Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.15, No.1&2, March-June, 2016, 19-30

HALL EFFECT ON MHD FLOW IN A ROTATING CHANNEL THROUGH POROUS MEDIUM IN THE PRESENCE OF AN INCLINED MAGNETIC FIELD

Sapna and Khem Chand
Department of Mathematics & Statistics
Himachal Pradesh University Shimla-171005.
Email: sapnamaths@gmail.com and khemthakur99@gmail.com

Abstract: An analysis of steady flow of a viscous, incompressible and electrically conducting fluid through porous medium in the presence of an inclined magnetic field in a rotating system is presented. A magnetic field of uniform strength is applied in a direction which is inclined at an angle θ to the positive direction of axis. The numerical value of velocity, electric field and shear stresses for different parameters involved in the problem are expressed through the graphs and tables. The results are discussed in detail.

Key Words: Magneto hydrodynamics, inclined magnetic field, porous medium.

Mathematical subject classification (2010): 76D05,76D10, 76S05.

1. Introduction

Theoretical and applied research in MHD flow in rotating system through porous medium has received increased attention during the past three decades. In view of increasing technical application using MHD effect, it is desirable to extend many of available viscous hydrodynamic solutions to include the effect of magnetic field when the fluid is electrically conducting. Rassow [5] studied the effect of transverse magnetic field on two dimensional flows past a flat plate. Further Singh [11] analyzed hydro magnetic effect on the three dimensional flow past a porous plate. MHD flow in rotating system through porous medium are of great use as it give rise to interesting structures and instabilities. Since we live on rotating earth, rotating flows are extremely important in geophysics- in the ocean and atmosphere. In a rapidly rotating fluid, particles moves across the pressure gradient due to Coriolis forces. In an engineering contexts, rotating flows are abundant e.g., in hydraulic turbo machinery. In recent years, many researchers, study the problems on rotating fluids, like Mazumder [4], has studied an exact solution of oscillatory Couette flow in rotating system. Singh and Mathew [12], investigated the injection/ suction effects on an oscillatory hydromagnetic flow in rotating horizontal channel. When magnetic field is strong in fluid flow and electron density of gas is low, the Hall effect is important. The interest in these problems generates from their importance in liquid metals, electrolytes and ionized gases. On the account of their varied importance, these flows have been studied by various authors Shercliff [13] and Vafai [14]. Hall current is of great importance in many astrophysical problems, Hall sensors and flows of plasma in MHD power generators. Garg [2] investigated the rotation and Hall current effect on MHD convective flow of second grade fluid through a porous medium in a porous vertical channel in slip flow regime with thermal radiation. Effect of Hall current on MHD flow in rotating channel partially filled with a porous medium has been analyzed by Chauhan [1]. The study of Hall effect in MHD flow in a rotating channel through porous medium has wide range of applications. The flow over a surface of fluid saturated porous medium is encountered in a variety of important situations. Due to porous medium the concept of relative permeability and assumption that it depends on saturation has remained one of longstanding criticism. In labs measurements and pore network simulation, it is observed that relative permeability also depend on flow direction. The study of the flow in a rotating channel in the presence of inclined magnetic field has many useful applications in science and technology. There exist a few study in literature which deals with this type of flow problems. Seth and Ghosh [8], analyzed the unsteady hydromagnetic flow in a rotating channel in the presence of inclined magnetic field. Ghosh [3] has studied the steady and unsteady hydromagnetic flow in a rotating channel in the presence of inclined magnetic field. Seth [9] has analyzed MHD Couette flow in a rotating system in the presence of an inclined magnetic field. Further Seth et al.[10] presented the effect of Hall current and rotation on unsteady MHD Couette flow in the presence of inclined magnetic field. Recently, Sarveshanand and Singh [7] studied the MHD free convection flow between parallel plates in the presence of inclined magnetic field.

In the present paper we have studied a steady flow of a viscous, incompressible and electrically conducting fluid through porous medium in the presence of an inclined magnetic field in a rotating system. It is an extension of work of Sarkar et al. [6] to porous medium.

2. Mathematical formulation and its solution

Consider the steady hydro magnetic flow of a viscous incompressible electrically conducting fluid between two parallel plates at $z = \pm L$, rotating with uniform angular velocity Ω about an axis perpendicular to the plates through porous medium. The plates and the fluid rotate in unison with uniform angular velocity. The fluid is permeated by a uniform magnetic field in a direction, which is inclined at an angle θ with the positive direction of z-axis in xz-plane. Since the plates are infinitely long along x and y-direction, all the physical quantities, except pressure, will be function of z only.

