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ANALYSIS OF TWO-LAYERED MODEL OF BLOOD FLOW THROUGH COMPOSITE STENOSED ARTERY IN POROUS MEDIUM UNDER THE EFFECT OF MAGNETIC FIELD

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Abstract: The present theoretical mathematical model is the representation of two-layered model of blood flow through stenotic tube in porous medium under the effect of magnetic field. In this mathematical model, the blood is considered as Newtonian fluid of variable viscosity in the central region and plasma fluid which is considered as Newtonian fluid of constant viscosity in the peripheral region of the stenotic tube. The governing equations representing the flow in central and peripheral layer are solved for the velocities of fluid, total flow rate, pressure gradient and shear rate by using Frobenius method. The expressions of pressure gradient and shear stress are discussed graphically for various flow parameters. This work may enhance to work upon the strength of magnetic field up to which we can control the blood flow in hypertensive patients and those who have blockage in their arteries.

Keywords: Stenosed vessel, Newtonian fluid, Peripheral layer, magnetic field, porous medium, Frobenius Method, axial velocity; flow rate, shear stress.

2010 Mathematics Subject Classification : Primary : 92B99; Secondary : 76599, 76Z05

1. Introduction

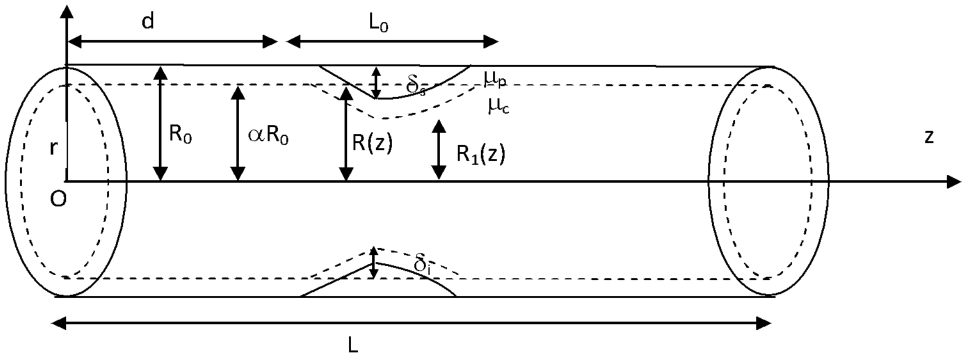
The study of blood is an object of scientific research for more than 100 years. The malfunctioning of arteries due the development of stenosis along the walls of tube is one of the serious problems related to circulatory disorders. Many researchers have paid attention upon flow characteristics of blood flow through atherosclerotic tube. The mathematical diagnosis of this problem has gone through many changes and modifications to account for new facts and unturned evidences. The actual reason for the development of this abnormal growth along the walls of artery is not clear but many researchers have pointed that the cause of this problem is transport of low density lipoproteins (LDL) molecules to walls of artery, which leads to formation of plaques and restricts the blood flow. Since the wall of artery is a porous connective tissue and deposition of LDL causes intimal thickening which makes it more stiffened and obstructs the natural flow of blood. A brief account of some recent and important contributions towards this field of research is presented here. Suri and Suri [16] had studied the effects of static transverse magnetic field on the stenosed bifurcated model of artery. They have observed that application of magnetic field reduces the strength of stenosis at the apex of bifurcation, shear stress and increases the velocity of blood flow. Haldar and Ghosh [5] discussed the effects of magnetic field on the blood flow with variable viscosity through stenosed tube and obtained analytic expressions for velocity, flow rate and shear stress and were discussed graphically. Haldar and Andersson [4] studied two-layered model of blood flow through stenosed arteries under the effect of magnetic field. In this model the central layer is represented by Casson fluid flow. Sanyal and Maiti [11] studied the effects of magnetic field on pulsatile flow of blood through constricted artery with variable viscosity and thes expressions of axial velocity and pressure gradient were discussed numerically. Dash and Mehta [3] examined the flow characteristics of Casson fluid flow in a tube filled with a homogeneous porous media. They solved momentum equation with Brinkman model in order to obtain analytical expressions of shear stress distribution, flow rate, frictional resistance and the results were also discussed graphically.

