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BIANCHI TYPE VI_0 INHOMOGENEOUS COSMOLOGICAL MODEL OF PERFECT FLUID DISTRIBUTION WITH TIME DEPENDENT COSMOLOGICAL TERM Λ

Barkha Rani Tripathi, Atul Tyagi and Swati Parikh
Email: barkha1808@gmail.com, tyagi.atul10@gmail.com,
parikh.swati1@gmail.com

Abstract: An inhomogeneous Bianchi type VI_0 cosmological model for perfect fluid distribution with time dependent cosmological term is investigated. Some physical and geometric aspects of the models are also discussed.

Key words: Cosmology, Inhomogeneous, Bianchi Type VI_0, Variable Cosmological term Λ.

1. Introduction

Inhomogeneous cosmological models play a significant role in understanding some essential features of the universe such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts on the construction of such model have been initiated by Tolman [14] and Bondi [4] who considered spherically symmetric models. Roy and Narain [10] have obtained solutions which generalize Bianchi types I, V and VI_0 models with perfect fluids. A class of inhomogeneous solutions corresponding to pressureless perfect fluid for more general type of orthogonal metric is obtained by Szekeres [13]. Collins and Szafron [6, 7] have done a systematic study of inhomogeneous cosmological models by introducing the concept of intrinsic space time. Pradhan and Bali [9] have investigated magnetized Bianchi type VI_0 barotropic massive string universe with decaying vacuum energy density. Bali et al. [2] have investigated some LRS Bianchi type VI_0 cosmological models with special free gravitational fields. Tyagi et al. [16] have obtained Bianchi type IX string cosmological models for perfect fluid distribution.

Verma and Shri Ram [17] have investigated Bianchi type VI_0 Bulk viscous fluid models with variable gravitational and cosmological constants. Bali and Tyagi [3] have obtained cylindrically symmetric inhomogeneous cosmological model with electromagnetic field for perfect fluid distribution. Bali and Tyagi [1] have obtained a plane symmetric inhomogeneous cosmological model for perfect fluid distribution with electromagnetic field. A large number of inhomogeneous cosmological model have been investigated by MacCallum [8]. Chhajed et al. [5] investigated Bianchi type I anisotropic inhomogeneous cosmological mode for perfect fluid distribution with electromagnetic field. Tyagi et al. [16] investigated inhomogeneous Bianchi Type VI_0 string dust cosmological model of perfect fluid distribution in general relativity. Sharma et al. [11] investigated inhomogeneous Bianchi Type VI_0 string cosmological model for stiff perfect fluid distribution in general relativity.

In this paper, we have investigated Bianchi type VI_0 inhomogeneous cosmological model for perfect fluid distribution in general relativity. Some physical and geometric aspects of the model are also discussed.

2. Metric and Solution of Field Equations

We consider the line-element in the form:

$$ds^2 = A^2(x,t)\{dt^2 - dx^2\} - e^{2x}B^2(t)dy^2 - e^{-2x}C^2(t)dz^2 \quad (1)$$

Einstein's field equations in the presence of the cosmological term (Λ) for perfect fluid distribution is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi[(\rho + p)v_i v_j - pg_{ij}] \quad (2)$$

where ρ is the matter density, p the isotropic pressure, and v_i is the unit flow vector. In the present scenario, the commoving co-ordinates are taken as

$$V_i = (0,0,0,A) \quad (3)$$

Equations (2) and (3) for the line element (1), lead to :

$$-8\pi p = \frac{1}{A^2}\left[\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) + \frac{B_4C_4}{BC} + 1\right] + \Lambda \quad (4)$$

$$-8\pi p = \frac{1}{A^2}\left[\left(\frac{A_4}{A}\right) - \left(\frac{A_1}{A}\right)_1 + \frac{C_{44}}{C} - 1\right] + \Lambda \quad (5)$$

$$-8\pi p = \frac{1}{A^2}\left[\left(\frac{A_4}{A}\right) - \left(\frac{A_1}{A}\right)_1 + \frac{B_{44}}{B} - 1\right] + \Lambda \quad (6)$$

$$8\pi\rho = \frac{1}{A^2}\left[1 - \frac{B_4C_4}{BC} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\right] - \Lambda \quad (7)$$

and

$$\left(\frac{B_4}{B} - \frac{C_4}{C}\right) - \frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \quad (8)$$

Where the sub indices 1 and 4 after *A*, *B* and *C* denote partial differentiation w.r.t. *x* and *t* respectively.