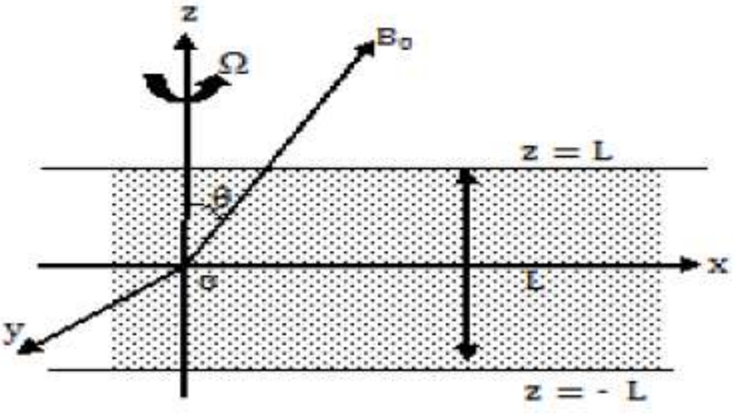

Fig.1. Geometrical configuration of the problem.

The basic equations of magneto hydrodynamic for steady flow are:

$$(\vec{q}.\nabla)\vec{q} + 2\Omega\hat{k}\times\vec{q} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{q} + \frac{1}{\rho}\vec{j}\times\vec{B} + \frac{\nu}{k}(\nabla\vec{q}) \tag{1}$$

$$\nabla.\vec{q} = 0 \tag{2}$$

$$\nabla\times\vec{B} = \mu_e\vec{j} \tag{3}$$

$$\nabla\times\vec{E} = 0 \tag{4}$$

$$\nabla.\vec{B} = 0 \tag{5}$$

$$\nabla.\vec{D} = \rho_e \tag{6}$$

and Ohm's law for a moving conductor taking Hall currents into account is

$$\vec{j} + \frac{\omega_e\tau_e}{B_0}\left(\vec{j}\times\vec{B}\right) = \sigma(\vec{E} + \vec{q}\times\vec{B}) \tag{7}$$

where $\vec{q}$, $\vec{B}$, $\vec{E}$, $\vec{j}$, $\vec{D}$, σ,ν, μ_e, ρ, ρ_e, p, B_0, ω_e,τ_e and k are the velocity vector, the magnetic field vector, the electric field vector, the current density vector, displacement vector, electric conductivity, kinematic viscosity, magnetic permeability, fluid density, charge density, modified fluid pressure including centrifugal force, applied magnetic field, cyclotron frequency, electron collision time and permeability of porous medium respectively.

It is assumed that the induced magnetic field may be neglected in comparison to applied magnetic field, since the magnetic Reynolds number is very small for liquid metals and partially ionized gases. The relation $\nabla.\vec{B} = 0$ gives $B_z = B_0\cos\theta$ throughout the fluid where $\vec{B} = (B_0\sin\theta, 0, B_0\cos\theta)$. The conservation of electric current $\nabla.\vec{j} = 0$ yields $j_z' =$ constant where $\vec{j} = (j_x', j_y', j_z')$. This constant is zero since $j_z' = 0$ at the plates which are electrically non- conducting. Hence, $j_z' = 0$ everywhere in the flow. Since the motion is steady, so from Maxwell equation $\nabla\times\vec{E}=0$ and we obtain $E_x=$ constant and $E_y=$ constant throughout the flow.

From equation (7) we get

$$j_x' + mj_y'\cos\theta = \sigma\left[E_x + B_0v'\cos\theta\right] \tag{8}$$

$$j_y' - mj_x'\cos\theta = \sigma\left[E_y - B_0u'\cos\theta\right] \tag{9}$$

$$-mj_y'\sin\theta = \sigma\left[E_z - B_0v'\sin\theta\right] \tag{10}$$

where $m=\omega_e\tau_e$ is the Hall parameter. For positive values of m, B_0 is upwards and the electrons of the conducting fluid gyrate in the same sense as the rotating system. For negative values of m, B_0 is downwards and the electrons gyrates in an opposite sense to the rotating system. In general, for an electrically conducting fluid, Hall current affects the flow in the presence of strong magnetic field and porous medium. The effect of Hall currents give rise to a force in the y-direction, which induces a cross flow in that direction. To simplify the problem, we assume that there is no variation of flow quantities in y- direction. This assumption is considered to be valid if the surface be of infinite extent in y- direction.

