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INJECTION/SUCTION EFFECT ON SPANWISE SINUSOIDAL FLUCTUATING MHD MIXED CONVECTION FLOW THROUGH POROUS MEDIUM IN A VERTICAL POROUS CHANNEL WITH THERMAL RADIATION

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Abstract : An analysis of magnetohydrodynamic (MHD) mixed convection flow of a viscous, incompressible and electrically conducting fluid through a porous medium filled in a vertical channel is carried out. The walls of the vertical channel lying in the planes $y^* = \pm\frac{d}{2}$ are porous and the fluid is injected through one of the porous plates of the channel with constant velocity and simultaneously sucked through the other plate with the same velocity. The temperature of the plate at $y^* = +\frac{d}{2}$ is assumed to be varying in space and time as $T^*(y^*, z^*, t^*) = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right)$. The temperature difference of the walls of the channel is assumed high enough to induce heat transfer due to radiation. A magnetic field of uniform strength is applied perpendicular to the planes of the channel walls. The magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. The fluid is acted upon by spanwise sinusoidal fluctuating pressure gradient in the vertically upward direction. It is also assumed that the conducting fluid is optically-thin gray gas, absorbing/ emitting radiation and non-scattering. The non-linear partial differential equations governing the flow problem along with its boundary conditions are non-dimensionalized by non-similar transformation and an exact analytical solution of the problem is obtained. The velocity field, the temperature field, the amplitude and the phase angle of the skin friction are shown graphically and discussed in detail.
Keywords : Magnetohydrodynamic (MHD), spanwise sinusoidal, mixed convection, heat radiation, porous medium.

1. Introduction

Unsteady magnetohydrodynamic (MHD) mixed convection flows through porous medium always remained of considerable interest because of their occurrence in nature and varied applications in many branches of science and technology. Magnetohydrodynamic (MHD) free convection flow through porous media are very important particularly in the fields of petroleum technology for the flow of oil through porous rocks, in chemical engineering for the purification and filtration processes and in the cases like drug permeation through human skin. A detailed account of the applications of the convection flows through porous media has been reported by Nield and Bejan [10]. A number of studies have appeared in the literature in view of numerous important engineering and geophysical applications of the channel flows through porous medium, for example in the fields of agriculture engineering for channel irrigation and to study the underground water resources, in petroleum technology to study the movement of natural gas, oil and water through the oil channels/reservoirs. Raptis et al. [11,13] analyzed free convection MHD flow through a porous medium between two parallel plates. An exact solution for the forced convection flow through porous medium filled in a channel is obtained by Vafai and Kim [24]. Sahin [14] studied a free convective flow in a vertical channel through a porous medium with heat transfer. Hakiem [3] analyzed MHD oscillatory flow on free convection radiation through porous medium with constant suction velocity. Singh et al. [15] studied heat and mass transfer in an unsteady MHD free convective flow through a porous medium bounded by vertical porous channel. Hassanien and Mansour [4] investigated unsteady magnetic flow through a porous medium between two infinite parallel plates.

Generally majority of authors have treated the permeability of the porous medium as constant. In fact a porous material containing the fluid is a non-homogeneous medium and there can be numerous in homogeneities present in a porous medium. In view of this Singh and Verma [16] analyzed an oscillatory flow through porous medium with periodic permeability where the permeability varies periodically in a direction and because of which the problem becomes three dimensional. Singh et al. [17] designed a mathematical model for the fluctuating flow and heat transfer in which the permeability of the porous medium is considered to be varying in space and time both. This three dimensional problem is solved adopting complex variable notations and employing series expansion method. Singh [18] studied injection/suction effect on convective oscillatory flow through porous medium bounded by two vertical porous plates.

Thermal radiation effects may play an important role in a number industrial and environmental heat transfer processes, for example, fossil fuel combustion, heating and cooling chambers, evaporation from larger open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids and power generation systems are some other important applications of heat radiation from a vertical wall to conductive fluids. Heat radiation has important applications in science and engineering, particularly in space technology and other high temperature processes. Actually, many processes in the areas of modern engineering occur at high temperatures and the knowledge of radiation heat transfer beside the convective heat transfer becomes very important for the design of the pertinent equipment such as nuclear power plants, gas turbines and various propulsion devices for jet aircrafts, missiles, satellites and space vehicles. During heat transfer due to high temperature the fluid involved can become electrically conducting. Raptis [12] analyzed radiation and free convection through porous medium. Makinde and Mhone [7] investigated heat transfer to MHD oscillatory flow in a channel filled with porous medium. Alagoa et al. [1] studied radiative and free convective effects of a MHD flow through a porous medium between infinite parallel plates with time-dependent suction. Radiation effects on MHD free convection flow over a vertical plate with heat and mass flux has been investigated by Sivaiah, et al. [22]. Mebine [9] studied the radiation effects on MHD Couette flow with heat transfer between two parallel plates. Singh and Garg [19] analyzed radiative heat transfer in MHD oscillatory flow through porous medium bounded by two vertical porous plates.