Chakravarty et al. [2] have taken two-layered model of blood flow in tapered flexible stenosed artery. However, the central layer is represented by Casson fluid and peripheral layer, free from cells, is a form of Newtonian fluid. The unsteady flow, which is subjected to pulsatile pressure gradient, is discussed using finite difference scheme. Ponalagusamy [9] have taken two-layered model of blood flow with variable thickness of peripheral layer and obtained expressions of slip velocity, core viscosity and thickness of peripheral layer. Rathod and Tanveer [10] have analyzed the pulsatile flow of couple stress blood through simple tube under the effect of magnetic field and body acceleration. They have determined expressions of flow rate, velocity, fluid acceleration and shear stress by using Laplace and finite Hankel transform. They have found that velocity of fluid increases with increase in body acceleration and permeability constant and decreases with increase in magnetic field. Joshi et al. [6] have investigated the two-layered model of blood flow through composite stenosed artery and explained the results of resistance to flow and wall shear stress graphically. Varshney et al. [17] have studied the effects of magnetic field on power law model of blood flow through multiple overlapping stenosed arteries. They have observed that magnetic field affects the various fluid properties like blood velocity, flow resistance, fluid acceleration and wall shear stress. This study is helpful in determining the various physiological factors such as back flow and low shear stress which are caused by high strength magnetic field. The governing equations are solved by making use of finite difference technique. Shah [12] has proposed a mathematical model for discussing the effects of magnetic field on power law fluid in stenosed artery. Musad and Khan [8] have discussed the effects of wall shear stress on the blood flow through stenosed region of two- layered model. Srivastava et al. [15] have presented a two-layered model of blood flow through an overlapping stenosis. They have taken the fluid as particle –fluid suspension in the central layer of tube and have obtained expressions for impedence, wall shear stress, shear stress at the peak of stenosis and critical height of stenosis. Singh and Rathee [14] have analysed the two-dimensional blood flow through stenosed artery due to LDL effect in the presence of magnetic field. Mekheimier et al. [7] have presented the mathematical model of blood flow through an elastic artery with overlapping stenosis under the effect of induced magnetic field and obtained the expressions for stream function, magnetic force function, axial velocity, axial induced magnetic field and current density analytically. Sharma et al. [13] discussed the role of heat transfer in blood flow through stenotic artery and numerically investigated the Navier-Stokes equations and energy equations using finite difference scheme. Bali and Awasthi [1] analysed the effect of magnetic field, height of stenosis, parameter determining the shape of profile on velocity, volumetric flow rate and shear stress through multistenosed artery.

In this paper, we consider the two-layered model of blood flow through composite stenosed blood vessel. The blood flowing in central layer is considered to be Newtonian fluid with variable viscosity. The viscosity of blood is varying according to Einstein relation. The periphery region of the vessel comprises of plasma layer whose flow is considered as Newtonian and of constant viscosity. The aim of our investigation is to study the effects of externally applied magnetic field on two-layered model of blood flow in composite stenosed vessel through porous medium. This theoretical study can model the real situation of a stenotic artery because the consideration of porous medium in blood flow through tissue is more appropriate, as it is a collection of dispersed cells and this makes the better understanding of this frequently occurring disease like atherosclerosis.

2. Mathematical Model

We consider steady, incompressible and fully developed flow of blood through two-layered model of composite stenosed artery. The blood in central layer of blood vessel is a suspension of erythrocytes and is considered as Newtonian fluid with variable viscosity which varies according to Einstein relation. The peripheral layer is filled with plasma fluid and is considered as Newtonian fluid of constant viscosity. The geometry of composite stenosed artery is



$$R_1(z)=\begin{cases}\alpha R_0-\dfrac{2\delta_i}{L_0}(z-d), & d\le z\le d+\dfrac{L_0}{2}\\ \alpha R_0-\dfrac{\delta_i}{2}\left\{1+\cos\dfrac{2\pi}{L_0}\left(z-d-\dfrac{L_0}{2}\right)\right\}, & d+\dfrac{L_0}{2}\le z\le d+L_0\\ \alpha R_0, & otherwise\end{cases} \quad \text{... (1)}$$

$$R(z)=\begin{cases}R_0-\dfrac{2\delta_s}{L_0}(z-d), & d\le z\le d+\dfrac{L_0}{2}\\ R_0-\dfrac{\delta_s}{2}\left\{1+\cos\dfrac{2\pi}{L_0}\left(z-d-\dfrac{L_0}{2}\right)\right\}, & d+\dfrac{L_0}{2}\le z\le d+L_0\\ R_0, & otherwise\end{cases} \quad \text{...(2)}$$

where $R_1(z)$ and $R(z)$ are respectively the radii of central layer and stenotic tube with peripheral layer, and R_0 is the radius of unobstructed blood vessel. L_0 is length of stenosis, *d* is the position of stenosis, δ_s is the height of stenosis, δ_i is the maximum bulging of the interface at $z=d+\dfrac{L_0}{2}$, α is the ratio of radius of central layer and radius of unobstructed artery.

The viscosity of blood in central layer is allowed to vary according to the Einstein relation

$$\mu_c = \mu_p\left[1+\beta h(r)\right] \qquad \text{... (3)}$$

where μ_c is viscosity of central layer, μ_p is viscosity of plasma, $h(r)$ is hematocrit and β is constant.

Hematocrit is described by the relation

$$h(r) = h_m\left[1-\left(\frac{r}{R_0}\right)^3\right] \qquad \text{... (4)}$$

where h_m is maximum hematocrit of blood.