Solving for j_x' and j_y' from (8) and (9), we have

$$j_x' = \frac{\sigma}{1+m^2\cos^2\theta}\left[\left(E_x - mE_y\cos\theta\right) + B_0\cos\theta(v' + mu'\cos\theta)\right] \tag{11}$$

$$j_y' = \frac{\sigma}{1+m^2\cos^2\theta}\left[\left(E_y + mE_x\cos\theta\right) + B_0\cos\theta(mv'\cos\theta - u')\right] \tag{12}$$

on the use of (11) and (12), equation of motion in rotating frame of reference are:

$$-2\Omega v' = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{d^2u'}{dz^2} + \frac{\sigma B_0\cos\theta}{\rho(1+m^2\cos^2\theta)}\left[\left(E_y + mE_x\cos\theta\right) + B_0\cos\theta(mv'\cos\theta - u')\right] - \frac{\nu u'}{k} \tag{13}$$

$$2\Omega u' = \nu\frac{d^2v'}{dz^2} - \frac{\sigma}{\rho(1+m^2\cos^2\theta)}\left[\left(E_x - mE_y\cos\theta\right) + B_0\cos\theta(v' + mu'\cos\theta)\right] - \frac{\nu v'}{k} \tag{14}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\sigma B_0\sin\theta}{\rho(1+m^2\cos^2\theta)}\left[\left(E_y + mE_x\cos\theta\right) + B_0\cos\theta(mv'\cos\theta - u')\right] \tag{15}$$

And the boundary conditions are:

$$u' = v' = 0 \text{ at } z = \pm L \tag{16}$$

Introducing the non-dimensional variables

$\eta = \frac{z}{L}, (u', v') = (u, v)\frac{\nu}{L}$, $(E_x, E_y, E_z) = \frac{\nu B_0}{L}(e_x, e_y, e_z)$, $j'_x, j'_y = \frac{\sigma \nu B_0}{L}(j_x, j_y)$, $K^2 = \frac{\Omega L^2}{\nu}$(rotation parameter), $M^2 = \frac{\sigma B_0^2 L^2}{\rho \nu}$ (magnetic parameter), $k_p = \frac{k}{L^2}$ (permeability parameter), $R = -\frac{\partial p^*}{\partial x}$ (pressure gradient) and $p^* = \frac{L^2 p}{\rho \nu^2}$.

Equations (13) to (15) become

$$-2K^2 v = R + \frac{d^2u}{d\eta^2} + \frac{M^2 \cos\theta}{1+m^2\cos^2\theta}\left[\left(e_y + me_x\cos\theta\right) + \cos\theta\,(mv\cos\theta - u)\right] - \frac{u}{k_p} \tag{17}$$

$$2K^2 u = \frac{d^2v}{d\eta^2} + \frac{M^2 \cos\theta}{1+m^2\cos^2\theta}\left[\left(e_x - me_y\cos\theta\right) + \cos\theta\,(v + mu\cos\theta)\right] - \frac{v}{k_p} \tag{18}$$

$$0 = -\frac{\partial p^*}{\partial \eta} - \frac{M^2 \sin\theta}{1+m^2\cos^2\theta}\left[\left(e_y + me_x\cos\theta\right) + \cos\theta\,(mv\cos\theta - u)\right] \tag{19}$$

The corresponding boundary conditions are:

$$u = v = 0 \text{ at } \eta = \pm 1 \tag{20}$$

3. Method of Solution

To solve the above equation (17) and (18) under the boundary condition given by equation (20), we assume the solution of the problem in the following form:

$$F(\eta) = u(\eta) + iv(\eta) \tag{21}$$

Combining equations (17) and (18) by using equation (21) the solution for the resulting equation is obtained in the following form:

$$F(\eta) = \frac{1}{a_0+ib_1}\left[R + \frac{E_0 M^2\cos\theta(1+im\cos\theta)}{1+m^2\cos^2\theta}\right]\left[1 - \frac{\cosh\sqrt{a_0+ib_1}\,\eta}{\cosh\sqrt{a_0+ib_1}}\right] \tag{22}$$

where $E_0 = e_y - ie_x$, $a_0 = a_1 + \frac{1}{k_p}$, $a_1 = \frac{M^2\cos^2\theta}{1+m^2\cos^2\theta}$, $b_1 = 2K^2 + \frac{mM^2\cos^3\theta}{1+m^2\cos^2\theta}$.

and $\overline{E_0}$ is the complex conjugate of E_0.