There are many material processing industrial operations where due to high temperatures all the three modes of heat transportation such as conduction, convection and radiation accompany together. The important areas in which thermal radiation heat transfer must be considered along with thermal convection heat transfer are direct flame impingement (DFI) furnace for rapid heating of metals in materials processing (Malikov et al. [8]), heating of a continuously moving load in the industrial radiant oven (Fedorov et al. [2]) and glass melting simulation (Lentes and Siedow [6]). The high temperatures in these industrial processes may not be uniform everywhere in the processing plants. In other words these high temperatures may not necessarily be constant. It may vary over a period of time and at the same time may be different at different locations. In view of this a few studies have been conducted by considering spanwise cosinusoidal temperatures of the surfaces. Singh [20] analyzed an unsteady free convection flow past a hot vertical porous plate with variable temperature. Singh and Khem Chand [21] discussed an unsteady free convective MHD flow past a vertical porous plate with such a variation of the temperature. Sumathi et al. [23] also attempted heat and mass transfer in an unsteady three dimensional mixed convection flow past an infinite vertical porous plate with cosinusoidally fluctuating temperature. Kumar and Singh [5] studied an unsteady MHD flow of radiating and reacting fluid past a vertical porous plate with cosinusoidally fluctuating temperature.

In the present paper, an unsteady MHD mixed convection flow of a viscous, incompressible and electrically conducting fluid through a porous medium filled in a vertical channel is studied in the presence of heat radiation when the temperature of one of the channel plates varies spanwise cosinusoidally as shown in figure 1b. The two porous plates of the channel are subjected to a constant injection and suction as shown in figure 1a. A magnetic field transverse to the flow is applied and the magnetic Reynolds number is assumed very small so that the induced magnetic field is neglected. It is also assumed that the conducting fluid is optically-thin gray gas, absorbing/ emitting radiation and non-scattering. An exact solution of the mathematical problem so formed is obtained and the final results for the velocity, temperature, shear stress and heat transfer coefficient in terms of their amplitudes and phase angles are discussed in the last section of the paper.

2. Mathematical analysis

Consider an oscillatory MHD convective flow of a viscous, incompressible and electrically conducting fluid through a porous medium in a vertical channel. The insulated plates of the channel are at distance 'd' apart. The porous walls of the vertical channel are lying in the $y^* = \pm\frac{d}{2}$ planes and the fluid is injected through the left porous plate with constant velocity (V) and simultaneously sucked through the other plate with the same velocity (V). The x^*- axis is oriented vertically upwards along the centreline of the channel. The y^*-axis taken perpendicular to the planes of the plates and a transverse magnetic field of uniform strength $\vec{B} = (0, B_0, 0)$ is applied along this axis. The non-uniform temperature of the plate at $y^* = +\frac{d}{2}$ is assumed to be varying spanwise cosinusoidally in space and time both as

$$T^* = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right). \qquad \text{... (1)}$$

Since the plates of the channel are of infinite extent in the x^*-direction, therefore, all the physical quantities except the pressure are independent of x^*. All fluid properties are assumed to be constant except that the influence of density variation with temperature is considered only in the body force term. The equation of continuity $\nabla.\vec{V} = 0$ for the constant injection/suction at the channel plates integrates to $v^* = V$ where $\vec{V} = (u^*, v^*, w^*)$ represents the velocity components in the directions (x^*, y^*, z^*) respectively. The physical configuration of the problem is shown in Figure 1a & 1b.

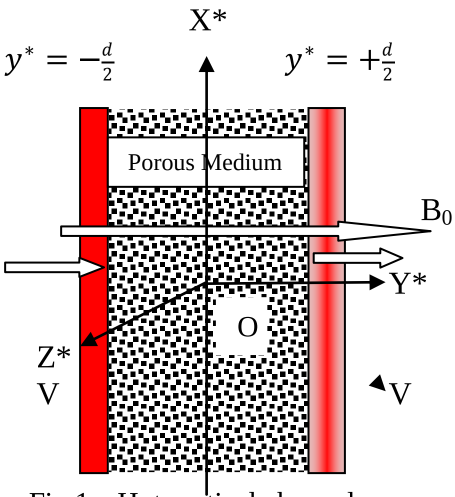

Fig.1a. Hot vertical channel.

