem of point explosion. Rangarao and Purohit [14] have studied the self-similar flows of a non-ideal gas driven by an expanding piston and obtained approximate analytic and numerical solutions by taking the equation of state suggested by Anisimov and Spiner [1]. Vishwakarma and Nath [23] have obtained similarity solutions for the flow behind an exponential shock by taking the same equation of state for the medium. Roberts and Wu [15, 16] have used an equivalent equation of state to study the shock wave theory of sonoluminescence. Vishwakarma et al. [22] too adopted this as their model of a non-ideal gas to obtain the self-similar solutions for the flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating gas in presence of an azimuthal magnetic field.

In the present work, we studied the propagation of diverging cylindrical shock waves in a low conducting gas as a result of time dependent energy input, under the influence of spatially variable axial and azimuthal components of magnetic induction. The medium ahead and behind the shock front are assumed to be an inviscid one and to behave as a non-ideal gas having the equation of state suggested by Anisimov and Spiner [1]. The initial density of the gas is constant. The gas ahead of the shock is assumed to be at rest. Effects of viscosity, heat-conduction, radiation and gravitation are not taken into account. Distribution of the flow variables in the flow-field behind the shock front are obtained, and the effects of the non-idealness of the gas and the variation of the ratio of specific heats are investigated.

2. Fundamental Equations and Boundary Conditions

The equation of state for a non-ideal gas is borrowed from the statistical physics (Landau and Lifshitz [9]) which has been simplified by Anisimov and Spiner [1] in the form

$$p = \rho R^* T(1 + \bar{b}\rho), \qquad \text{... (1)}$$

where $\bar{b}\,(<< 1)$ is internal volume of the molecules, R^* is the gas constant, p, ρ and T are pressure, density and temperature of the gas, respectively.

The internal energy 'e' per unit mass is given by (Ojha [12], Wu and Roberts [27])

$$e = \frac{p}{\rho(\gamma - 1)(1 + \bar{b}\rho)} \simeq \frac{p(1 - \bar{b}\rho)}{(\gamma - 1)\rho}, \qquad \text{... (2)}$$

which implies that

$$C_p - C_v = R^* \left(1 + \frac{\bar{b}^2 \rho^2}{1 + 2\bar{b}\rho} \right) \simeq R^*, \qquad \text{... (3)}$$

neglecting the term $\bar{b}^2 \rho^2$. Here C_p, C_v are the specific heats of the gas at constant pressure and constant volume, respectively, and $\gamma = \dfrac{C_p}{C_v}$.

The basic equations governing the unsteady and cylindrically symmetric motion of a low conducting non-ideal gas in presence of axial and azimuthal components of magnetic induction are given by (Tyl [20], Sakurai [18] and Vishwakarma [26])

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0, \quad \text{...(4)}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) + \frac{\partial p}{\partial r} = -\sigma u\,(B_{z_0}^2 + B_{\theta_0}^2), \quad \text{...(5)}$$

$$\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r}\right) - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = (\gamma - 1)\sigma u^2 (B_{z_0}^2 + B_{\theta_0}^2), \quad \text{...(6)}$$

$$\frac{\partial B_z}{\partial r} = \mu\sigma B_{z_0} u, \quad \text{...(7)}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu\sigma B_{\theta_0} u, \quad \text{...(8)}$$

where u, B_z, B_θ are the velocity, axial magnetic induction and azimuthal magnetic induction, respectively, at distance r from the axis of symmetry and at the time t, B_{z_0} is the initial axial magnetic induction, B_{θ_0} the initial azimuthal magnetic induction, μ the magnetic permeability, σ the electrical conductivity and 'a' the speed of sound given by

$$a^2 = \frac{\gamma p}{\rho}\left(\frac{1 + 2\bar{b}\rho}{1 + \bar{b}\rho}\right). \quad \text{...(9)}$$

It is assumed that, due to explosion along the axis of symmetry, a cylindrical shock is produced and propagates into the low conducting non-ideal gas of constant density ρ_0 in presence of the axial magnetic induction B_{z_0} and azimuthal magnetic induction B_{θ_0}.