Substituting the value of $h(r)$ from equation (4) in equation (3), we get

$$\mu_c = \mu_p\left[a-k\left(\frac{r}{R_0}\right)^3\right] \qquad \text{...(5)}$$

where $a = 1+k$ and $k = \beta h_m$.

The governing equations for the flow in central and peripheral layer for the present problem, are given by

$$\mu_c\left[\frac{\partial^2 w_c}{\partial r^2}+\frac{1}{r}\frac{\partial w_c}{\partial r}\right]+\left(\frac{\partial \mu_c}{\partial r}\right)\frac{\partial w_c}{\partial r}-H^2\sigma_e^c w_c-\frac{\partial p}{\partial z}=0 \cdot \qquad \text{...(6)}$$

and $$\mu_p\left[\frac{\partial^2 w_p}{\partial r^2}+\frac{1}{r}\frac{\partial w_p}{\partial r}\right]-H^2\sigma_e^p w_p-\frac{\mu_0}{K}w_p-\frac{\partial p}{\partial z}=0 \cdot \qquad \text{...(7)}$$

where H is applied transverse magnetic field, K is permeability constant, $\frac{\partial p}{\partial z}$ is pressure gradient, w_c and w_p are the velocities of fluid, σ_e^c and σ_e^p are the electrical conductivities, of central and peripheral layers respectively.

The boundary conditions are

$$\frac{\partial w_c}{\partial r} = 0 \text{ at } r = 0. \qquad ...(8)$$

$$w_p = 0 \text{ at } r = R(z) . \qquad ... (9)$$

$$w_c = w_p \text{ at } r = R_1(z). \qquad ... (10)$$

$$\tau_c = \tau_p \text{ at } r = R_1(z). \qquad ... (11)$$

Let us assume the transformation

$$x = \frac{r}{R_0} \qquad ... (12)$$

to make the variable r dimensionless.

Therefore, using equation (12) in equations (5 - 7), we obtain

$$\mu_c = \mu_p(a - kx^3). \qquad ... (13)$$

$$\left(a - k x^3\right)\left[x\frac{\partial^2 w_c}{\partial x^2} + \frac{\partial w_c}{\partial x}\right] - 3k x^3 \frac{\partial w_c}{\partial x} - M_1^2 x w_c = x\frac{R_0^2}{\mu_0}\frac{dp}{dz} \qquad ...(14)$$

$$x\frac{\partial^2 w_p}{\partial x^2} + \frac{\partial w_p}{\partial x} - M_2^2 x w_p - \frac{R_0^2}{K} x w_p = x\frac{R_0^2}{\mu_0}\frac{dp}{dz} \qquad ... (15)$$

where,

$M_1^2 = \dfrac{R_0^2 H^2 \sigma_e^c}{\mu_0}$, M_1 is Hartmann number for central layer.

$M_2^2 = \dfrac{R_0^2 H^2 \sigma_e^p}{\mu_0}$, M_2 is Hartmann number for peripheral layer.

Using the transformation (12), the boundary conditions (8 - 11) take the form

$$\frac{\partial w_c}{\partial x} = 0 \text{ at } x = 0. \qquad ... (16)$$

$$w_p = 0 \text{ at } x = \frac{R(z)}{R_0}. \qquad ... (17)$$

$$w_c = w_p \text{ at } x = \frac{R_1(z)}{R_0}. \quad \text{... (18)}$$

$$\tau_c = \tau_p \text{ at } x = \frac{R_1(z)}{R_0}. \quad \text{... (19)}$$

3. Solution of the Problem

We solve equations (14) and (15) by using Frobenius method for second order differential equation. Therefore, the complete solutions of equations (14) and (13) are

$$w_c(x) = A\sum_{m=0}^{\infty} D_m x^m + \frac{R_0^2}{4a\mu_0}\frac{dp}{dz}\sum_{m=0}^{\infty}\overline{D}_m x^{m+2} \quad \text{... (20)}$$

$$w_p(x) = C\sum_{m=0}^{\infty} F_m x^m + D\left[\log x\sum_{m=0}^{\infty} F_m x^m - \left\{\frac{1}{4}\left(M_2^2+\frac{R_0^2}{K}\right)x^2 + \frac{3}{256}\left(M_2^2+\frac{R_0^2}{K}\right)^2 x^4 .....\right\}\right]$$

$$+\frac{R_0^2}{4\mu_0}\frac{dp}{dz}\sum_{m=0}^{\infty}\overline{F}_m x^{m+2} \quad \text{... (21)}$$

where,

$$D_m = \frac{k\,m\,(m-3)D_{m-3} + M_1^2 D_{m-2}}{a\,m^2} \quad \text{... (22)}$$

$$\overline{D}_m = \frac{k(m+2)(m-1)\overline{D}_{m-3} + M_1^2\overline{D}_{m-2}}{a(m+2)^2} \quad \text{... (23)}$$

$$F_m = \frac{\left(M_2^2 + \frac{R_0^2}{K}\right)}{m^2}F_{m-2} \quad \text{... (24)}$$

$$\overline{F}_m = \frac{\left(M_2^2 + \frac{R_0^2}{K}\right)}{(m+2)^2}\overline{F}_{m-2} \quad \text{... (25)}$$

$$D_0 = \overline{D}_0 = F_0 = \overline{F}_0 = 1 \text{ and } D_{m+1} = \overline{D}_{m+1} = F_{m+1} = \overline{F}_{m+1} = 0 \quad \text{... (26)}$$

To obtain the value of constant D in equation (21), we apply boundary condition (18) on equations (20) and (21).

