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HALL EFFECT ON CHEMICALLY REACTING AND RADIATING MHD OSCILLATORY VISCOELASTIC FLOW THROUGH POROUS VERTICAL CHANNEL

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Abstract. The effect of the hall current and chemical reaction on MHD oscillatory free convective, incompressible, electrically conducting viscoelastic fluid in an infinite vertical porous channel has been analysed. A uniform injection/ suction velocity is applied at the plates. The magnetic field of uniform strength is applied in the direction normal to plane of the plates. Analytic solution of the equations governing the flow are obtained for velocity, temperature and concentration profile. The influence of the parameters entering in the governing equations on flow are evaluated numerically and discussed with the help of graphs and tables.

Keywords: Hall effect, chemically reacting, radiating, oscillatory and viscoelastic.

2010 Mathematical Subject Classification : 76D05, 76V05.

1. Introduction

Many fluids such as blood, dyes, ketchup, shampoo, paint, mud, clay coating polymer melts, certain oil and greases etc. have complicated relations between stress and strain. Such fluids do not obey the Newton's law of viscosity and are usually called non-Newtonian fluids. The flows of such fluids occur in a wide range of practical applications and have a key importance in polymer depolarization, bubble columns, fermentation, composite processing, boiling, plastic foam processing, bubble absorption and many others. Therefore, non-Newtonian fluids have attracted the attention of large variety of researchers including the interest of experimentalists and theoreticians like engineers, modellers, physicists, computer scientists and mathematicians however these fluids are varied in nature, the constitutive equations which govern them are many taking account of the variations of rheological properties. The model and hence, the arising equations, are much more complicated and are of higher order than the well known Navier- Stokes equations.

Since the flow of an electrically conducting viscous incompressible fluid between two parallel plates in the presence of the transversely applied magnetic field has applications in many devices such as (MHD) power generator, MHD pumps, accelerators, aerodynamics heating, electrostatics precipitations, polymers technology, petrol industry, purification of molten metals from non metallic inclusion and fluid droplets sprays transfer, subjected to different physical effect, have been studied by many authors such as Alpher [1], Attia and Kotb [2] and Tani [15] . These results are important for the design of the duct wall and cooling arrangement. The rectangular channel problem has later been extended also to fluids obeying non-Newtonian constitutive equations. The hydrodynamic flow of viscoelastic fluid has attracted the attention of many scholars like Hartnett [6] and Skelland [14] due to its industrial applications. Ever increasing industrial applications in the manufacture of plastic film and artificial fiber materials, in recent years, has led to a renewed interest in study of viscoelastic fluid. Good list of references on this problem can be found in Khan *et al.* [9] and Sadeghy *et al.* [13].

In most of the above studies, the Hall current term was ignored in applying Ohm's law as it has no marked effect for small and moderate value of magnetic field. However, the current trend for application of MHD is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable Attia and Kotb [2]. Under these conditions, the Hall current is important and has a marked effect on the magnitude and the direction of current density and consequently on magnetic force term. The inclusion of the hall current as well as the non-Newtonian fluid characteristics lead to the some interesting effects both, on velocity and temperature field as reported by the Attia [3] in his investigation.

The convective flow with simultaneous heat and mass transfer under the influence of the magnetic field and chemical reaction have attracted a considerable attention of researchers because such process exists in many branches of science and technology. In many chemical engineering processes, there is a chemical reaction between a foreign mass and fluid. These processes take place in many industrial applications such as a food processing, manufacture of ceramics, polymer production, dying evaporation at the surface of a water body and electric power industry. The effect of the chemical reaction on various problems had been examined by Chamhka [4] and Prakash *et al.* [12].

On the hand the role of the thermal radiation is of major importance in engineering areas occurring at high temperature and knowledge of radiative heat transfer became very important in a nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles. In view of these applications Olajuwan [10] and Pal [11] has studied and reported the significance of thermal radiations. Flows through porous media with heat and mass transfer are seen in the wide application such as chemical catalytic reactors, solar power collectors, insulation of nuclear reactors, fluid films lubrication and analysis of polymer in chemical engineering. The representative study in this area can be found in Hayat *et al.* [7, 8].

Motivated by above studies we intended to investigate an MHD oscillatory free convective flow of viscoelastic, incompressible, electrically conducting and chemically reacting fluid through a highly porous medium bounded by two infinite vertical porous plate in the presence of the Hall current.

2. Formulation of the problem

Consider an oscillatory free convective flow of a viscoelastic, incompressible and electrically conducting fluid through a highly porous medium bounded between two infinite vertical porous plates distance '*d*' apart. A constant injection velocity, w_0 is applied at the stationary plate $z^* = 0$ and the same constant suction velocity, w_0 is applied at the plate $z^* = 2$ which is oscillating in its own plane with velocity $U^*(t^*)$ about non zero constant mean velocity U_0. A uniform magnetic field with magnetic flux density vector B_0 is applied perpendicular to the plane of the plates. Using relation $\Delta.\vec{B} = 0$ for the magnetic field $\vec{B} = (B_x^*, B_y^*, B_z^*), B_z^* = B_0$ is obtained everywhere in the fluid (B_0 is a constant). If $\vec{J} = (J_x^*, J_y^*, J_z^*)$ is the current density, from the relation $\nabla.\vec{J} = 0$, one have $J_z^* =$ constant. Since the channel is non-conducting, $J_z^* = 0$ everywhere.

The generalized Ohm's law, in the absence of the electric field (Meyer), is of the form:

$$\vec{J} + \frac{\omega_e \tau_e}{B_0(\vec{J} \times \vec{B})} = \sigma\left(\mu_e \vec{V} \times \vec{B} + \frac{1}{e\, n_e \nabla p_e}\right)$$

where $\vec{V}$, σ, μ_e, ω_e, τ_e, e, n_e and p_e are respectively the velocity, the electrical conductivity, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron and the electron pressure. Under the usual assumptions that the electron pressure (for a weakly ionized gas), the thermoelectric pressure and ion slip are negligible, one has from the Ohm's law

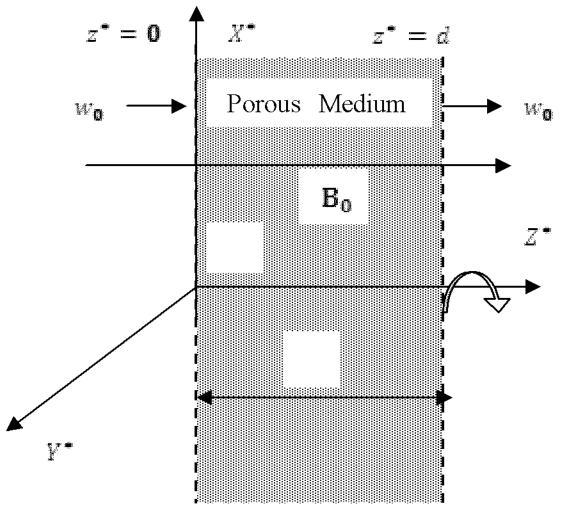


Fig.1: Geometrical configuration of the physical problem.

$J_x^* + \omega_e \tau_e J_y^* = \sigma \mu_e B_0 v^*$ and $J_y^* - \omega_e \tau_e J_x^* = -\sigma \mu_e B_0 u^*$

from these one can obtain

$$J_x^* = \frac{\sigma \mu_e B_0}{1+m^2}(mu^* + v^*) \quad \text{and} \quad J_y^* = \frac{\sigma \mu_e B_0}{1+m^2}(mv^* - u^*)$$

where $m = \omega_e \tau_e$ is the Hall parameter.