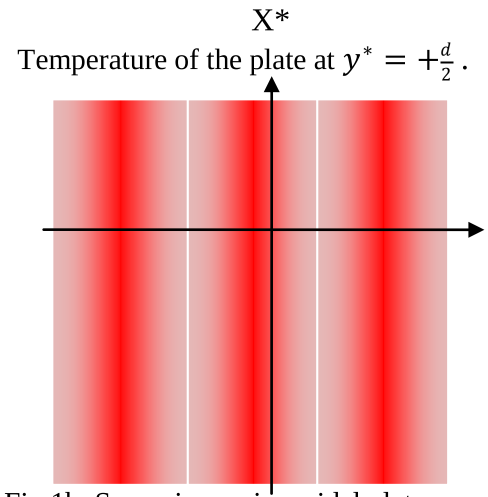

Fig.1b. Spanwise cosinusoidal plate temperature.

Using the velocity and the magnetic field distributions as stated above and taking into account the usual Boussinsq's approximation the magnetohydrodynamic (MHD) forced and free convection flow in the vertical channel is governed by the following momentum and energy differential equations:

$$\frac{\partial u^*}{\partial t^*} + V\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) - \frac{\sigma B_0^2}{\rho}u^* - \frac{\vartheta}{K^*}u^* + g\beta(T^* - T_1), \quad \text{... (2)}$$

$$\rho c_p\left(\frac{\partial T^*}{\partial t^*} + V\frac{\partial T^*}{\partial y^*}\right) = k\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) - \frac{\partial q^*}{\partial y^*}, \quad \text{... (3)}$$

where ρ is the density, ϑ is the kinematic viscosity, β is the coefficient of volume expansion, g is the acceleration due to gravity, k is the thermal conductivity, c_p is the specific heat at constant pressure and p^* is the pressure.

The last term in equation (3) stands for heat due to radiation in the y^*-direction. Following Raptis et al. (2003) the local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q^*}{\partial y^*} = 4a'\sigma'(T^{*4} - T_1^4), \quad \text{... (4)}$$

where a' is the mean absorption coefficient and σ' is Stefan- Boltzmann constant. We assume that the temperature differences within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding T^{*4} in a Taylor series about T_1 and neglecting higher order terms, thus

$$T^{*4} \cong 4T_1^{*3}T^* - 3T_1^4. \quad \text{... (5)}$$

Substituting (5) into (4) and simplifying, we obtain

$$\frac{\partial q^*}{\partial y^*} = 16a'\sigma' T_1^3(T^* - T_1). \quad \text{... (6)}$$

The substitution of equation (6) into the energy equation (3) for the heat due to radiation, we get

$$\rho c_p\left(\frac{\partial T^*}{\partial t^*} + V\frac{\partial T^*}{\partial y^*}\right) = k\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) - 16a'\sigma' T_1^3(T^* - T_1), \quad \text{... (7)}$$

The boundary conditions for the problem are

$$y^* = -\frac{d}{2}: \quad u^* = w^* = 0,\ v^* = V,\ T^* = T_1, \quad \text{... (8)}$$

$$y^* = \frac{d}{2}: \quad u^* = w^* = 0,\ v^* = V,\ T^* = T_1 + (T_2 - T_1)\cos\left(\frac{\pi z^*}{d} - \omega^* t^*\right), \quad \text{... (9)}$$

where ω^* is the frequency of oscillations.

Introducing the following non-dimensional quantities

$$x, y, z = \frac{x^*, y^*, z^*}{d},\ t = \omega^* t^*,\ u = \frac{u^*}{V},\ \theta = \frac{T^* - T_1}{T_2 - T_1}, \omega = \frac{\omega^* d^2}{\vartheta}, p = \frac{p^*}{\rho V^2}, \quad \text{... (10)}$$

into equations (2) and (7) we get

$$\omega\frac{\partial u}{\partial t}+\lambda\frac{\partial u}{\partial y}=-\lambda\frac{\partial p}{\partial x}+\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-M^2u-K^{-1}u+Gr\,\theta\ , \qquad \text{... (11)}$$