By using non dimensional parameters the equations (11) and (12), transform to

$$J = \frac{1+im\cos\theta}{1+m^2\cos^2\theta}(E_0 - F\cos\theta) \tag{23}$$

where $J = j'_y - ij'_x$.

To determine the electric field E_0, we use the condition $\int_{-1}^{1} J\,\partial\eta = 0$ (24)

$$\text{Therefore } E_0 = \frac{R(\sqrt{a_0+ib_1} - \tanh\sqrt{a_0+ib_1})\cos\theta}{\sqrt[3]{a_0+ib_1} - \frac{M^2\cos\theta(1+im\cos\theta)}{\sqrt{a_0+ib_1}(1+m^2\cos^2\theta)} + \frac{M^2\cos\theta(1+im\cos\theta)}{1+m^2\cos^2\theta}\tanh\sqrt{a_0+ib_1}} \tag{25}$$

The shear stresses at the channel are important physical characteristic in the context of MHD flows. The non- dimensional shear stress components τ_x and τ_y at the plates $(\eta = 1)$ and $(\eta = -1)$ due to flow in non-dimensional flow is given by

$$\tau_x = \frac{\partial F}{\partial \eta}\Big]_{\eta=1} = \frac{1}{a_0+ib_1}\left[R + \frac{E_0M^2\cos\theta(1+im\cos\theta)}{1+m^2\cos^2\theta}\right]\tanh\sqrt{a_0+ib_1}$$

and $$\tau_y = \frac{\partial F}{\partial \eta}\Big]_{\eta=-1} = \frac{-1}{a_0+ib_1}\left[R + \frac{E_0M^2\cos\theta(1+im\cos\theta)}{1+m^2\cos^2\theta}\right]\tanh\sqrt{a_0+ib_1}$$

4. Results and Discussion

To study the effects of Hall currents, rotation, angle of inclination and permeability parameter on the velocity, the numerical values of fluid velocity components are shown graphically against η for several values of Hall parameter (m), rotation parameter (K^2), angle of inclination (θ) and permeability parameter (k_p) in figures 2-3 when $M^2 = 10$ and $R = 1$. For $M^2 = 0$ the hydromagnetic drag forces vanishes and we have purely hydrodynamic channel flow in a rotating system. Here we consider solid line for primary velocities i.e., for u and dotted lines for secondary velocities i.e., for v in all the graphs.

It is seen from Fig.2 that the primary velocity decreases with an increase in rotation parameter K^2. The primary velocity is more near the plates than that at the middle of the channel. It is also seen that the magnitude of the secondary velocity decreases with increasing values of K^2. This means that the rotation had retarding influence on both the primary and secondary velocities. As we know that Ekman number expresses the relative significance of viscous hydrodynamic forces and rotational forces, so for $K^2 = 1$, the viscous and rotational forces are of same order of magnitude. When $K^2 > 1$, rotational effect dominates viscous effect and reversible effect for $K^2 < 1$, which shows that larger values of Ekman number implies lesser rotational effect. This rotational drag force is a positive body force therefore accelerates the flow. However as K^2 increases the magnitude of this force is reduced which cause a reduction in fluid velocities.

It is analyzed from Fig.3 that the primary and secondary velocities decrease with the increase in Hall parameter. It means that Hall current has a retarding influence on primary and secondary velocities.

From Fig.4 it is observed that the primary velocity decreases whereas secondary velocity increases with the increase in angle of inclination θ of the magnetic field, this means that angle of inclination has a retarding influence on primary velocity whereas it accelerates the secondary velocity. From the above calculations we come to know that the influence of θ is experienced in the term containing $\cos\theta$. Maximum primary velocity in the channel when $\theta = 0^0$,however maximum secondary velocity when $\theta = 60^0$.Which shows that variable inclination of the applied magnetic field is the best control mechanism for the flow velocity.