The origin is assumed to be at the plate $z^* = 0$ and the channel is oriented vertically upward along the X^*-axis. Since the plates are infinite in extent, all the physical quantities except the pressure depend only on z^* and t^*. Denoting the velocity components by u^*, v^*, w^* in x^*, y^*, z^* direction respectively and temperature by T^* the flow in system under the following assumptions

1 A uniform magnetic field is applied normal to the plane of plates.

2 Boussinesq approximation is applied.

3 The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is negligible.

4 The effect of viscous and Joule's dissipation is assumed to be negligible in the energy equation as small velocity is usually encountered in free convection flows.

5 There exists a first order chemical reaction between the fluid and species concentration.

6 The level of species concentration is very low so that the heat generated during chemical reaction can be neglected.

is governed by the following equations

$$w_z^* = 0 \qquad \text{....(1)}$$

$$\frac{\partial u^*}{\partial t^*} + w_0 \frac{\partial u^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mv^* - u^*) + g\beta(T^* - T_d)$$

$$+g\beta_c(C^* - C_d) - v\frac{u^*}{K^*} + \frac{1}{\rho}\frac{\partial \tau^*_{x^*z^*}}{\partial z^*} \qquad \text{....(2)}$$

$$\frac{\partial v^*}{\partial t^*} + w_0 \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial y^*} - \frac{\sigma B_0^2}{\rho(1+m^2)}(mu^* + v^*) - v\frac{v^*}{K^*} + \frac{1}{\rho}\frac{\partial \tau^*_{x^*z^*}}{\partial z^*} \qquad \text{...(3)}$$

where $\tau^*_{x^*z^*}$ and $\tau^*_{y^*z^*}$ are the component of shear stress of the viscoelastic fluid.

$$\tau^*_{x^*z^*} = \mu\frac{\partial u^*}{\partial z^*} - \frac{\mu}{\alpha}\frac{\partial \tau^*_{x^*z^*}}{\partial t^*} \quad \text{and} \quad \tau^*_{y^*z^*} = \mu\frac{\partial v^*}{\partial z^*} - \frac{\mu}{\alpha}\frac{\partial \tau^*_{y^*z^*}}{\partial t^*}$$

where μ is the coefficient of viscosity and α is the modulus of rigidity. In the limit α tend to infinity or at the steady state, the fluid behaves like a viscous fluid without elasticity. Solving equation for $\tau^*_{x^*z^*}$ and $\partial\tau^*_{y^*z^*}$ in terms of velocity component u and v we obtain

$$\frac{\partial \tau^*_{x^*z^*}}{\partial z^*} = \frac{\partial}{\partial z^*}\left(\mu\frac{\partial u^*}{\partial z^*}\right) - \frac{1}{\alpha}\frac{\partial}{\partial z^*}\left(\mu\frac{\partial}{\partial t^*}\left(\mu\frac{\partial u^*}{\partial z^*}\right)\right)$$

and $$\frac{\partial\tau^*_{y^*z^*}}{\partial z^*}=\frac{\partial}{\partial z^*}\left(\mu\frac{\partial v^*}{\partial z^*}\right)-\frac{1}{\alpha}\frac{\partial}{\partial z^*}\left(\mu\frac{\partial}{\partial t^*}\left(\mu\frac{\partial v^*}{\partial z^*}\right)\right)$$

where the terms $\frac{1}{\alpha^2}\frac{\partial}{\partial y^*}\left(\mu\frac{\partial}{\partial t^*}\left(\mu\frac{\partial u^*}{\partial z^*}\right)\right)$ and $\frac{1}{\alpha^2}\frac{\partial}{\partial y^*}\left(\mu\frac{\partial}{\partial t^*}\left(\mu\frac{\partial v^*}{\partial z^*}\right)\right)$ which are proportional to $\frac{1}{\alpha^2}$ have been neglected. Substituting $\frac{\partial\tau^*_{x^*z^*}}{\partial z^*}$ and $\frac{\partial\tau^*_{y^*z^*}}{\partial z^*}$ in the momentum equations, we get

$$\frac{\partial u^*}{\partial t^*}+w_0\frac{\partial u^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial P^*}{\partial x^*}+\nu\frac{\partial^2 u}{\partial z^{*2}}+\frac{\sigma B_0^2}{\rho(1+m^2)}(mv^*-u^*)+g\beta(T^*-T_d)$$
$$+g\beta_c(C^*-C_d)-K_0^*\frac{\partial^3 u^*}{\partial t^*\partial z^{*2}}-\nu\frac{u^*}{K^*} \qquad \text{.... (4)}$$

$$\frac{\partial v^*}{\partial t^*}+w_0\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial P^*}{\partial y^*}+\nu\frac{\partial^2 v^*}{\partial z^{*2}}-\frac{\sigma B_0^2}{\rho(1+m^2)}(mu^*+v^*)-K_0^*\frac{\partial^3 v^*}{\partial t^*\partial z^{*2}}-\nu\frac{v^*}{K^*} \qquad \text{...(5)}$$

$$\rho C_P\left(\frac{\partial T^*}{\partial t^*}+w_0\frac{\partial T^*}{\partial z^*}\right)=\kappa\frac{\partial^2 T^*}{\partial t^{*2}}-Q_0(T^*-T_d)\frac{\partial q}{\partial z^*} \qquad \text{...(6)}$$

$$\frac{\partial C^*}{\partial t^*}+w_0\frac{\partial C^*}{\partial z^*}=D\frac{\partial^2 C^*}{\partial z^{*2}}-K_1(C^*-C_d) \qquad \text{...(7)}$$

where P^* is the modified pressure, ρ is density, B_0 is the electromagnetic induction, g is the acceleration due to gravity, β is coefficient of volume expansion, β_c is the coefficient of expansion with concentration, μ is the coefficient of viscosity, ν is kinematic viscosity, $K_0^*=\mu^2/\rho\alpha$ is the viscoelasticity, K^* is the permeability of the porous medium, C_p is the specific heat at constant pressure, κ is the thermal conductivity, q is the radiative heat flux, Q_0 is the heat absorption, D is the molecular diffusivity and K_1 is the chemical reaction parameter.

The boundary conditions are

$$\left.\begin{array}{l} u^* = v^* = 0, T^* = T_0 + \in (T_0 - T_d)\cos\omega^* t^*,\ C^* = C_0 + \in (C_0 - C_d)\cos\omega^* t^* \text{ at } z^* = 0 \\ u^* = U(t) = U_0(1 + \in \cos\omega^* t^*),\ v^* = 0,\ T^* = T_d, C^* = C_d \text{ at } z^* = d \end{array}\right\} \quad ....(8)$$

where ω^* is the frequency of oscillations and ε is very small positive constant.

Now following Cogley *et al.* [5], it is assumed that fluid is optically thin with a relative low density and radiative heat flux is given by

$$\frac{\partial q}{\partial z} = 4a^2(T^* - T_d) \quad ....(9)$$

where 'a' is the mean absorption coefficient.

Introducing the following non-dimensional quantities

$\eta = \frac{z^*}{d}, t = \omega^* t^*, u = \frac{u^*}{U_0}, v = \frac{v^*}{U_0}, \theta = \frac{T^* - T_d}{T_0 - T_d}, \phi = \frac{C^* - C_d}{C_0 - C_d}, \lambda = \frac{w_0 d}{v}$ (injection suction parameter), $G_r = \frac{vg\beta(T_0 - T_d)}{U_0 w_0^2}$ (the Grashof number), $G_m = \frac{vg\beta(C_0 - C_d)}{U_0 w_0^2}$ (modified Grashof number), $H = B_0\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number), $K = K^*/d^2$ (permeability parameter), $P_r = \mu C_p/\kappa$ (Prandtl number), $S_c = \upsilon/d$ (Schmidt number), $N = \frac{4\alpha^2 d^2}{\kappa}$ (radiation parameter), $Q = \frac{Q_0 d^2}{\mu C_p}$ (heat absorption number), $\chi = \frac{K_1 d^2}{v}$ (chemical reaction parameter), $\omega = \frac{\omega^* d^2}{v}$ (frequency of oscillation parameter) and $K_0 = K_0^* \frac{w_0^2}{v^2}$ (viscoelastic parameter).