Hence, we get $D = 0$.

Therefore, equation (21) becomes

$$w_p = C\sum_{m=0}^{\infty} F_m x^m + \frac{R_0^2}{4\mu_0}\frac{dp}{dz}\sum_{m=0}^{\infty} \overline{F}_m x^{m+2} \qquad \text{... (27)}$$

To determine the constants A and C, we use boundary conditions (17) and (19) on equations (20) and (21), we obtain

$$A = \frac{R_0^2}{4\mu_0}\frac{dp}{dz}\frac{1}{\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty} mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty} F_m\left(\frac{R}{R_0}\right)^m}\times$$

$$\left[\sum_{m=0}^{\infty} F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}(m+2)\overline{F}_m\left(\frac{R_1}{R_0}\right)^{m+1} - \sum_{m=0}^{\infty}\overline{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty} mF_m\left(\frac{R_1}{R_0}\right)^{m-1}\right.$$

$$\left.-\frac{1}{a}\left\{a-k\left(\frac{R_1}{R_0}\right)^3\right\}\sum_{m=0}^{\infty}(m+2)\overline{D}_m\left(\frac{R_1}{R_0}\right)^{m+1}\sum_{m=0}^{\infty} F_m\left(\frac{R}{R_0}\right)^m\right] \qquad \text{... (28)}$$

$$C = \frac{\dfrac{-R_0^2}{4\mu_0}\dfrac{dp}{dz}\sum_{m=0}^{\infty}\overline{F}_m\left(\dfrac{R}{R_0}\right)^{m+2}}{\sum_{m=0}^{\infty} F_m\left(\dfrac{R}{R_0}\right)^m} \qquad \text{... (29)}$$

Putting the value of A and C in equations (20) and (27), we find

$$w_p(x) = \frac{\dfrac{R_0^2}{4\mu_0}\dfrac{dp}{dz}\left[\sum_{m=0}^{\infty} F_m\left(\dfrac{R}{R_0}\right)^m\sum_{m=0}^{\infty}\overline{F}_m x^{m+2} - \sum_{m=0}^{\infty}\overline{F}_m\left(\dfrac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}\overline{F}_m x^m\right]}{\sum_{m=0}^{\infty} F_m\left(\dfrac{R}{R_0}\right)^m} \qquad \text{... (30)}$$

$$w_c(x)=\frac{R_0^2}{4\mu_0}\frac{dp}{dz}\frac{1}{\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]}\frac{1}{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}}\times$$

$$\left[\left\{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}(m+2)\overline{F}_m\left(\frac{R_1}{R_0}\right)^{m+1}-\sum_{m=0}^{\infty}\overline{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}mF_m\left(\frac{R_1}{R_0}\right)^{m-1}\right.\right.$$

$$-\frac{1}{a}\left\{a-k\left(\frac{R_1}{R_0}\right)^3\right\}\sum_{m=0}^{\infty}(m+2)\overline{D}_m\left(\frac{R_1}{R_0}\right)^{m+1}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\Bigg\}\sum_{m=0}^{\infty}D_m x^m$$

$$\left.+\frac{1}{a}\left\{a-k\left(\frac{R_1}{R_0}\right)^3\right\}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}\overline{D}_m x^{m+2}\right] \quad \text{... (31)}$$

The total flow rate Q is given by $Q=Q_c+Q_p$,

where Q_c and Q_p are flow rates corresponding to central and peripheral layers respectively, given by

$$Q_c=2\pi R_0^2\int_0^{R_1/R_0}x.w_c\,dx. \quad \text{... (32)}$$

and $$Q_p=2\pi R_0^2\int_{R_1/R_0}^{R/R_0}x.w_p\,dx. \quad \text{... (33)}$$

Thus, the total flow rate is found to be

$$Q=\frac{\pi R_0^4}{2\mu_0}\frac{dp}{dz}\frac{1}{\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}}\left[\sum_{m=0}^{\infty}\frac{D_m}{m+2}\left(\frac{R_1}{R_0}\right)^{m+2}\right.$$

$$\left\{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}(m+2)\overline{F}_m\left(\frac{R_1}{R_0}\right)^{m+1}-\sum_{m=0}^{\infty}\overline{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}mF_m\left(\frac{R_1}{R_0}\right)^{m-1}\right.$$

