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C-FIELD COSMOLOGICAL MODEL WITH BULK VISCOSITY AND TIME DEPENDENT Λ IN BIANCHI TYPE I SPACE TIME

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Abstract: C-field cosmological model for dust distribution with time dependent cosmological term **(Λ)** and bulk viscosity in the frame work of Bianchi Type I space-time is investigated. The creation field increases with time which matches with the result as obtained by Hoyle and Narlikar [13]. The vacuum energy density (Λ) decreases with time $\Lambda = \frac{1}{t^2}$ which is the same as obtained by Beesham [7]. The model is free from Particle Horizon. The pair of state finder parameter {γ,s} agrees with ΛCDM model. The spatial volume (R^3) increases with time. The model admits uniform expansion and reality condition in presence of C-field, is satisfied.

Key words and phrases. C-field, Cosmological, dust, bulk viscosity, vacuum energy, Bianchi I

1. Introduction

The prediction of Friedmann-Robertson-Walker (FRW) models that universe starts with a big-bang. But the big-bang models have the following problems:

(i) The model has singularity in the past and one in future (ii) the conservation of energy is violated (iii) it leads to a very small particle horizon in the early epoch of universe (iv) no consistent big-bang model that explains the origin and evolution of universe (v) it has flatness problem. Also FRW are unstable near the singularity and fails to describe early universe as pointed out by Patridge and Wilkinson [17]. Therefore, spatially homogeneous and anisotropic Bianchi Type I space time is undertaken to study the universe in its early stages of evolution.

Seeing the failure of big-bang model to describe early universe, the alternative theories of gravity have been proposed from time to time. The best well known theory is Steady-State Theory established by Bondi and Gold [10]. In this theory, a very flow but continuous creation of matter in contrast to explosive creation of the standard FRW model, was considered. But this theory was discarded by not giving any physical justification for continuous creation of matter. To over come this difficulty, Hoyle and Narlikar [13] adopted a field theoretic approach introducing a massless and charge less scalar field. In C-field theory there is no big-bang type singularity as in Steady-State Theory of Bondi and Gold [10]. Narlikar [15] has explained that matter creation is accomplished at the expense of negative energy C-field. Creation field cosmological models have been investigated by Narlikar and Padmanabhan [16], Bali and Tikekar [1], Bali and Kumawat [2] indifferent contexts.

The introduction of viscosity in the cosmic fluid context has been very useful in explaining many physical aspects of dynamics of homogeneous cosmological models. The remarkable degree of isotropy of the cosmic microwave background radiation (CMBR) reveals the significance of dissipative effects in cosmology. The effect of viscosity on cosmological models has been investigated by many authors viz. Bali [3], Chimento et al. [12], Sahni and Starobinsky [19], Saha [18], Bali et al. [4]. The present day observations of the smallness of the cosmological constant $(\Lambda \sim 10^{-122})$ support the assumption that the cosmological constant is time dependent. Several authorx viz. Bertolami [9], Beesham [7], Berman [8], Bali and Singh [5], Singh et al. [20] have investigated cosmological models in which Λ decays with time. Recently Bali and Saraf [6] have investigated C-field cosmological model for barotropic fluid distribution with variable bulk viscosity and vacuum energy (Λ) in FRW space-time.

In this paper, we have investigated C-field cosmological model for dust distribution with bulk viscosity and time dependent cosmological term (Λ) in Bianchi Type I space-time. The creation field increases with time and the model admits uniform expansion.

2. The Metric and Field Equations

We consider Bianchi Type I space-time in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2 \tag{1}$$

where A, B, C are metric potentials and are functions of t-alone.

The Einstein's modified field equations by introduction of C-field with bulk viscosity are given by

$$R_i^j - \frac{1}{2} R g_i^j = -\left[\underset{(m)}{T}{}_i^j + \underset{(c)}{T}{}_i^j \right] - \Lambda g_i^j \tag{2}$$

(in the geometrized unit $8\pi G=1$, c = 1)

where

$$\underset{(m)}{T}{}_i^j = \rho v_i v^j - \zeta\theta (v_i v^j - g_i^j) \tag{3}$$

ζ being the coefficient of bulk viscosity given by Landau and Lifshitz [ ] and

$$\underset{(c)}{T}{}_i^j = -f\left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right) \tag{4}$$

given by Hoyle and Narlikar [13] with f > 0, a coupling constant between matter and creation field (C). We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1, v_4 = 1$$

and ρ being energy density and $C_i = \dfrac{dc}{dx^i}$.

The modified Einstein field equation (2) for the metric (1) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -\zeta\theta + \frac{1}{2} f\dot{C}^2 + \Lambda(t) \tag{5}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = --\zeta\theta + \frac{1}{2} f\dot{C}^2 + \Lambda(t) \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -\zeta\theta + \frac{1}{2} f\dot{C}^2 + \Lambda(t) \tag{7}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} = -\rho - \frac{1}{2} f\dot{C}^2 + \Lambda(t) \tag{8}$$

where $\dot{C} = C_4 = \dfrac{dC}{dt}$.