In equations (4) to (9), we get the following non-dimensional equations

$$\omega\frac{\partial u}{\partial t} + \lambda\frac{\partial u}{\partial \eta} = \omega\frac{\partial U}{\partial t} + \frac{\partial^2 U}{\partial \eta^2} + \frac{H^2}{(1+m^2)}\{mv - (u - U)\} + G_r\theta\lambda^2 + G_m\lambda^2\phi$$

$$-\frac{\omega K_0}{\lambda^2}\frac{\partial^3 u}{\partial t\partial \eta^2} - \frac{1}{K}(U - u) \quad ...(10)$$

$$\omega\frac{\partial v}{\partial t}+\frac{\partial v}{\partial \eta}=\frac{\partial^2 v}{\partial \eta^2}-\frac{H^2}{(1+m^2)}\{m(u-U)+v\}-\frac{\omega K_0}{\lambda^2}\frac{\partial^3 v}{\partial t\partial \eta^2}-\frac{v}{K} \qquad \text{... (11)}$$

$$\omega\frac{\partial \theta}{\partial t}+\lambda\frac{\partial \theta}{\partial \eta}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial \eta^2}-\left(Q+\frac{N^2}{P_r}\right)\theta \qquad \text{....(12)}$$

$$\omega\frac{\partial \phi}{\partial t}+\lambda\frac{\partial \phi}{\partial \eta}=\frac{1}{S_c}\frac{\partial^2 \phi}{\partial \eta^2}-\chi\phi \qquad \text{.....(13)}$$

The boundary conditions in non-dimensional form become

$$\left.\begin{array}{l} u=v=0,\ \theta=(1+\in\cos t),\ \Phi=(1+\in\cos t)\ \text{at}\ \eta=0 \\ u=U(t)=(1+\in\cos t),\ v=0, \theta=0,\ \Phi=0\ \text{at}\ \eta=1 \end{array}\right\} \qquad \text{....(14)}$$

3. Method of solution

Introducing a complex velocity function of the form:

$$q(\eta,t)=u(\eta,t)+iv(\eta,t)$$

equations (10) and (11) combine to give to following single equation

$$\omega\frac{\partial q}{\partial t}+\lambda\frac{\partial q}{\partial \eta}=\omega\frac{\partial U}{\partial t}+\frac{\partial^2 q}{\partial \eta^2}-\frac{H^2}{(1+m^2)}(1+im)(q-U)-\frac{\omega K_0}{\lambda^2}\frac{\partial^3 q}{\partial t\partial \eta^2}$$

$$-\frac{1}{K}(q-U)+G_r\theta\lambda^2+G_m\lambda^2\phi \qquad \text{...(15)}$$

The boundary conditions (14) in complex notation is written as

$$\left.\begin{array}{ll} q=0,\ \theta=1+\frac{\in}{2}(e^{it}+e^{-it}),\ \Phi=1+\frac{\in}{2}(e^{it}+e^{-it})\,\text{at}\ \eta=0 \\ q=U(t)=1+\frac{\in}{2}(e^{it}+e^{-it}),\ \theta=0,\ \Phi=0 \qquad \text{at}\ \eta=1 \end{array}\right\} \qquad \text{... (16)}$$

In view of the periodic variations of the plate ($\eta = 1$) velocity and the plate ($\eta = 0$) temperature taken in the boundary conditions (16) the solution of the problem is assumed to be of the form

$$q(\eta,t) = q_0(\eta) + \frac{\in}{2}[q_1(\eta)e^{it} + q_2(\eta)e^{iit}] \quad .... (17)$$

$$\theta(\eta,t) = \theta_0(\eta) + \frac{\in}{2}[\theta_1(\eta)e^{it} + \theta_2(\eta)e^{iit}] \quad .... (18)$$

$$\phi(\eta,t) = \phi_0(\eta) + \frac{\in}{2}[\phi_1(\eta)e^{it} + \phi_2(\eta)e^{iit}] \quad ....(19)$$

Substituting equations (17), (18) and (19) into equations (12), (13) and (15) and comparing the harmonic and non-harmonic terms, we get

$$\frac{d^2q_0}{d\eta^2} - \lambda\frac{dq_0}{d\eta} - l_1^2 q_0 = -l_1^2 - G_r\lambda^2\theta_0 - G_m\lambda^2\phi_0 \quad .... (20)$$

$$r^2\frac{d^2q_1}{d\eta^2} - \lambda\frac{dq_1}{d\eta} - m_1^2 q_1 = -m_1^2 - G_r\lambda^2\theta_1 - G_m\lambda^2\phi_1 \quad .... (21)$$

$$s^2\frac{d^2q_2}{d\eta^2} - \lambda\frac{dq_2}{d\eta} - m_1^2 q_2 = -m_1^2 - G_r\lambda^2\theta_2 - G_m\lambda^2\phi_2 \quad .... (22)$$

$$\frac{d^2\theta_0}{d\eta^2} - \mathrm{P_r}\lambda\frac{d\theta_0}{d\eta} - l_1^2\theta_0 = 0 \quad .... (23)$$

$$\frac{d^2\theta_1}{d\eta^2} - \mathrm{P_r}\lambda\frac{d\theta_1}{d\eta} - m_1^2\theta_1 = 0 \quad .... (24)$$

$$\frac{d^2\theta_2}{d\eta^2} - \mathrm{P_r}\lambda\frac{d\theta_2}{d\eta} - n_1^2\theta_2 = 0 \quad .... (25)$$

$$\frac{d^2\phi_0}{d\eta^2} - S_c\lambda\frac{d\phi_0}{d\eta} - S_c\chi\phi_0 = 0 \quad .... (26)$$

$$\frac{d^2\phi_1}{d\eta^2} - S_c\lambda\frac{d\phi_1}{d\eta} - m_1^2\phi_1 = 0 \quad ....(27)$$

$$\frac{d^2\phi_2}{d\eta^2} - S_c\lambda\frac{d\phi_2}{d\eta} - n_1^2\phi_2 = 0 \quad .... (28)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{llll} q_0 = q_1 = q_2 = 0 & \theta_0 = \theta_1 = \theta_2 = 1 & \phi_0 = \phi_1 = \phi_2 = 1 & at\ \eta = 0 \\ & \text{and} & & \\ q_0 = q_1 = q_2 = 1 & \theta_0 = \theta_1 = \theta_2 = 0 & \phi_0 = \phi_1 = \phi_2 = 0 & at\ \eta = 1 \end{array}\right\} \quad ....(29)$$