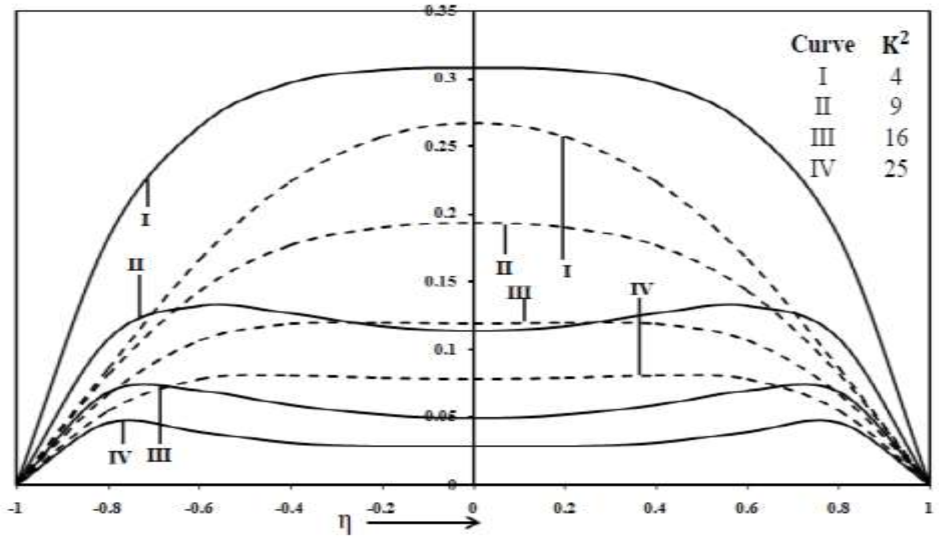

Fig.2. Primary and secondary velocities for different K^2 when $\theta = 45^0$, $m = 0.5$ and $k_p = 2$.

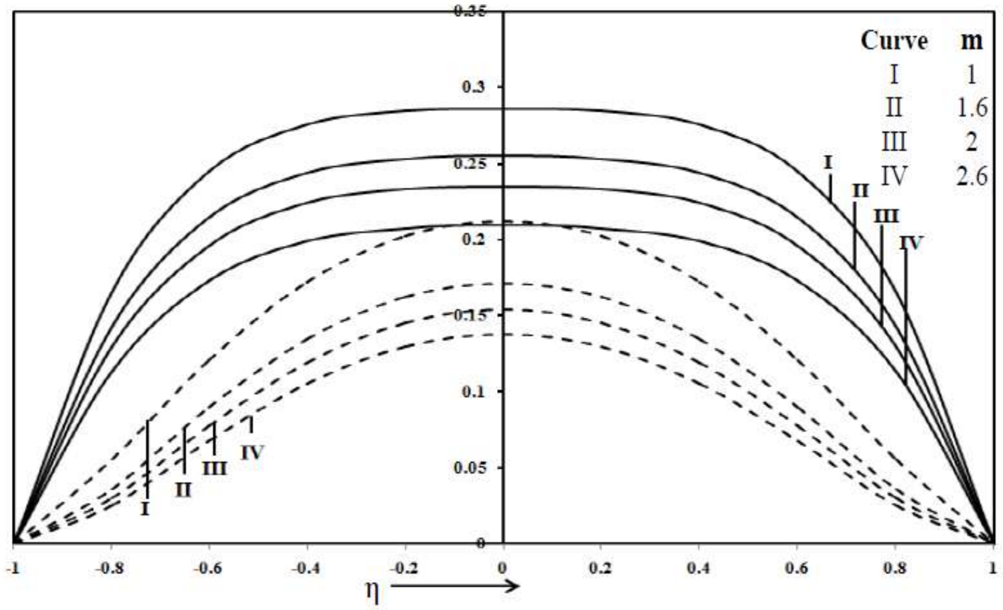

Fig.3. Primary and secondary velocities for different m when $\theta = 45^0, K^2 = 0.5$ and $k_p = 2$.