3. Solution of Field Equations

To get the deterministic solution in terms of cosmic time t, we assume three conditions

(i) $\zeta\theta = k$ (constant) (9)
as considered by Zimdahl [21]

(ii) $R^3 = ABC = t^n$ (10)

(Power law inflation)

(iii) $\Lambda = \dfrac{1}{R^2}$ (11)

as considered by Chen and Wu [11].

Equation (5) and (6) and (6) and (7) lead to

$$\frac{A_4}{A} - \frac{B_4}{B} = \frac{\ell}{t^n} \tag{12}$$

and

$$\frac{A_4}{A} - \frac{C_4}{C} = \frac{m}{t^n} \tag{13}$$

where ℓ and m are constants. Also equation (10) leads to

$$\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{n}{t} \tag{14}$$

Equations (12), (13) and (14) lead to

$$\frac{A_4}{A} = \frac{\ell + m}{3t^n} + \frac{n}{3t} \tag{15}$$

Thus, we have

$$A = a\, t^{n/3} \exp\,(L^{1-n}) \tag{16}$$

where

$$L = \frac{\ell + m}{3(1-n)} \tag{17}$$

and a is constant of integration.

Similarly equations (12), (13) and (15) lead to

$$B = C_2\, t^{n/3} \exp\,(M\, t^{1-n}) \tag{18}$$

and

$$C = C_3 t^{n/3} \exp\,(N\, t^{1-n}) \tag{19}$$

Therefore, the metric (1) leads to

$$ds^2 = -dt^2 + t^{2n/3} \exp[2(L\, t^{1-n})]\, dx^2$$

$$+ t^{2n/3} \exp[2(M\, t^{1-n})]\, dy^2 + t^{2n/3} \exp[2(N\, t^{1-n})]\, dz^2 \tag{20}$$

4. Physical Aspects

The matter density ρ, vacuum energy density (a), the expansion (θ), the Hubble parameter (p) for the model (20) are given by

$$\rho = \frac{n^2}{3t^2} - \frac{1}{t^{2n/3}} + \frac{1}{2}f \tag{21}$$

where $\ell^2 + m^2 = \ell m$ is assumed.

$$\Lambda = \frac{1}{R^2} = \frac{1}{t^2} \tag{22}$$

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{n}{t} \tag{23}$$

$$H = \frac{\theta}{3} = \frac{n}{3t} \tag{24}$$

Conservation equation

$$\left(\underset{(m)}{T}{}^{j}_{i} + \underset{(c)}{T}{}^{j}_{i} + \Lambda g^{j}_{i} \right)_{;j} = 0$$

leads to

$$\frac{d}{dt}\dot{C}^2 + 2\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\dot{C}^2$$

$$= \frac{2\dot{\rho}}{f} + \frac{2}{f}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)(\rho - \zeta\theta) + \frac{2\dot{\Lambda}}{f} \tag{25}$$

To satisfy conservation equation (25), we take n = 3. Thus equation (25) leads to

$$\frac{d}{dt}\dot{C}^2 + \frac{6}{t}\dot{C}^2 = \frac{6}{t}\left(1 - \frac{k}{f}\right) \tag{26}$$

From equation (26), we have

$$\dot{C}^2 = \beta, \beta = \left(1 - \frac{k}{f}\right), \zeta\theta = k$$

which leads to

$$C = \sqrt{\beta}\, t$$

Thus Creation field increases with time which matches with the result obtained by Hoyle and Narlikar [13].

5. Particle Horizon

The particle horizon for the model (20) is given by

$$P_H = \int_{-\infty}^{\infty} \frac{dt}{R^3}$$

$$= \int_{0}^{\infty} \frac{dt}{t^3} = \infty$$

Thus particle Horizon does not exist i.e. the model has Event Horizon.

6. State Finder Parameter {γ,s}

The state finder parameters effectively differentiate between forms of dark energy and provide simple diagnosis whether a particular model fits into the basic observational data. Following Sahni et al. [19], the state finder diagnostic pair {γ,s} is given by

$$\gamma = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} = 1 + 3(-1) + 2 = 0 \text{ as } H = \frac{1}{t}$$

$$s = \frac{\gamma - 1}{3\left(q - \frac{1}{2}\right)} = 0$$

which agrees with ΛCDM model.

The deceleration parameter (q) for the model (20) when n = 3, is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2}$$

$$= 0$$

7. Conclusion

The reality condition $\rho > 0$ for n = 3 leads to

$$\frac{2}{t^2} + \frac{1}{2} f > 0$$

The model (20) for n = 3, has uniform expansion. Also the model is free from particle Horizon. The state finder parameter {γ,s} agrees with ΛCDM model. The vacuum energy density $\Lambda = \frac{1}{t^2}$ which agrees with the result as obtained by Beesham [7]. The Creation field increases with time which matches with the result as obtained by Hoyle and Narlikar [13]. The bulk viscosity ζ increases with time. The expansion $\theta = \frac{3}{t}$. Thus $\zeta\theta = k$ (constant) is maintained.

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