The solution of the equations (20) to (28) under the boundary conditions (29) are given by

$$q_0(\eta) = B_1 \cosh A_{13}\eta + B_2 \sinh A_{14}\eta - \frac{G_r\lambda^2}{e^{A_8} - e^{A_7}}\left(\frac{e^{A_7\eta + A_8}}{A_7^2 - \lambda A_7 - l_1^2} - \frac{e^{A_8\eta + A_7}}{A_8^2 - \lambda A_8 - l_1^2}\right)$$
$$- \frac{G_m\lambda^2}{e^{A_2} - e^{A_1}}\left(\frac{e^{A_1\eta + A_2}}{A_1^2 - \lambda A_1 - l_1^2} - \frac{e^{A_2\eta + A_1}}{A_2^2 - \lambda A_2 - l_1^2}\right) + \frac{1}{Kl_1^2} \quad ...(30)$$

$$q_1(\eta) = B_3 \cosh A_{15} + B_4 \sinh A_{16} + \frac{K^{-1} + i\omega}{m_1^2} - \frac{G_r\lambda^2}{e^{A_{10}} - e^{A_9}}\left(\frac{e^{A_9\eta + A_{10}}}{r^2 A_9^2 - \lambda A_9 - m_1^2}\right.$$
$$\left. - \frac{e^{A_{10}\eta + A_9}}{r^2 A_{10}^2 - \lambda A_{10} - m_1^2}\right) - \frac{G_m\lambda^2}{e^{A_4} - e^{A_3}}\left(\frac{e^{A_3\eta + A_4}}{r^2 A_3^2 - \lambda A_3 - m_1^2} - \frac{e^{A_4\eta + A_3}}{r^2 A_4^2 - \lambda A_4 - m_1^2}\right) ...(31)$$

$$q_2(\eta) = B_5 \cosh A_{17} + B_6 \sinh A_{18} + \frac{K^{-1} - i\omega}{n_1^2} - \frac{G_r\lambda^2}{e^{A_{12}} - e^{A_{11}}}\left(\frac{e^{A_{11}\eta + A_{12}}}{s^2 A_{11}^2 - \lambda A_{11} - n_1^2}\right.$$
$$\left. - \frac{e^{A_{12}\eta + A_{11}}}{s^2 A_{12}^2 - \lambda A_{12} - n_1^2}\right) - \frac{G_m\lambda^2}{e^{A_6} - e^{A_5}}\left(\frac{e^{A_5\eta + A_6}}{sr^2 A_5^2 - \lambda A_5 - n_1^2} - \frac{e^{A_6\eta + A_5}}{sr^2 A_6^2 - \lambda A_6 - n_1^2}\right) ...(32)$$

$$\theta_0(\eta) = \frac{e^{A_7\eta + A_8} - e^{A_8\eta + A_7}}{e^{A_8} - e^{A_7}} \quad ...\ (33)$$

$$\theta_1(\eta) = \frac{e^{A_9\eta + A_{10}} - e^{A_{10}\eta + A_9}}{e^{A_{10}} - e^{A_9}} \quad ...(34)$$

$$\theta_2(\eta) = \frac{e^{A_{11}\eta + A_{12}} - e^{A_{12}\eta + A_{11}}}{e^{A_{12}} - e^{A_{11}}} \quad ...\ (35)$$

$$\phi_0(\eta) = \frac{e^{A_1\eta + A_2} - e^{A_2\eta + A_1}}{e^{A_2} - e^{A_1}} \quad ...\ (36)$$

$$\phi_1(\eta) = \frac{e^{A_3\eta + A_4} - e^{A_4\eta + A_3}}{e^{A_4} - e^{A_3}} \quad ...\ (37)$$

$$\phi_2(\eta) = \frac{e^{A_5\eta + A_6} - e^{A_6\eta + A_5}}{e^{A_6} - e^{A_5}} \quad ...\ (38)$$

All constants used above are listed in appendix.

4. Results and Discussion

We write the resultant velocities of the steady and unsteady flow as

$$u_0(\eta)+iv_0(\eta)=q_0(\eta) \text{ and } u_1(\eta)+iv_1(\eta)=q_1(\eta)e^{it}+q_2e^{-it} \quad \text{... (39)}$$

The solution (30) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and the phase difference due to the primary and secondary velocities for the steady flow are given by

$$R_0=\sqrt{u_0^2+v_0^2} \text{ and } \psi_0=\tan^{-1}(v_0/u_0) \quad \text{...(40)}$$

Solutions (31) and (32) together give the unsteady part of the flow. The unsteady primary and secondary velocity components, $u_1(\eta)$ and $v_1(\eta)$ respectively, for the fluctuating flow can be obtained as:

$$u_1(\eta)=\{\mathrm{Re}\,q_1(\eta)+\mathrm{Re}\,q_2(\eta)\}\cos t-\{\mathrm{Im}\,q_1(\eta)-\mathrm{Im}\,q_2(\eta)\}\sin t \quad \text{... (41)}$$

$$v_1(\eta)=\{\mathrm{Re}\,q_1(\eta)-\mathrm{Re}\,q_2(\eta)\}\sin t+\{\mathrm{Im}\,q_1(\eta)+\mathrm{Im}\,q_2(\eta)\}\cos t \quad \text{... (42)}$$

The resultant velocity and the phase difference of the unsteady flow are given by

$$R_1=\sqrt{u_1^2+v_1^2} \text{ and } \psi_1=\tan^{-1}(v_1/u_1) \quad \text{... (43)}$$

The amplitude of resultant velocities R_0 and R_1 for steady and unsteady part respectively is presented in figures (2-5). These figures depict that with increasing value of suction parameter λ, the Grashof number G_r, the modified Grashof number G_m, Hartmann number, and chemical reaction parameter χ result in acceleration while all other parameter decelerate the fluid motion.

Physically if (G_r >0), it means cooling of the plate (or heating of the fluid) i.e. in free convection current which transfer heat away from the plate into the boundary layer region, therefore increasing value of Grashof number G_r accelerates the flow also increasing value of chemical reaction parameter χ represent more heat absorption hence accelerate the fluid motion. The increasing value radiation parameter N implies less interaction of radiation with momentum boundary layer hence flow get decelerated. The S_c Schmidt number characterises the relative effectiveness of momentum and mass transport by diffusion, higher value of S_c Schmidt number lead to species diffusivity rate exceed the momentum diffusivity will diminish the concentration in the boundary layer. The values 0.7 and 7, represent air and water respectively, the fluids with higher values of Prandtl number P_r possess higher viscosities and lower thermal conductivity such fluid flow slow than lower Prandtl number P_r, as a result fluid velocity will be decreased with increasing Prandtl number P_r. Further as the viscoelastic K_0 parameter increases, the hydrodynamics boundary layer adheres strongly to the surface which in turn retards the fluid motion.

The phase difference ψ_0 for the steady flow is presented in figures (6-7). These figures illustrate that ψ_0 diminished with increase of the suction parameter λ, Grashof number G_r, permeability parameter K, the Schmidt number S_c and heat absorption number Q while enhanced with the increase of all other parameters.

For unsteady part, the phase difference ψ_1 is presented in the figures (8-9). It is observed from these figures that ψ_1 increase with the increase the suction parameter λ., Grashof number G_r, the Schmidt number S_c and decreases with the increase of all other parameters.

The amplitude and the phase difference of shear stress at the stationary plate ($\eta = 0$) for a steady flow can be obtained as

$$\tau_{0r} = \sqrt{\tau_{0x}^2 + \tau_{0y}^2} \text{ and } \psi_{0r} = \tan^{-1}\left(\frac{\tau_{0y}^2}{\tau_{0x}^2}\right) \quad \text{... (44)}$$

where $\tau_{1x} + i\tau_{1y} = \left(\frac{\partial u_1}{\partial y}\right)_{\eta=0}$

Here τ_{0x} and τ_{0y} are, respectively, the shear stresses at the stationary plate due to the primary and secondary velocity components. The numerical value of the amplitude and the phase difference of the shear stresses ψ_{0r} at the stationary plate ($\eta = 0$) for the steady flow are presented in Table 1.