$$\left.-\frac{1}{a}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty}(m+2)\overline{D}_m\left(\frac{R_1}{R_0}\right)^{m+1}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\right\}$$

$$\frac{1}{a}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}\frac{\overline{D}_m}{m+4}\left(\frac{R_1}{R_0}\right)^{m+4}$$

$$+\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\left[\left\{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}\frac{\bar{F}_m}{m+4}\left(\left(\frac{R}{R_0}\right)^{m+4}-\left(\frac{R_1}{R_0}\right)^{m+4}\right)\right.\right.$$

$$\left.\left.-\sum_{m=0}^{\infty}\bar{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}\frac{F_m}{m+2}\left(\left(\frac{R}{R_0}\right)^{m+2}-\left(\frac{R_1}{R_0}\right)^{m+2}\right)\right\}\right] \quad \ldots (34)$$

The system of blood flow is closed, hence the total flow rate is constant. So, we can assume $Q=\pi$. Therefore, the pressure gradient is given by

$$\frac{dp}{dz}=\frac{2\mu_0}{R_0^4}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\Bigg/\left[\sum_{m=0}^{\infty}\frac{D_m}{m+2}\left(\frac{R_1}{R_0}\right)^{m+2}\times\right.$$

$$\left\{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}(m+2)\bar{F}_m\left(\frac{R_1}{R_0}\right)^{m+1}-\sum_{m=0}^{\infty}\bar{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}mF_m\left(\frac{R_1}{R_0}\right)^{m-1}\right.$$

$$\left.-\frac{1}{a}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty}(m+2)\bar{D}_m\left(\frac{R_1}{R_0}\right)^{m+1}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\right\}$$

$$+\frac{1}{a}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}\frac{\overline{D}_m}{m+4}\left(\frac{R_1}{R_0}\right)^{m+4}$$

$$+\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\left[\left\{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}\frac{\bar{F}_m}{m+4}\left(\left(\frac{R}{R_0}\right)^{m+4}-\left(\frac{R_1}{R_0}\right)^{m+4}\right)\right.\right.$$

$$\left.\left.\left.-\sum_{m=0}^{\infty}\bar{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}\frac{F_m}{m+2}\left(\left(\frac{R}{R_0}\right)^{m+2}-\left(\frac{R_1}{R_0}\right)^{m+2}\right)\right\}\right]\right] \quad \ldots(35)$$

The shear stress on the wall is given by

$$\tau_{rz}=\frac{R}{2}\frac{dp}{dz}$$

$$=\frac{R\mu_0}{R_0^4}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\Bigg/\left[\sum_{m=0}^{\infty}\frac{D_m}{m+2}\left(\frac{R_1}{R_0}\right)^{m+2}\times\right.$$

$$\left\{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}(m+2)\bar{F}_m\left(\frac{R_1}{R_0}\right)^{m+1}-\sum_{m=0}^{\infty}\bar{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}mF_m\left(\frac{R_1}{R_0}\right)^{m-1}\right.$$

$$-\frac{1}{a}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\left[\sum_{m=0}^{\infty}(m+2)\bar{D}_m\left(\frac{R_1}{R_0}\right)^{m+1}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\right\}$$

$$+\frac{1}{a}\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\left[\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}\frac{\overline{D_m}}{m+4}\left(\frac{R_1}{R_0}\right)^{m+4}\right.$$

$$+\left[a-k\left(\frac{R_1}{R_0}\right)^3\right]\left[\left\{\sum_{m=0}^{\infty}F_m\left(\frac{R}{R_0}\right)^m\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}\frac{\bar{F}_m}{m+4}\left(\left(\frac{R}{R_0}\right)^{m+4}-\left(\frac{R_1}{R_0}\right)^{m+4}\right)\right.\right.$$

$$\left.\left.-\sum_{m=0}^{\infty}\bar{F}_m\left(\frac{R}{R_0}\right)^{m+2}\sum_{m=0}^{\infty}mD_m\left(\frac{R_1}{R_0}\right)^{m-1}\sum_{m=0}^{\infty}\frac{F_m}{m+2}\left(\left(\frac{R}{R_0}\right)^{m+2}-\left(\frac{R_1}{R_0}\right)^{m+2}\right)\right\}\right] \quad \text{... (36)}$$