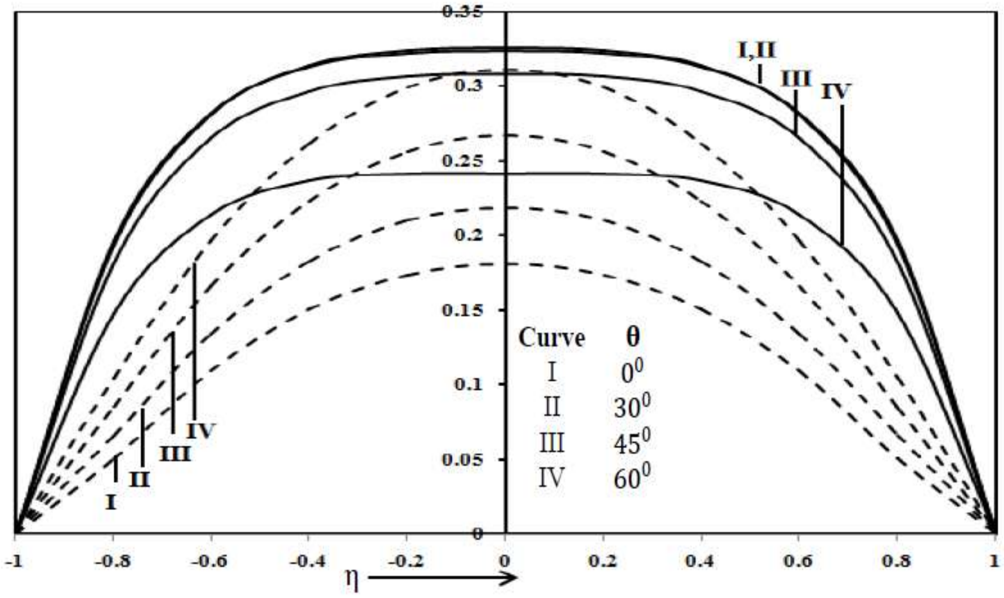

Fig.4. Primary and secondary velocities for different θ when $K^2 = 4$, $m = 0.5$ and $k_p = 2$.

Table-1. Primary velocity for different values of k_p when $K^2 = 4, m = 0.5$ and $\theta = 45^0$.

k_p/η	-1	0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
∞	0	.1827	.2658	.2968	.3054	.3067	.3054	.2968	.2658	.1827	0
1	0	.1807	.2637	.2962	.3063	.3082	.3063	.2962	.2637	.1807	0
2	0	.1819	.2651	.2969	.3064	.3081	.3064	.2969	.2651	.1819	0
3	0	.1822	.2654	.2970	.3062	.3077	.3062	.2970	.2654	.1822	0

Table-2. Secondary velocity for different values of k_p when $K^2 = 4, m = 0.5$ and $\theta = 45^0$.

k_p/η	-1	0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
∞	0	.0920	.1778	.2392	.2743	.2856	.2743	.2392	.1778	.0920	0
1	0	.0798	.1551	.2089	.2394	.2492	.2394	.2089	.1551	.0798	0
2	0	.0857	.1661	.2237	.2564	.2670	.2564	.2237	.1661	.0857	0
3	0	.0878	.170	.2288	.2623	.2730	.2623	.2288	.170	.0878	0

Table-3. Electric field $|E_0|$ for different values of K^2 when $M^2 = 10$, $k_p = 2$ and $\theta = 45^0$.

m/K^2	4	9	16	25
0	.3450	.6452	1.017	1.4305
0.5	.3057	.4929	.6884	.8790
1	.2503	.3872	.5235	.6545
1.5	.2053	.3165	.4257	.5312
2	.1666	.2593	.3502	.4389

Table-4. Electric field $|E_0|$ for different values of θ when $M^2 = 10$, $k_p = 2$ and $K^2 = 4$.

$\mathbf{m/\theta}$	$\mathbf{0^0}$	$\mathbf{30^0}$	$\mathbf{45^0}$	$\mathbf{60^0}$
0	.2598	.2828	.3450	.5759
0.5	.3852	.3441	.3057	.2753
1	.7038	.4020	.2503	.1719
1.5	1.302	.4211	.2053	.1210
2	1.945	.4052	.1666	.0893

Table-5. Electric field $|E_0|$ for different values of k_p when $M^2 = 10$, $\theta = 45^0$and $K^2 = 4$.

$\mathbf{m/k_p}$	∞	**1**	**2**	**3**
0	.3362	.3544	.3450	.3420
0.5	.3007	.3112	.3057	.3039
1	.2483	.2528	.2503	.2496
1.5	.2048	.2063	.2053	.2051
2	.1668	.1668	.1666	.1665

Table-6. Shear stresses $\left(\tau_x = -\tau_y\right)$ for different values of K^2 when $k_p = 2$ and $\theta = 45^0$.