λ	Pr	Gr	Gm	K	H	N	Sc	χ	Q	m	τ_{0r}	ψ_{0r}
1	0.7	5	5	1	2	2	0.22	0.2	0.1	0.5	13.753	-0.7289
.5	0.7	5	5	1	2	2	0.22	0.2	0.1	0.5	4.2112	-0.1239
1	07	5	5	1	2	2	0.22	0.2	0.1	0.5	14.731	-0.0698
1	0.7	10	5	1	2	2	0.22	0.2	0.1	0.5	21.384	-1.0322
1	0.7	5	10	1	2	2	0.22	0.2	0.1	0.5	18.19	-0.5843
1	0.7	5	5	2	2	2	0.22	0.2	0.1	0.5	11.325	-0.7406
1	0.7	5	5	1	4	2	0.22	0.2	0.1	0.5	15.221	0.044
1	0.7	5	5	1	2	4	0.22	0.2	0.1	0.5	9.2333	-0.0383
1	0.7	5	5	1	2	2	0.66	0.2	0.1	0.5	13.568	-0.7313
1	0.7	10	5	1	2	2	0.22	02	0.1	0.5	14.291	-0.7282
1	0.7	5	5	1	2	2	0.22	0.2	0.5	0.5	12.996	-0.7520
1	0.7	5	5	1	2	2	0.22	0.2	0.1	01	9.9575	-0.56560

Table 1. Values for steady shear stress τ_{0r} and phase angle ψ_{0r} of steady shear stress.

It is observed from the table 1 that, the increase of permeability parameter K, radiation parameter N, Schmidt number S_c, heat absorption number Q and the Hall current parameter m leads to decrease while all other parameters lead to the increase the value of τ_{0r} . The values of ψ_{0r} , the steady phase difference decreases with the increase of Grashof numberG_r, permeability parameter K, the Schmidt number S_c and heat absorption number Q while all other parameter lead to decrease of it.

For the unsteady part of flow, the amplitude and the phase difference of the shear stresses at the stationary plate ($\eta = 0$) can be obtained as

$$\tau_{1x} + i\tau_{1y} = \left(\frac{\partial u_1}{\partial y}\right)_{\eta=0} + i\left(\frac{\partial v_1}{\partial y}\right)_{\eta=0} \qquad \text{... (45)}$$

which gives $\tau_{1r} = \sqrt{\tau_{1x}^2 + \tau_{1y}^2}$ and $\psi_{1r} = \tan^{-1}\left(\frac{\tau_{1y}}{\tau_{1x}}\right)$ ... (46)

here τ_{1x} and τ_{1y} are, respectively, the shear stresses at the stationary plate due to the primary and secondary velocity components. The numerical value of the amplitude and the phase difference of the shear stresses ψ_{1r} at the stationary plate (η = 0) for the unsteady flow are presented in Table 2&3.

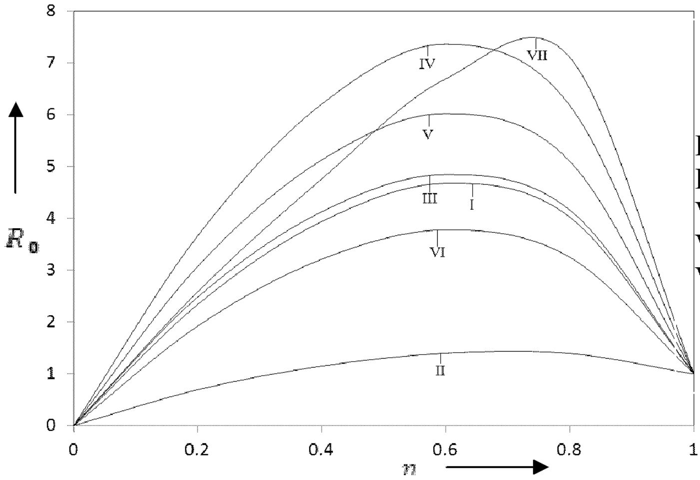
It is observed from table 2 that the unsteady shear stress τ_{1r} increases with increasing frequency of oscillations for all parameter. Table 3 depict that phase lag increases with the increasing frequency of oscillation for all parameter.

λ	P_r	G_r	G_m	K	H	N	S_c	χ	Q	m	K_0	ω=10	ω=20
1	0.7	5	5	1	2	2	0.22	0.2	0.1	0.5	.05	1.5604	32.799
.5	0.7	5	5	1	2	2	0.22	0.2	0.1	0.5	.05	0.4784	1.1684
1	07	5	5	1	2	2	0.22	0.2	0.1	0.5	.05	6.9614	37.164
1	0.7	10	5	1	2	2	0.22	0.2	0.1	0.5	.05	9.4406	27.374
1	0.7	5	10	1	2	2	0.22	0.2	0.1	0.5	.05	2.2887	37.726
1	0.7	5	5	2	2	2	0.22	0.2	0.1	0.5	.05	1.4113	36.989
1	0.7	5	5	1	4	2	0.22	0.2	0.1	0.5	.05	8.1401	3.8907
1	0.7	5	5	1	2	4	0.22	0.2	0.1	0.5	.05	4.4118	36.137
1	0.7	5	5	1	2	2	0.66	0.2	0.1	0.5	.05	9.3261	19.814
1	0.7	10	5	1	2	2	0.22	02	0.1	0.5	.05	1.7678	32.368
.1	0.7	5	5	1	2	2	0.22	0.2	0.5	0.5	.05	1.4586	32.885
1	0.7	5	5	1	2	2	0.22	0.2	0.1	1	.05	1.7519	48.452
1	0.7	5	5	1	2	2	0.22	0.2	0.1	.5	0.2	2.6705	0.2669

Table 2. Values for unsteady shear stress τ_{1r} at $t = \pi/4$

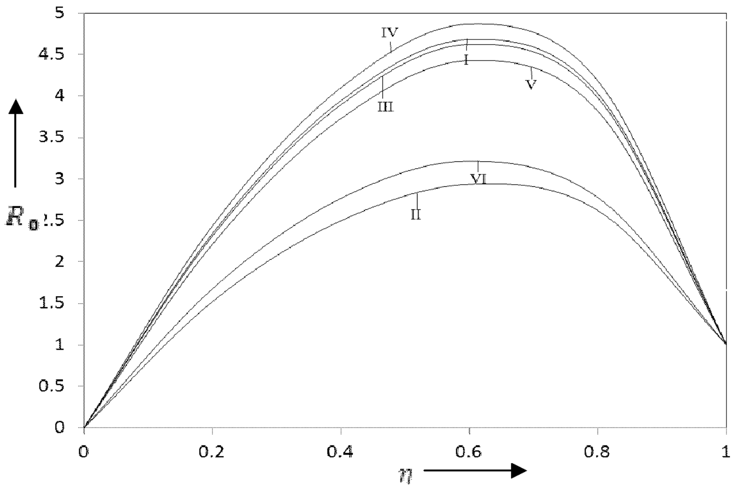
λ	P_r	G_r	G_m	K	H	N	S_c	χ	Q	m	K_0	ω=10	ω=20
1	0.7	5	5	1	2	2	0.22	0.2	0.1	0.5	.05	1.0223	-0.2554
.5	0.7	5	5	1	2	2	0.22	0.2	0.1	0.5	.05	0.7604	-0.0713
1	07	5	5	1	2	2	0.22	0.2	0.1	0.5	.05	-0.0056	-0.0901
1	0.7	10	5	1	2	2	0.22	0.2	0.1	0.5	.05	0.2585	-0.3990
1	0.7	5	10	1	2	2	0.22	0.2	0.1	0.5	.05	0.3441	-0.33151
1	0.7	5	5	2	2	2	0.22	0.2	0.1	0.5	.05	1.3338	-0.0544
1	0.7	5	5	1	4	2	0.22	0.2	0.1	0.5	.05	-0.1616	-0.1404
1	0.7	5	5	1	2	4	0.22	0.2	0.1	0.5	.05	-0.0916	-0.1607
1	0.7	5	5	1	2	2	0.66	0.2	0.1	0.5	.05	0.7501	-0.3910
1	0.7	10	5	1	2	2	0.22	02	0.1	0.5	.05	0.8251	-0.2241
.1	0.7	5	5	1	2	2	0.22	0.2	0.5	0.5	.05	1.1225	-0.2237
1	0.7	5	5	1	2	2	0.22	0.2	0.1	01	.05	1.4046	0.4205
1	0.7	5	5	1	2	2	0.22	0.2	0.1	0.5	0.2	0.1971	-0.5522

Table 3. Values for phase angle ψ_{1r} of unsteady shear stress at $t = \pi/4$.