4. Results and Discussion

We have studied the two-layered model of blood flow in composite stenosed artery under the effect of magnetic field through porous medium. The numerical computations have been carried out by making use of the parameters $L_0 = 40$ mm, $d = 30$ mm, $\mu = 0.35$ P, and z =50mm. The figure 1 illustrates the behavior of pressure gradient with increase in stenosis size for different values of Hartmann number (M_1 and M_2) for central and peripheral layer respectively. It shows that pressure gradient increases for an increase in stenosis size. It also depicts that slight increase in magnitude of pressure gradient with increase in values of Hartmann numbers from M_1 = 2 and M_2 = 4 to M_1 = 3 and M_2 = 5and is due to the effect of porosity and then it decreases with further increase in values of Hartmann number at fixed stenosis size which proves that application of external magnetic field on stenosed arteries control the blood flow. It is observed from figure 2 that for an increase in values of k, the pressure gradient increases. Therefore, it can be concluded that the increase in concentration of hematocrit can be dangerous for a diseased heart. These observations are in good agreement with those of Haldar and Ghosh [5]. The figure 3 shows the variations of pressure gradient with stenosis size for different values of permeability constant *K*. It is seen that for fixed value of stenosis size, the pressure gradient rises with rise in the values of permeability constant which leads to more and more deposition of LDL molecules along the walls of artery and ultimately forms the arteriosclerotic plaques, causing disturbance in the flow of blood. This result is in good agreement with the findings of Dash and Mehta[3]. The figure 4 describes the combined behavior of the ratio of radius of central layer to the radius of unobstructed artery (α), magnetic field and porosity. The increase in value of α results in decrease of thickness of peripheral layer, hence it is reported that decrease in thickness of peripheral layer causes reduction in magnitude of pressure gradient under the effect magnetic field. The similar trends are observed for shear stress from figure (5-7) for increase in magnetic field, k and permeability constant for fixed stenosis size. But the slightly flattening of curves is noted in case of shear stress at the wall of constricted artery which proves that it is more effective at higher values of k, indicating the possibility of rupture of stenosis.

5. Conclusions

The main findings of this paper can be summarized as:

1. The trends of pressure gradient and shear stress are similar. However the values of shear stress are lower in magnitude in comparison to pressure gradient.
2. The presence of peripheral layer causes reduction in flow characteristics of blood flow in stenosed artery under the effect of magnetic field through porous medium.
3. The pressure gradient and shear stress show an increase for slight increase in strength of magnetic field and then decrease for further increase in magnetic field which shows that magnetic field can be used to control the blood flow of hypertensive patients.
4. The rise in shear stress at the walls of stenosed artery with increase in values of permeability constant, results in increase of net uptake of LDL along the walls of blood vessel leads to formation of stenosis.

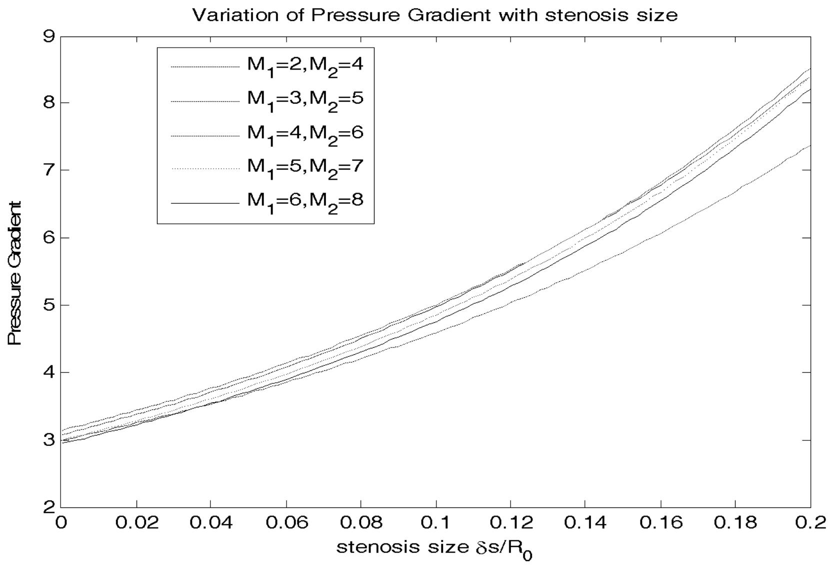


Figure 1: Variation of Pressure Gradient with stenosis size for different values of Hartmann number.