$\mathbf{m/K^2}$	**4**	**9**	**16**	**25**
0	1.2997	.83861	.60869	.47630
0.5	1.2622	.87793	.66297	.52979
1	1.1223	.80007	.61457	.49599
1.5	.97277	.69494	.53631	.43411
2	.84878	.60289	.46517	.37657

Table.7. Shear stresses $(\tau_x = -\tau_y)$ for different values of k_p when $K^2 = 4$ and $\theta = 45^0$.

$\mathbf{m/k_p}$	∞	**1**	**2**	**3**
0	1.3045	1.2928	1.2997	1.3016
0.5	1.2677	1.2550	1.2622	1.2643
1	1.1268	1.1161	1.1223	1.124
1.5	.97611	.96779	.97277	.97408
2	.85108	.84486	.84878	.84974

5. Conclusion

Numerical results for important parameters are presented in tables 1-7. From table 1& 2, it is observed that with the increase in permeability parameter primary velocity and secondary velocities increases. From table 3, it is clear that with the increase in value of rotation parameter K^2, the value of electric field increases and reverse trend is observed for hall parameter m. From table 4 it is observed that with increasing value of θ for $m = 0$ value of electric field increases and opposite pattern observed for value of m. Table 5 depicts that with the increasing value of permeability parameter(k_p), the electric field and Hall current parameter(m) decreases. Numerical results of shear stress components $(\tau_x = -\tau_y)$ are being presented in tables 6 & 7. From table 6 & 7 it is observed that with the increase in rotation parameter K^2 and permeability parameter k_p shear stress components $(\tau_x = -\tau_y)$ decreases and increases respectively.

References

[1] Chauhan, D.S. and Agrawal, R. (2010). Effect of Hall current on MHD flow in rotating channel partially filled with a porous medium, *Chem. Engg. Comm.* 1197, 848-863.

[2] Garg, B.P., Singh, K.D and Bansal, A.K. (2014). Rotation and Hall current effects on MHD convective flow of second grade fluid through a porous medium in a porous vertical channel in slip flow regime with thermal radiation, *International Journal of Innovations in Engineering and Technology*. 3, 164-182.

[3] Ghosh, S.K. (1991). A note on steady and unsteady hydromagnetic flow in a rotating channel in the presence of inclined magnetic field, *International Journal of Engineering Science*. 29, 1013-1016.

[4] Mazumder, B.S. (1991). An exact solution of oscillatory Couette flow in a rotating system, *ASME.J. App. Mech.*58, 1104-1107.

[5] Rassow, V.J. (1958). Effect of transverse magnetic field on two dimensional flow past a flat plate, *NASA*, Rept.1358.

[6] Sarkar, B.C., Das, S. and Jana, R.N. (2014). Hall effects on MHD flow in a rotating channel in the presence of an inclined magnetic field, *Journal of Applied Science and Engineering.* 17, 243-252.

[7] Sarveshanand and Singh, A.K. (2015). MHD free convection between parallel porous plates in the presence of inclined magnetic field, *Springer Plus*. 4, 333.

[8] Seth, G.S. and Ghosh, S.K. (1986). Unsteady hydromagnetic flow in a rotating channel in the presence of inclined magnetic field, *International Journal of Engineering Science*. 24, 1183-1193.

[9] Seth, G.S. and Nandkeolyar, R. (2009). MHD Couette flow in a rotating system in the presence of inclined magnetic field, Applied *Mathematical Sciences*. 3, 2919-2932.

[10] Seth, G.S., Nandkeolyar,R. and Ansari, Md. (2012). Effect of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field, *Journal of Applied Fluid Mechanics.* 5, 67-74.

[11] Singh, K.D. (1991). Hydromagnetic effect on the three dimensional flow past a porous plate, *ZAMP*, 41, 441-446.

[12] Singh, K.D. and Mathew, A. (2008). Injection/suction effect of an oscillatory hydromagnetic flow in a rotating horizontal porous channel, *Indian Journal of Physics*. 82, 435-445.

[13] Shercliff, J.A. (1965), *A text book of magnetohydrodynamics*, Pergamon Press, London.

[14] Vafai, K. (2015), *Handbook of porous media*, CRC Press, Third Edition.