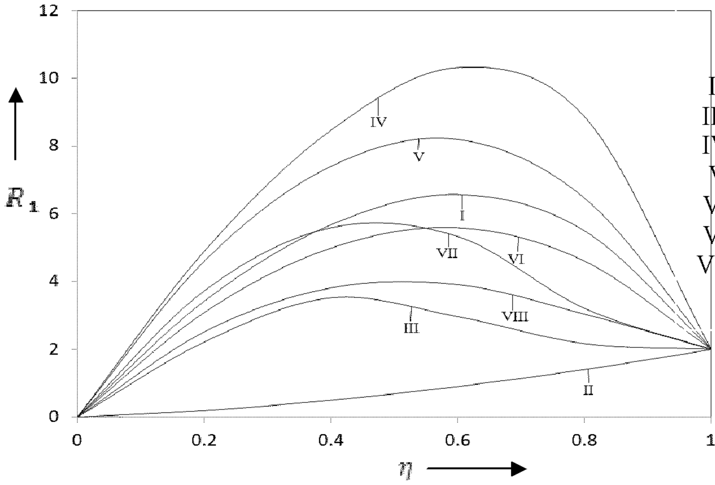
	λ	Pr	Gr	Gm	K	H
I	1	.7	5	5	1	2
II	.5	.7	5	5	1	2
III	1	.7	5	5	1	2
IV	1	.7	10	5	1	2
V	1	.7	5	10	1	2
VI	1	.7	5	5	2	2
VII	1	.7	5	5	1	4

Fig. 2: Resultant velocity R_0 due to u_0 and v_0 for N =2, S_c =.22, χ =.2 , Q =.1 and m =.5.



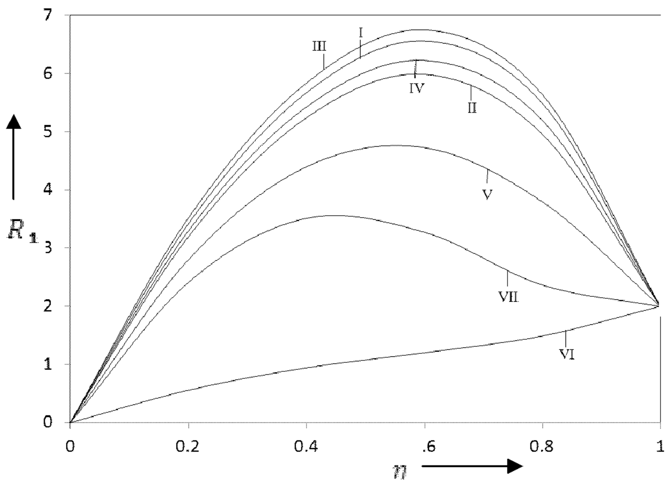
	λ	N	Sc	χ	Q	m
I	1	2	.22	.2	.1	.5
II	1	4	.22	.2	.1	.5
III	1	2	.66	.2	.1	.5
IV	1	2	.22	2	.1	.5
V	1	2	.22	.2	.3	.5
VI	1	2	.22	.2	.1	1

Fig. 3:Resultant velocity R_0 due to u_0 and v_0 for P_r = 0.7, G_r=5, G_m = 5, and K=1, H =2.



	λ	Pr	Gr	Gm	K	H	N
I	1	.7	5	5	1	2	2
II	.5	.7	5	5	1	2	2
III	1	.7	5	5	1	2	2
IV	1	.7	10	5	1	2	2
V	1	.7	5	10	1	2	2
VI	1	.7	5	5	2	2	2
VII	1	.7	5	5	1	4	2
VIII	1	.7	5	5	1	2	4

Fig. 4: Resultant Velocity R_1 due to u_1 and v_1 for S_c =.22, χ =.2 Q =.1, m =.5, K_0 = .05 and ω = 5



	λ	S_c	χ	Q	m	k_0	ω
I	1	.22	.2	.1	.5	.05	5
II	1	.66	.2	.1	.5	.05	5
III	1	.22	2	.1	.5	.05	5
IV	1	.22	.2	.3	.5	.05	5
V	1	.22	.2	.1	1	.05	5
VI	1	.22	.2	.1	.5	0.2	5
VII	1	.22	.2	.1	.5	.05	10

Fig. 5: Resultant Velocity R_1 due to u_1 and v_1 for for P_r = 0.7, G_r=5, G_m = 5, K=1, H =2 and N =2.

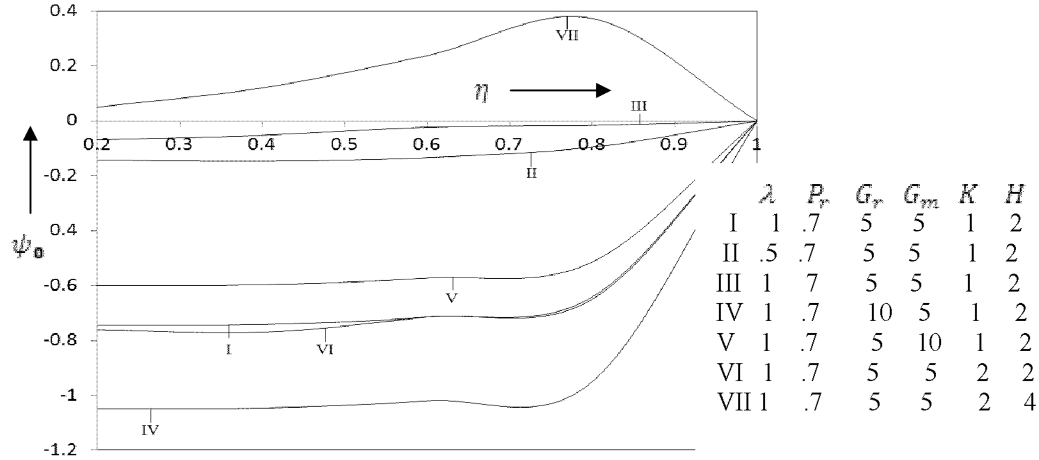


Fig. 6: Phase angle ψ_0 due to u_0 and v_0 for N = 2, S_c =.22, χ =.2 Q =.1 and m =.5.

$N = 2$,$S_c = .22$, $\chi = .2$ $Q = .1$ and $m = .5$.