The axial and azimuthal components of magnetic induction ahead of the shock are assumed to be varying and obeying the laws:

$$B_{z_0} = SR^{-m} \text{ and } B_{\theta_0} = QR^{-n}, \qquad \text{... (10)}$$

where R is the shock radius and S, Q, m, n are constants.

Since σ is small, the magnetic induction may be taken continuous across the shock front (Sakurai [18]). Neglecting the counter pressure, the shock conditions may be written as

$$u_s = \frac{2(1-\delta)}{(\gamma+1)}V, \quad \rho_s = \left(\frac{\gamma+1}{2\delta+\gamma-1}\right)\rho_0, \; p_s = \frac{2(1-\delta)}{(\gamma+1)}\rho_0 V^2,$$

$$B_{z_s} = B_{z_0}, \; B_{\theta_s} = B_{\theta_0} \qquad \text{... (11)}$$

where the subscript 's' refers the conditions immediately behind the shock front, $V = dR/dt$ denotes the velocity of the shock and $\delta = \bar{b}\rho_0$ is the parameter of non-idealness of the gas.

3. Similarity Transformations

To obtain similarity solutions, we write the unknown variables in the following form (Vishwakarma and Yadav [26])

$$u = Vf(x), \; \rho = \rho_0 D(x), \; p = \rho_0 V^2 P(x),$$

$$B_z = \sqrt{\rho_0 \mu}\, V b_z(x), \; B_\theta = \sqrt{\rho_0 \mu}\, V b_\theta(x), \qquad \text{... (12)}$$

where f, D, P, b_z and b_θ are the functions of the non-dimensional variable $x = \frac{r}{R(t)}$ only. The shock front is represented by $x = 1$.

The total energy of the flow-field behind the shock may be assumed to be time dependent and varying as (Rogers [17], Freeman [8], Director and Dabora [7])

$$E = E_0 t^k, \qquad \text{... (13)}$$

where E_0 and k are constants. The value $k = 0$ corresponds to the case of instantaneous energy release. The positive values of k corresponds to the class in which the total energy increases with time.

The total energy of the flow-field behind the shock front is, therefore, expressed as

$$E_0 t^k = 2\pi \int_0^R \left[\frac{p(1 - b\rho)}{\gamma - 1} + \frac{1}{2}\rho u^2 + \frac{B_z^2 + B_\theta^2}{2\mu} \right] r\, dr. \qquad \text{... (14)}$$

Applying the similarity transformations (12) in the relation (14), we find that the motion of the shock front is given by the equation

$$V = \frac{dR}{dt} = \left(\frac{E_0}{2\pi J \rho_0} \right)^{1/2} t^{k/2} R^{-1}, \qquad \text{... (15)}$$

where

$$J = \int_0^1 \left[\frac{\left[1 - b\rho_0 D\right]}{\gamma - 1} + \frac{1}{2} f^2 D + \frac{1}{2}(b_z^2 + b_\theta^2) \right] x\, dx.$$

Equation (15), on integration, yields

$$R = \left(\frac{4}{k+2}\right)^{1/2}\left(\frac{E_0}{2\pi J \rho_0}\right)^{1/4} t^{\left(\frac{k+2}{4}\right)}, \qquad \text{... (16)}$$

and therefore,

$$V = \left(\frac{k+2}{4}\right)\frac{R}{t}. \qquad \text{...(17)}$$

After using the similarity transformations, the equations (4) to (8) change into the following set of ordinary differential equations

$$(f - x)\frac{dD}{dx} + D\frac{df}{dx} + \frac{Df}{x} = 0 \qquad \text{... (18)}$$

$$(f - x)\frac{df}{dx} + \frac{1}{D}\frac{dP}{dx} - f = -R_m(M_{Az}^{-2} + M_{A\theta}^{-2})\frac{f}{D}, \qquad \text{... (19)}$$

$$(f - x)\frac{dP}{dx} - \gamma(f - x)\left(\frac{1+2\delta D}{1+\delta D}\right)\frac{P}{D}\frac{dD}{dx} - 2P$$