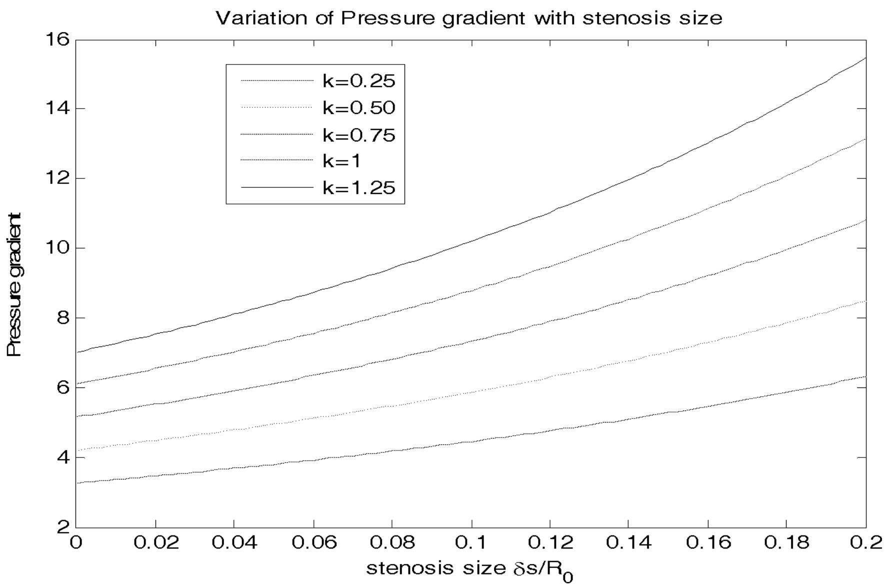


Figure 2: Variation of Pressure Gradient with stenosis size for different values of k.

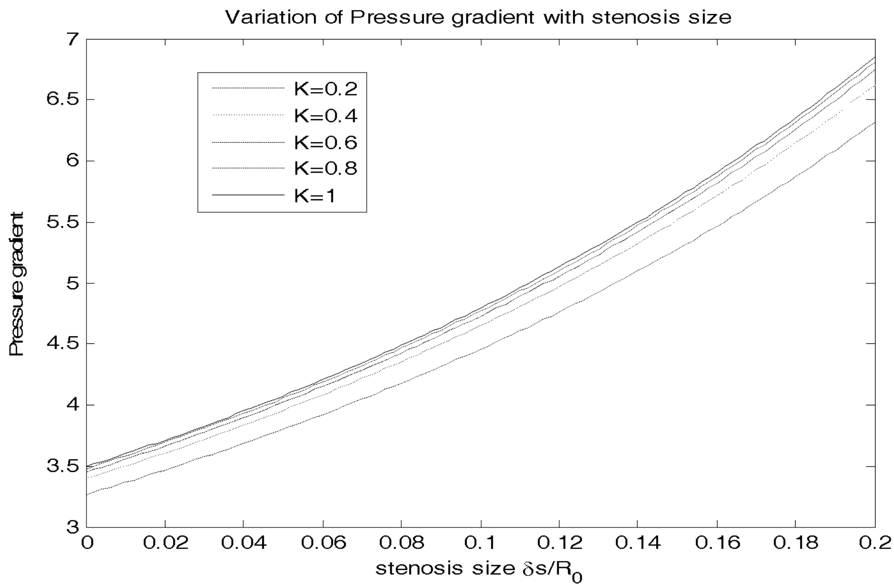


Figure 3: Variation of Pressure Gradient with stenosis size for different values of *K*.

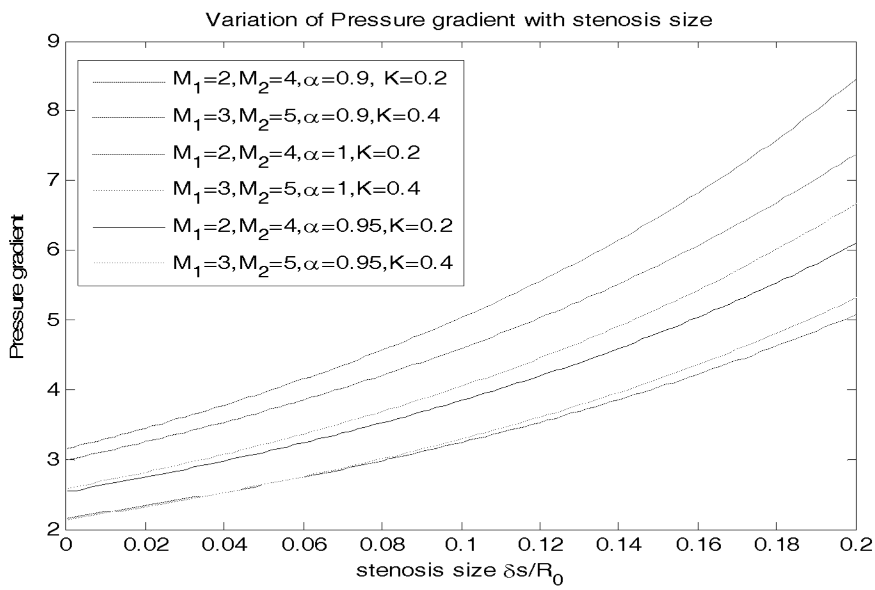


Figure 4: Variation of Pressure Gradient with Stenosis Size.