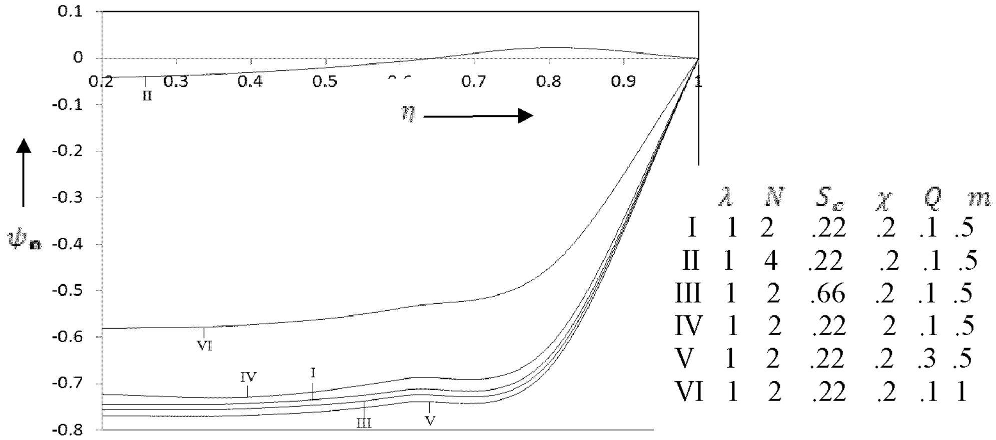
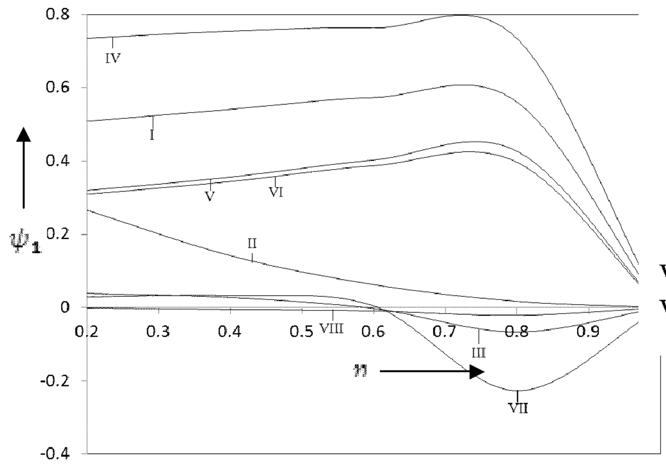
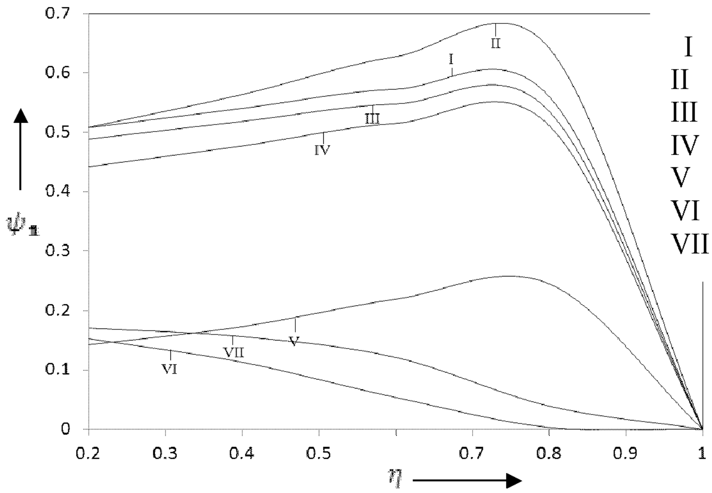


Fig. 7: Phase angle ψ_0 due to u_0 and v_0 for for P_r = 0.7, G_r=5, G_m = 5, K=1, and N =2.



	λ	P_r	G_r	G_m	K	H	N
I	1	.7	5	5	1	2	2
II	.5	.7	5	5	1	2	2
III	1	7	5	5	1	2	2
IV	1	.7	10	5	1	2	2
V	1	.7	5	10	1	2	2
VI	1	.7	5	5	2	2	2
VII	1	.7	5	5	1	4	2
VIII	1	.7	5	5	1	2	4

Fig. 8: Phase angle ψ_1 due to u_1 and v_1 for S_c =.22, χ =.2 Q =.1, m =.5, K_0 = .05 and ω = 5.



	λ	S_c	χ	Q	m	k_0	ω
I	1	.22	.2	.1	.5	.05	5
II	1	.66	.2	.1	.5	.05	5
III	1	.22	2	.1	.5	.05	5
IV	1	.22	.2	.3	.5	.05	5
V	1	.22	.2	.1	1	.05	5
VI	1	.22	.2	.1	.5	0.2	5
VII	1	.22	.2	.1	.5	.05	10

Fig. 9: Phase angle ψ_1 due to u_1 and v_1 for for P_r = 0.7, G_r=5, G_m = 5, K=1, H =2 and N =2.

5. Conclusion

In the present paper the Hall effect on chemically reacting and radiating MHD oscillatory viscoelastic fluid through porous channel is studied. The analytic solution of the governing equation under the prescribed boundary conditions is obtained. The conclusion of the study is as follows:

1. The amplitude of the steady and unsteady velocity increases significantly with the increase of suction parameter, the Grashof number, the modified Grashof number, Hartmann number and the chemical reaction parameter where as they decrease with the Hall current and radiation parameter.
2. The increase in viscoelastic parameter retards the fluid motion.
3. The amplitude of steady and unsteady shear stress at the stationary plate is enhanced with the Hall current parameter.

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Appendix

$$l_1^2 = \left(K^{-1} + \frac{H^2}{(1+m^2)}(1+im)\right), \qquad m_1^2 = \left(K^{-1} + i\omega + \frac{H^2}{(1+m^2)}(1+im)\right),$$

$$n_1^2 = \left(K^{-1} - i\omega + \frac{H^2}{(1+m^2)}(1+im)\right), \qquad r^2 = \left(1 + \frac{i\alpha\omega}{\lambda^2}\right),$$

$$s^2 = \left(1 + \frac{i\alpha\omega}{\lambda^2}\right), \qquad l_2^2 = (QP_r + N^2),$$

$$m_2^2 = \{(Q + i\omega)P_r + N^2), \qquad n_2^2 = \{(Q - i\omega)P_r + N^2),$$

$$m_3^2 = (\chi + i\omega)S_c, \qquad n_3^2 = (\chi - i\omega)S_c,$$

$$A_1 = \frac{\lambda S_c + \sqrt{(\lambda S_c)^2 + 4\chi S_c}}{2}, \qquad A_2 = \frac{\lambda S_c - \sqrt{(\lambda S_c)^2 + 4\chi S_c}}{2},$$

$$A_3 = \frac{\lambda S_c + \sqrt{(\lambda S_c)^2 + 4(\chi + i\omega) S_c}}{2}, \qquad A_4 = \frac{\lambda S_c - \sqrt{(\lambda S_c)^2 + 4(\chi + i\omega) S_c}}{2},$$

$$A_5 = \frac{\lambda S_c + \sqrt{(\lambda S_c)^2 + 4(\chi - i\omega) S_c}}{2}, \qquad A_6 = \frac{\lambda S_c - \sqrt{(\lambda S_c)^2 + 4(\chi - i\omega) S_c}}{2},$$

$$A_7 = \frac{\lambda P_r + \sqrt{(\lambda P_r)^2 + 4 l_2^2}}{2}, \qquad A_8 = \frac{\lambda P_r - \sqrt{(\lambda P_r)^2 + 4 l_2^2}}{2},$$

$$A_9 = \frac{\lambda P_r + \sqrt{(\lambda P_r)^2 + 4 m_2^2}}{2}, \qquad A_{10} = \frac{\lambda P_r - \sqrt{(\lambda P_r)^2 + 4 m_2^2}}{2},$$

$$A_{11} = \frac{\lambda P_r + \sqrt{(\lambda P_r)^2 + 4 n_2^2}}{2}, \qquad A_{12} = \frac{\lambda P_r - \sqrt{(\lambda P_r)^2 + 4 n_2^2}}{2},$$

$$A_{13} = \frac{\lambda + \sqrt{\lambda^2 + 4 l_1^2}}{2}, \qquad A_{14} = \frac{\lambda - \sqrt{\lambda^2 + 4 l_1^2}}{2},$$