$$= (\gamma - 1) f^2 R_m (M_{Az}^{-2} + M_{A\theta}^{-2}), \qquad \text{... (20)}$$

$$\frac{db_z}{dx} = f R_m M_{Az}^{-1}, \qquad \text{... (21)}$$

$$\frac{db_\theta}{dx} + \frac{1}{x} b_\theta = f R_m M_{A\theta}^{-1}, \qquad \text{... (22)}$$

where R_m, M_{Az} and $M_{A\theta}$ are respectively, the magnetic Reynolds number, axial Alfven-Mach number and azimuthal Alfven-Mach number, and they are given by

$$R_m = \sigma\mu V R, \qquad M_{Az} = \left(\frac{\mu\rho_0 V^2}{B_{z_0}^2}\right)^{1/2} \quad \text{and} \quad M_{A\theta} = \left(\frac{\mu\rho_0 V^2}{B_{\theta_0}^2}\right)^{1/2}.$$

Using the self-similarity transformations (13), the boundary conditions (12) can be written as

$$f(1)=\frac{2(1-\delta)}{\gamma+1};\quad D(1)=\frac{\gamma+1}{\gamma-1+2\delta};\quad P(1)=\frac{2(1-\delta)}{\gamma+1};$$

$$b_z(1)=\frac{1}{M_{Az}};\quad b_\theta(1)=\frac{1}{M_{A\theta}}. \qquad \text{... (23)}$$

For the existence of similarity solutions magnetic Reynolds number R_m, axial Alfven-Mach number M_{Az} and azimuthal Alfven-Mach number $M_{A\theta}$ should be constants. Therefore,

$k=0,\quad m=1\quad$ and $\quad n=1.$ ...(24)

This (k = 0) shows that, in the present study, the similarity solution exists only in the case when there is instantaneous release of a finite amount of energy E_0 at the axis of symmetry.

By solving equations (18) to (20) for $\frac{dD}{dx}, \frac{dP}{dx}, \frac{df}{dx}$, we get

$$\frac{dD}{dx}=-\frac{D}{(f-x)}\left[\frac{df}{dx}+\frac{f}{x}\right], \qquad \text{... (25)}$$

$$\frac{dP}{dx}=-D(f-x)\frac{df}{dx}+fD-(M_{Az}^{-2}+M_{A\theta}^{-2})R_m f, \qquad \text{... (26)}$$

$$\frac{df}{dx}=\frac{f}{\left[\gamma P\left(\frac{1+2\delta D}{1+\delta D}\right)-D(f-x)^2\right]}\left[(\gamma f-x)(M_{Az}^{-2}+M_{A\theta}^{-2})R_m -D(f-x)-\frac{P}{f}\left\{\frac{\gamma f}{x}\left(\frac{1+2\delta D}{1+\delta D}\right)-2\right\}\right]. \qquad \text{... (27)}$$

For exhibiting the numerical solutions, it is convenient to write field variables in the following form:

$$\frac{u}{u_s} = \frac{f(x)}{f(1)}, \quad \frac{\rho}{\rho_s} = \frac{D(x)}{D(1)}, \quad \frac{P}{P_s} = \frac{P(x)}{P(1)}, \quad \frac{B_z}{B_{z_s}} = \frac{b_z(x)}{b_z(1)}, \quad \frac{B_\theta}{B_{\theta_s}} = \frac{b_\theta(x)}{b_\theta(1)}. \qquad ...(28)$$

The shock-boundary conditions in terms of these variables are

$$\frac{u}{u_s} = 1, \quad \frac{\rho}{\rho_s} = 1, \quad \frac{P}{P_s} = 1, \quad \frac{B_z}{B_{z_s}} = 1, \quad \frac{B_\theta}{B_{\theta_s}} = 1. \qquad ...(29)$$

Now, the differential equations (21), (22) and (25) to (27) may be numerically integrated with the boundary conditions (22), to obtain the flow-field behind the shock front.