From equations (5) and (6), we have

$$\frac{B_{44}}{B} = \frac{C_{44}}{C} \tag{9}$$

Equation (8) leads to

$$\frac{A_1}{A} = \left(\frac{\frac{B_4}{B} - \frac{C_4}{C}}{\frac{B_4}{B} + \frac{C_4}{C}} \right) = F(t) \tag{10}$$

where *F(t)* is some function of time alone . Integrating equation (10), we have

$$\log A = x\, F(t) + G(t) \tag{11}$$

From equation (9) we have

$$CB_4 - BC_4 = b \tag{12}$$

where *b* is a constant. Equations (10) and (12) give

$$F = \frac{b}{(BC)_4} \tag{13}$$

From Equations (4), (5), (11) and (12), we have

$$-xF_4\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - x\ \mathrm{F}_{44} + \frac{B_{44}}{B} + \frac{B_4C_4}{BC} + 2 = 0$$

$$x\left[F_{44} + F_4\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\right] - \left[\frac{B_{44}}{B} + \frac{B_4C_4}{BC} - 2\right] = 0 \tag{14}$$

From which we conclude that

$$F_{44} + F_4\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \tag{15}$$

and

$$\frac{\mathrm{B}_{44}}{B} + \frac{B_4C_4}{BC} - 2 = 0 \tag{16}$$

Equation (15) on integration yields

$$F_4 = \left(\frac{a}{BC}\right) \tag{17}$$

where a is constant. From Equations (17) and (13), we get

$$F = \beta (BC)^{a/b} \quad , \tag{18}$$

Where β is a constant.

From equations (13) and (12), we get

$$BC = LT^{\frac{b}{a+b}} \tag{19}$$

where $L = \left(\frac{a+b}{\beta}\right)^{\frac{b}{a+b}}$, T = t + t_o , t_o being a constant. Equations (12) and (19) give

$$\frac{B}{C} = \alpha \exp.\left[\frac{\mathrm{b(a+b)}}{\mathrm{aL}} T^{\frac{a}{a+b}}\right] \tag{20}$$

where α is a constant. From Equations (11), (19) and (20), we have

$$B = (\alpha L)^{1/2} T^{n/2} \ \exp.\left[\frac{\mathrm{b}}{\mathrm{2L(1-n)}} T^{1-n}\right],$$

$$C = \left(\frac{L}{\alpha}\right)^{1/2} T^{n/2} \exp.\left[\frac{-\mathrm{b}}{\mathrm{2L(1-n)}} \mathrm{T}^{1-n}\right]$$

and

$$A = \exp.[XmT^{1-n} + G]$$

Where $X = x$, $m = \beta L^{a/b}$, $n = \frac{b}{a+b}$

By suitable transformation of coordinates and remaining constants the line element (1) reduces to the form

$$ds^2 = \exp.[2XmT^{1-n} + 2G][\mathrm{dT}^2 - dX^2] - \alpha L T^n \exp\left[2X + \frac{b}{L(1-n)} T^{1-n}\right] dY^2$$

$$- \alpha L T^n \exp\left[-2X - \frac{b}{L(1-n)} T^{1-n}\right] dZ^2 \tag{21}$$

3. Some Geometrical and Physical Properties of the Model

In this case the physical parameters i.e the pressure *p*, the energy density ρ scalar expansion θ and shear scalar σ for the model (25) are given by :

$$8\pi p = \Lambda - \frac{1}{\exp\,(2XmT^{1-n} + G)}\left[- Xmn(1-n)T^{-(1+n)} + \frac{n}{2}\left(\frac{n}{2} - 1\right)\frac{1}{T^2} + \frac{nbT^{-(1+n)}}{2L} - 1 \right] \quad (22)$$

$$8\pi\rho = -\Lambda + \frac{1}{\exp(2XmT^{1-n} + G)}\left[1 - \left(\frac{n^2}{4T^2} - \frac{b^2}{4L^2T^{2n}} \right) + \frac{Xmn(1-n)}{T^{1+n}} + \frac{n}{T} \right] \quad (23)$$

$$\theta = \frac{1}{\exp\left[XmT^{1-n} + G\right]}\left[\frac{-Xm(1-n)}{T^n} + \frac{n}{T} \right] \quad (24)$$

$$\sigma^2 = \frac{\left\{ \frac{Xm(1-n)}{T^n} + \frac{n}{T} \right\}}{3\exp\{2XmT^{1-n} + G\}} - \frac{\left\{ \frac{Xm(1-n)^n}{T^{n+1}} + \frac{n^2}{4T^2} - \frac{b^2}{4L^2T^{2n}} \right\}}{\exp\{2XmT^{1-n} + G\}} \quad (25)$$

Magnitude of rotation ω is zero.

For the specification of Cosmological Term Λ, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho \quad (26)$$

where γ is a constant. From Equations (26) , (27) and (30), we obtain

$$\Lambda(1+\gamma) = \frac{\left[\frac{(1-y)Xmn(1-n)}{T^{1+n}} + \frac{n}{2}\left(\frac{n}{2} - 1\right)\frac{1}{T^2} + \frac{nb}{2LT^{n+1}} - \left(\frac{n^2}{4T^2} - \frac{b^2}{4T^{2n}L^2} \right) + \frac{n\gamma}{T} + \gamma - 1 \right]}{\exp\{2XmT^{1-n} + G\}} \quad (25)$$

4. Conclusion

The model (25) starts with a big bang at T = 0 and goes on expanding till T = ∞ where θ becomes zero. It is clear that as T increases, the ratio of the shear scalar σ and expansion θ tends to constant i.e. $\frac{\sigma}{\theta} \rightarrow$ constant. From equation (29) we see that the cosmological term is a decreasing function of time and it approaches a small positive value at later time. Hence model does not approach isotropy for large value of T. In general the model represents expanding, shearing and non rotating universe.

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