$$A_{15} = \frac{\lambda + \sqrt{\lambda^2 + 4 m_1^2}}{2}, \qquad A_{16} = \frac{\lambda - \sqrt{\lambda^2 + 4 m_1^2}}{2},$$

$$A_{17} = \frac{\lambda + \sqrt{\lambda^2 + 4 n_1^2}}{2}, \qquad A_{18} = \frac{\lambda - \sqrt{\lambda^2 + 4 n_1^2}}{2},$$

$$B_1 = -1 + \frac{G_r \lambda^2}{e^{A_8} - e^{A_7}} \left(\frac{e^{A_8}}{A_7^2 - \lambda A_7 - l_1^2} - \frac{e^{A_7}}{A_8^2 - \lambda A_8 - l_1^2} \right)$$
$$+ \frac{G_m \lambda^2}{e^{A_2} - e^{A_1}} \left(\frac{e^{A_2}}{A_1^2 - \lambda A_1 - l_1^2} - \frac{e^{A_1}}{A_2^2 - \lambda A_2 - l_1^2} \right)$$

$$B_2 = \frac{1}{\sinh A_{14}}\Big[-B_1 \cosh A_{13} + \frac{G_r \lambda^2}{e^{A_8} - e^{A_7}} \left(\frac{e^{A_7 + A_8}}{A_7^2 - \lambda A_7 - l_1^2} - \frac{e^{A_7 + A_8}}{A_8^2 - \lambda A_8 - l_1^2} \right)$$

$$+ \frac{G_m \lambda^2}{e^{A_2} - e^{A_1}} \left(\frac{e^{A_1 + A_2}}{A_1^2 - \lambda A_1 - l_1^2} - \frac{e^{A_1 + A_2}}{A_2^2 - \lambda A_2 - l_1^2} \right)$$

$$B_3 = -1 + \frac{G_r \lambda^2}{e^{A_{10}} - e^{A_9}} \left(\frac{e^{A_{10}}}{r^2 A_9^2 - \lambda A_9 - m_1^2} - \frac{e^{A_9}}{r^2 A_{10}^2 - \lambda A_{10} - m_1^2} \right)$$

$$+ \frac{G_m \lambda^2}{e^{A_4} - e^{A_3}} \left(\frac{e^{A_4}}{r^2 A_3^2 - \lambda A_3 - m_1^2} - \frac{e^{A_3}}{r^2 A_4^2 - \lambda A_4 - m_1^2} \right)$$

$$B_4 = \frac{1}{\sinh A_{16}} \Bigg[1 - \frac{K^{-1} + i\omega}{m_1^2} - B_3 \cosh A_{15} + \frac{G_r \lambda^2}{e^{A_{10}} - e^{A_9}} \left(\frac{e^{A_9 + A_{10}}}{A_9^2 - \lambda A_9 - m_1^2} - \frac{e^{A_9 + A_{10}}}{A_{10}^2 - \lambda A_{10} - m_1^2} \right)$$

$$+ \frac{G_m \lambda^2}{e^{A_4} - e^{A_3}} \left(\frac{e^{A_3 + A_4}}{r^2 A_3^2 - \lambda A_3 - m_1^2} - \frac{e^{A_3 + A_4}}{r^2 A_4^2 - \lambda A_4 - m_1^2} \right) \Bigg]$$

$$B_5 = -1 + \frac{G_r \lambda^2}{e^{A_{12}} - e^{A_{11}}} \left(\frac{e^{A_{12}}}{s^2 A_{11}^2 - \lambda A_{11} - n_1^2} - \frac{e^{A_{11}}}{s^2 A_{12}^2 - \lambda A_{12} - n_1^2} \right)$$

$$+ \frac{G_m \lambda^2}{e^{A_6} - e^{A_5}} \left(\frac{e^{A_6}}{s^2 A_5^2 - \lambda A_5 - n_1^2} - \frac{e^{A_5}}{s^2 A_6^2 - \lambda A_6 - n_1^2} \right)$$

$$B_6 = \frac{1}{\sinh A_{18}} \Bigg[1 - \frac{K^{-1} - i\omega}{n_1^2} - B_5 \cosh A_{17} + \frac{G_r \lambda^2}{e^{A_{12}} - e^{A_{11}}} \left(\frac{e^{A_{11} + A_{12}}}{s^2 A_{11}^2 - \lambda A_{11} - n_1^2} - \frac{e^{A_{11} + A_{12}}}{s^2 A_{12}^2 - \lambda A_{12} - n_1^2} \right)$$

$$+ \frac{G_m \lambda^2}{e^{A_6} - e^{A_5}} \left(\frac{e^{A_5 + A_6}}{s r^2 A_5^2 - \lambda A_5 - n_1^2} - \frac{e^{A_5 + A_6}}{s^2 A_6^2 - \lambda A_6 - n_1^2} \right)$$

References

[1] Alpher, R. A. (1961). Heat transfer in magnetohydrodynamic flow between parallel plates, *Int. J. Heat and Mass Transfer*. **3(2),** 108-112.

[2] Attia, H.A. and Kotb, N.A. (1996). MHD flow between two parallel plates with heat transfer, *Acta Mechanica* **117**(1-4), 215-220.

[3] Attia, H.A., (2008). Effect of Hall current on transient hydro magnetic couette- poiseuille flow of viscoelastic fluid with heat transfer, *App. Mathematical modeling* **32**, 375-388.

[4] Chamkha, A.J. (2003). MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction, *Int. Comm. Heat and Mass Transfer* , **30**(3), 413-422.

[5] Cogley, A.C.L., Vinvent, W.G. and Giles, E.S. (1968). Differential approximation for radiative transfer in Non-linear equations-grey gas near equilibrium, *American Institute of Aeronautics and Astronautics* **6**, 551-553.

[6] Hartnett, J.P. (1992). Viscoelastic fluid: A new challenge in heat transfer, *ASME Trans*. **114**.

[7] Hayat, T., Ahmed, N., Sajid, M. and Asghar, S. (2007). On the MHD flow of a second grade fluid in a porous channel**,** *Comput. Math Appl*. **54**, 407-414.

[8] Hayat, T., Sajjad, R., Abbas, Z., Sajid M and Hendi, A.A. (2011). Radiation effects on MHD flow of Maxwell fluid in a channel with porous medium, *Int. J. Heat Mass Transfer* **54**, 854-862.

[9] Khan, K., Abel, M.S. and Sonth, R.M. (2003). Visco-elastic MHD flow, heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work, *Heat Mass Transfer* **40**, 47-57.

[10] Olajuwon, B.I. (2011). Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion, *Int. Commun. Heat Mass Transfer* **38**(3), 377-382.

[11] Pal, D. (2011). Combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching permeable surface, *Commun Nonlin. Sci. Numer Simulat*. **16**, 1890-1904.

[12] Prakash, J., Sivaraj, R. and Kumar, B.R. (2011). Influence of chemical reaction on unsteady MHD mixed convective flow over a moving vertical porous plate, *Int. J. Fluid Mech,* **3**, 1-14.

[13] Sadeghy, K. and Sharifi, M. (2004). Local similarity solution for the flow of a second grade viscoelastic fluid above a moving plate, *Int. J. Non-Linear Mech.* **39**, 1265-1273.

[14] Singh, K.D. and Kumar, R. (2009). Combine effect of Hall current and rotation on free convection MHD flow in porous channel, Indian journal of pure and applied physics 47, 617-623.

[15] Skelland, A.H.P. (1976). Non-Newtonian flow and heat transfer, *John Wiley Sons New York*.

[16] Tani, I. (1962). Steady motion of conducting fluids in channels under transverse magnetic fields with consideration of Hall effect, J. of Aerospace Sci. **29**, 287-296.