4. Results and Discussion

The reduced flow variables $\frac{u}{u_s}, \frac{\rho}{\rho_s}, \frac{p}{p_s}, \frac{B_z}{B_{z_s}}$ and $\frac{B_\theta}{B_{\theta_s}}$ are obtained by numerical integration of the differential equations (21), (22) and (25) to (27) with the boundary conditions (23). For the purpose of numerical integration, the values of the constant parameters are taken as (Vishwakarma and Singh [25], Ranga Rao and Purohit [14]) $\gamma = 1.4, 1.66$; $M_{Az}^{-2} = 0.01$; $M_{A\theta}^{-2} = 0.01$; $R_m = 0.01$; $\delta = 0, 0.025, 0.05, 0.1$. The value $\delta = 0$ corresponds to the case of ideal gas.

Figures 1-5 show the variation of the flow variables $\frac{u}{u_s}$, $\frac{\rho}{\rho_s}$, $\frac{p}{p_s}$, $\frac{B_z}{B_{z_s}}$ and $\frac{B_\theta}{B_{\theta_s}}$ with x at various values of the parameters δ and γ. It is shown that, as we move inward from the shock front towards the axis of symmetry, the reduced fluid velocity $\frac{u}{u_s}$, reduced density $\frac{\rho}{\rho_s}$, reduced pressure $\frac{p}{p_s}$ and reduced axial magnetic induction $\frac{B_z}{B_{z_s}}$ decrease, whereas reduced azimuthal magnetic induction $\frac{B_\theta}{B_{\theta_s}}$ increases.

The effects of an increase in the value of non-idealness parameters δ are (from figures 1-5)

(i) to increase the density $\frac{\rho}{\rho_s}$ and the axial magnetic induction $\frac{B_z}{B_{z_s}}$ at any point in the flow-field behind the shock;

(ii) to decrease the velocity $\frac{u}{u_s}$;

(iii) to decrease the slope of profiles of density and axial magnetic induction and to increase that of the profiles of the velocity; and

(iv) to increase the density and pressure in the flow field near the shock front. It means that an increase in the parameter of non-idealness δ, results in an increase in the shock strength.

The effects of an increase in the γ are (from figures 1-5)

(i) to decrease the velocity $\frac{u}{u_s}$ and to increase the density $\frac{\rho}{\rho_s}$ and the axial magnetic induction $\frac{B_z}{B_{z_s}}$ at any point in the flow-field behind the shock; and

(ii) to decrease the slope of profiles of density and axial magnetic induction and to increase that of the velocity.

Variation of δ or γ has no effect on the azimuthal magnetic field B_θ / B_{θ_s} in the flow-field behind the shock.

Equations (16) and (24) show that the shock velocity $\dot{R}$ decreases with time and varies as $t^{-1/2}$.

5. Conclusion

In the present paper, the similarity solutions have been obtained for the propagation of cylindrical shock waves in a low conducting non-ideal gas under the influence of variable initial axial and azimuthal components of magnetic induction. On the basis of the present study, one may draw the following conclusions:

(i) The shock velocity decreases with time as $t^{-1/2}$.

(ii) An increase in the value of non-idealness parameter of the gas δ gives rise an increase in the density $\frac{\rho}{\rho_s}$ and in the axial magnetic induction $\frac{B_z}{B_{z_s}}$, and a decrease in the velocity $\frac{u}{u_s}$ at any point in the flow-field behind the shock.

(iii) An increase in the non-idealness parameter δ, increases the shock strength.

(iv) An increase in the value of γ gives rise an increase in the density $\frac{\rho}{\rho_s}$ and in the axial magnetic induction $\frac{B_z}{B_{z_s}}$, and a decrease in the velocity $\frac{u}{u_s}$ at any point in the flow-field behind the shock.