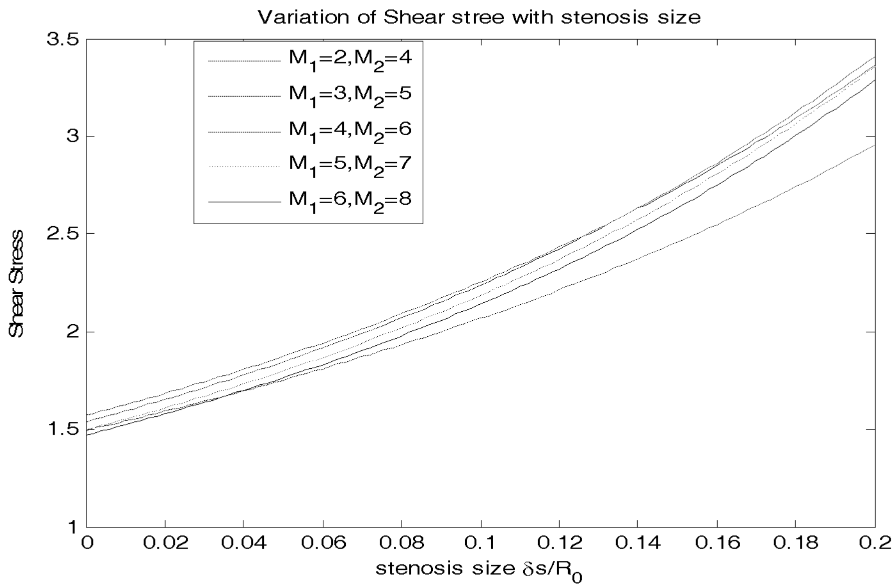


Figure 5: Variation of Shear Stress with stenosis size for different values of Hartmann number.

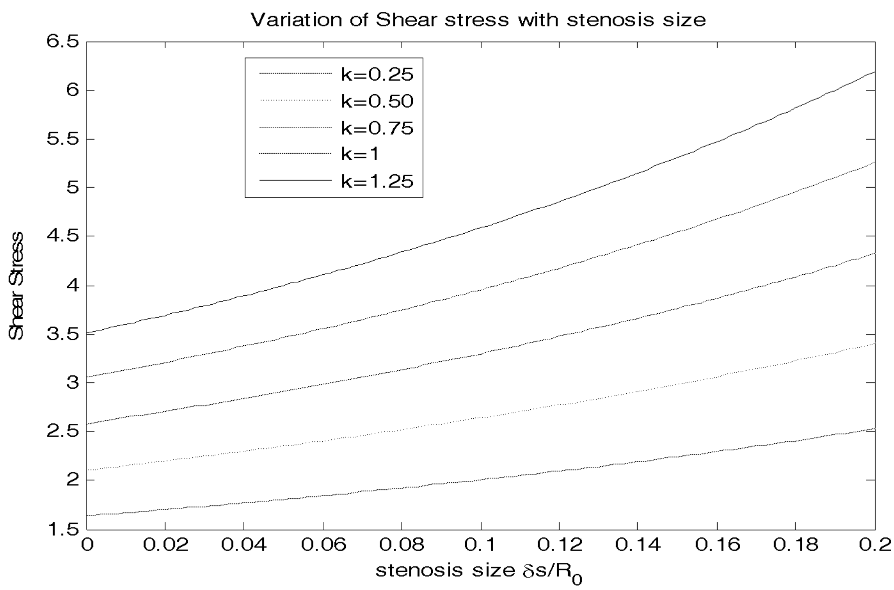


Figure 6: Variation of Shear Stress with stenosis size for different values of *k*.

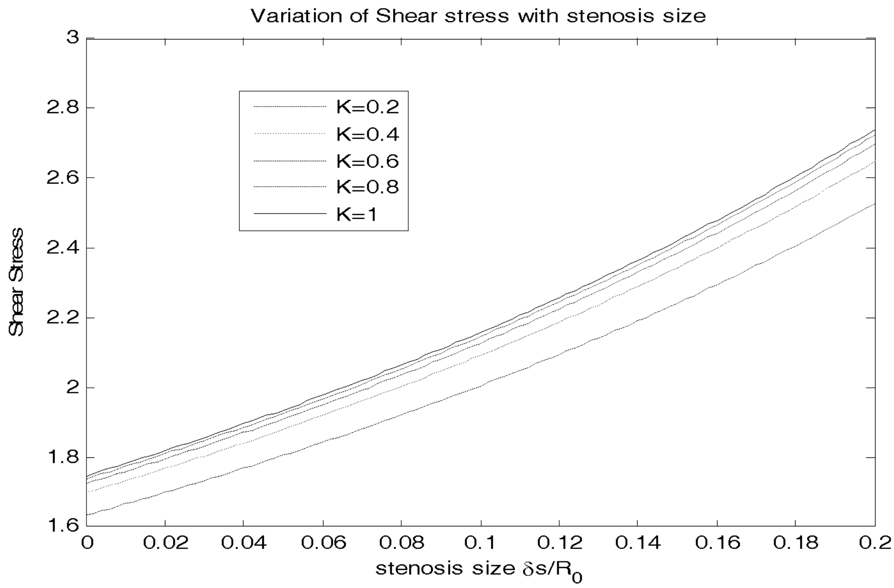


Figure 7: Variation of Shear stress with stenosis size for different values of *K*.

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