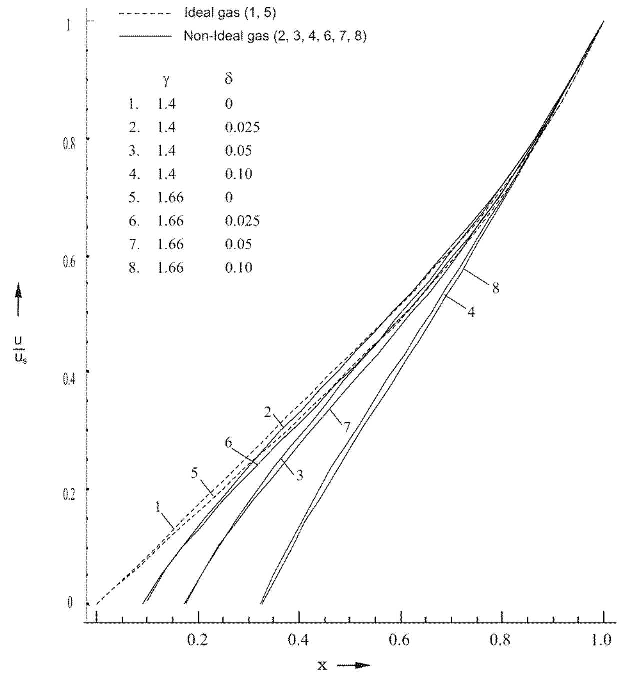

Figure 1: Variation of reduced velocity in the region behind the shock front for $M_{AZ}^{-2} = 0.01$, $M_{A\theta}^{-2} = 0.01$ and $R_m = 0.01$

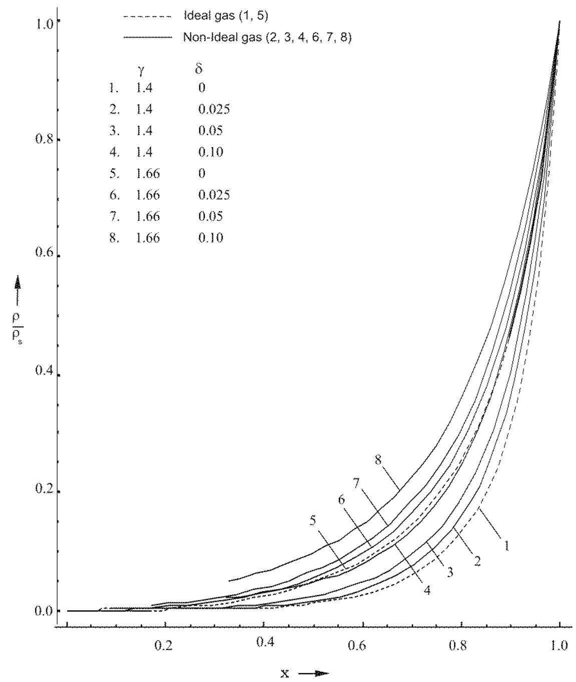

Figure 2: Variation of reduced density in the region behind the shock front for $M_{AZ}^{-2} = 0.01$, $M_{A\theta}^{-2} = 0.01$ and $R_m = 0.01$

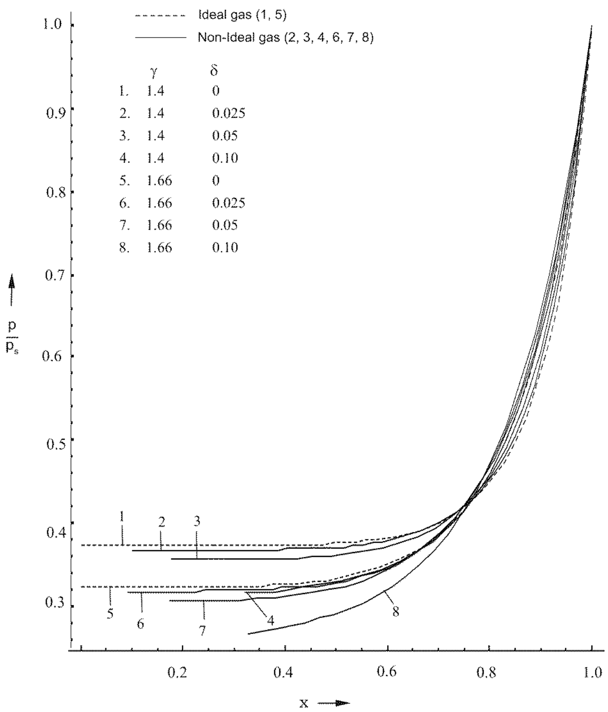

Figure 3: Variation of reduced pressure in the region behind the shock front for $M_{AZ}^{-2} = 0.01$, $M_{A\theta}^{-2} = 0.01$ and $R_m = 0.01$

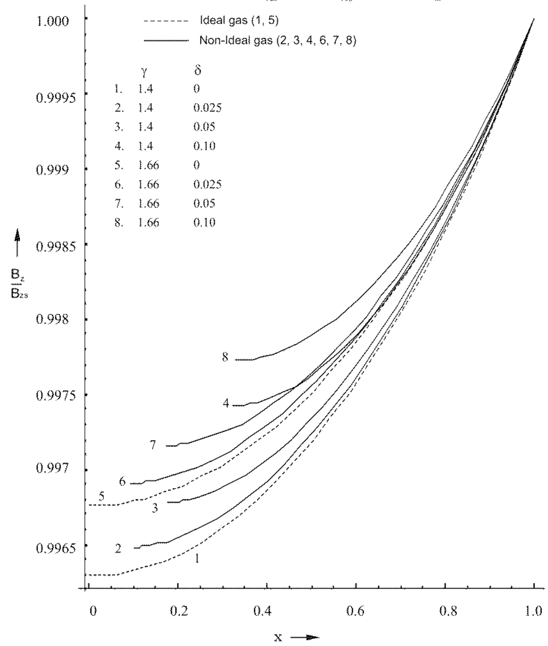

Figure 4: Variation of reduced axial magnetic induction in the region behind the shock front for $M_{AZ}^{-2} = 0.01$, $M_{A\theta}^{-2} = 0.01$ and $R_m = 0.01$

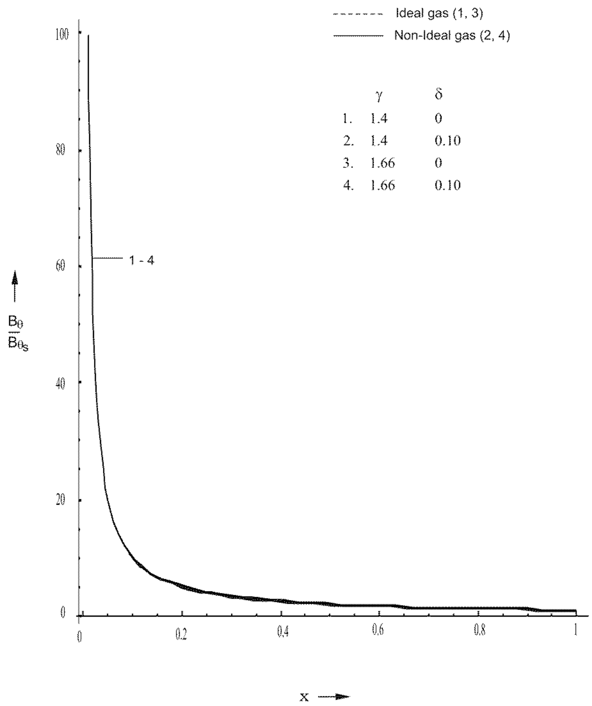

Figure 5: Variation of reduced azimuthal magnetic induction in the region behind the shock front for $M_{AZ}^{-2} = 0.01$, $M_{A\theta}^{-2} = 0.01$ and $R_m = 0.01$

References

[1] Anisimov, S. I. and Spiner, O. M. (1972). Motion of an almost ideal gas in the presence of a strong point explosion, *J. Appl. Math. Mech.*, **36,** 883-887.

[2] Arora, R. and Sharma, V. D. (2006). Convergence of strong shock in Van der Waals gas, SIAM J. *Appl. Math.* **66**, 1825-1837.

[3] Bhutani, O. P. (1966). Propagation and attenuation of a cylindrical blast wave in magnetogasdynamics, *J. Math. Anal. Appl.* **13**, 565-576.

[4] Christer, A. H. and Helliwell, J. B. (1969). Cylindrical shock and detonation waves in magnetogasdynamics, *J. Fluid Mech.* **39**, 705-725.

[5] Cole, J. D. and Greifinger, C. (1962). Similarity solution for cylindrical magnetohydrodynamic blast waves, *Phys. Fluids* **5**, 1597-1607.

[6] Deb Ray, G. (1973). Similarity solutions for cylindrical blast waves in magnetogasdynamics, *Phys. Fluids* **16**, 559-560.

[7] Director, M. N. and Dabora, E. K. (1977). An experimental investigation of variable energy blast waves, *Acta Astronautica* **4**, 391-407.

[8] Freeman, R. A. (1968). Variable energy blast waves, *Brit. J. Appl. Phys. (J. Phys. D)* **1**, 1697-1710.

[9] Landau, L. D. and Lifshitz, E. M. (1958). A Course of Theoretical Physics, Statistical Physics, *Pergamon Press* **5**, 225-250.

[10] Lin, S. C. (1954). Cylindrical shock waves produced by instantaneous energy release, *J. Appl. Phys.* **25**, 54-57.

[11] Madhumita, G. and Sharma, V. D. (2004). Imploding cylindrical and spherical shock waves in a non-ideal medium, *J. Hyperbolic Differ. Equ.* **1**, 521-530.

[12] Ojha, S. N. (2002). Shock waves in non-ideal fluids, *Int. J. Appl. Mech. Engng.* 7, 445-464.

[13] Pai, S. I. (1958). Cylindrical shock waves produced by instantaneous energy release in magnetogasdynamics, *Proc. Theo. Appl. Mech.* **81**, 89-100.

[14] Ranga Rao, M. P. and Purohit, N. K. (1976). Self-similar piston problem in non-ideal gas, *Int. J. Eng. Sci.* **14**, 91-97.

[15] Roberts, P. H. and Wu, C. C. (1996). Structure and stability of a spherical implosion, *Phys. Lett.* **213**, 59-64.

[16] Roberts, P. H. and Wu, C. C. (2003). *The shock wave theory of sonoluminescene in Shock Focussing Effects in Medical Science and Sonoluminescene (Eds. R. C. Srivastavad et. al)*, Springer Verlag, 1-27.

[17] Rogers, M. H. (1958). Similarity flows behind strong shock waves, *Quart. J. Mech. Appl. Math.* **11**, 411-422.

[18] Sakurai, A. (1965). *Blast wave theory, Basic Developments in Fluid Dynamics* (Ed. M. Holt), Academic Press, New York, 309-375

[19] Taylor, G. I. (1950). The formation of a blast wave by a very intense explosion, Proc. *Royal Soc. Lond. A.* **201**, 159-174.

[20] Tyl, J. (1992). The influence of the action of magnetic field on the phenomenon of implosion of a cylindrical shock wave in gas, *J. Tech. Phys.* **33**, 205-218.

[21] Vishwakarma, J. P. (2002). *Current Trends in Industrial and Applied Mathematics*, (Eds. P. Manchanda et at.), Anamaya Publishers, New Delhi, 109-122.

[22] Vishwakarma, J. P., Maurya, A. K. and Singh, K. K. (2007). Self-similar adiabatic flow headed by magnetogasdynamics cylindrical shock wave in a rotating non-ideal gas, *Geophys. Astrophys. Fluid Dyn.* **101**, 115-167.

[23] Vishwakarma, J. P. and Nath, G. (2006). Similarity solutions for unsteady flow behind an exponential shock in a dusty gas,*Phys. Scr.* **74**, 493-498.

[24] Vishwakarma, J. P. and Nath, G. (2009). Self-similar solution of a shock propagation in a mixture of a non-ideal gas and small solid particles, *Meccanica* **44**, 239-254.

[25] Vishwakarma, J. P. and Singh, A. K. (2009). Self-similar solutions for cylindrical shock waves in a low conducting gas, *Aligarh Bull. Maths*. **28**, 5-14.

[26] Vishwakarma, J. P. and Yadav, A. K. (2003). Self-similar analytical solutions for blast waves in inhomogeneous atmospheres with frozen-in-magnetic field, Eur. *Phys. J. B* **34**, 247-253.

[27] Wu, C. C. and Roberts P. H. (1993). Shock wave propagation in a sonoluminescing gas bubble, *Phys. Rev. Lett.* **70**, 3423-3427.