$$\omega Pr\frac{\partial \theta}{\partial t}+\lambda Pr\frac{\partial \theta}{\partial y}=\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)-N^2\theta\ , \qquad \text{... (12)}$$

where ‘*’ represents the dimensional physical quantities,

$\lambda=\frac{Vd}{\vartheta}$ is the injection/suction parameter,

$M=B_0d\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number,

$K=\frac{K^*}{d^2}$ is the permeability of the porous medium,

$Gr=\frac{g\beta\, d^2(T_2-T_1)}{\vartheta V}$ is the Grashof number,

$Pr=\frac{\mu c p}{k}$ is the Prandtl number,

$N=4\sqrt{\frac{a'\sigma' T_1^3 d^2}{k}}$ is the radiation parameter.

The boundary conditions in the dimensionless form become

$$y=-\frac{1}{2}: u=w=0,\ \theta=0, \qquad \text{... (13)}$$

$$y=\frac{1}{2}: u=w=0,\ \theta=\cos(\pi z-t)\ . \qquad \text{... (14)}$$

3. Solution of the problem

In order to obtain the solution of this flow in the porous channel when the fluid is acted upon by an unsteady periodic drop in pressure, we assume the solution in complex variable notations as

$$u(y,z,t)=u_0(y)e^{i(\pi z-t)},\quad T(y,z,t)=\theta_0(y)e^{i(\pi z-t)},\quad -\frac{\partial p}{\partial x}=Ae^{i(\pi z-t)}, \qquad \text{... (15)}$$

where A is a constant. The real part of the solution will have physical significance.

The boundary conditions (13) and (14) can also be written in complex notations as

$$y=-\frac{1}{2}: u=0,\ \theta=0, \qquad \text{... (16)}$$

Substituting expressions (15) into equations (11) and (12), we obtain following equations

$$u_0''-\lambda u_0'-(\pi^2+M^2+K^{-1}-i\omega)u_0=-\lambda A-Gr\theta_0\ , \qquad \text{... (18)}$$

$$\theta_0''-\lambda Pr\theta_0'-(\pi^2+N^2-i\omega Pr)\theta_0=0\ , \qquad \text{... (19)}$$

where the primes in these ordinary differential equations denote differentiation with respect to y. The boundary conditions (16) and (17) reduce to

$$y = -\tfrac{1}{2}:\ u_0 = 0,\quad \theta_0 = 0, \qquad \text{... (20)}$$

$$y = \tfrac{1}{2}:\ u_0 = 0,\quad \theta_0 = 1\ . \qquad \text{... (21)}$$

The solution of equation (18) for the velocity field under the boundary conditions (20) and (21) is obtained as

$$u(y,z,t) = \left[\frac{1}{2sinh\left(\frac{m-n}{2}\right)}\left[\frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l}\left(\frac{e^{\frac{r-s}{2}}}{C_1}-\frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(e^{my-\frac{n}{2}}-e^{ny-\frac{m}{2}}\right)\\ +\left(\frac{C_1-C_2}{C_1C_2}\right)\left(e^{my+\frac{n}{2}}-e^{ny+\frac{m}{2}}\right)e^{-\frac{\lambda Pr}{2}}\end{array}\right\} + \frac{2\lambda A}{C_3}\left(e^{my}\, sinh\tfrac{n}{2} - e^{ny}\, sinh\tfrac{m}{2}\right)\right] + \frac{\lambda A}{C_3} - \frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left(\frac{e^{ry-\frac{s}{2}}}{C_1}-\frac{e^{sy-\frac{r}{2}}}{C_2}\right)\right] e^{i(\pi z-t)}, \qquad \text{... (22)}$$

where $C_1 = r^2 - \lambda r - C_3$, $C_2 = s^2 - \lambda s - C_3$, $C_3 = (\pi^2 + M^2 + K^{-1} - i\omega)$,

$$m = \frac{\lambda + \sqrt{\lambda^2 + 4(\pi^2 + M^2 + K^{-1} - i\omega)}}{2}, \qquad n = \frac{\lambda - \sqrt{\lambda^2 + 4(\pi^2 + M^2 + K^{-1} - i\omega)}}{2},$$

$$r = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(\pi^2 + N^2 - i\omega Pr)}}{2}, \qquad s = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(\pi^2 + N^2 - i\omega Pr)}}{2}.$$

Similarly, the solution of equation (19) for the temperature field under the boundary conditions (20) and (21) is obtained as

$$\theta(y,z,t) = \left(\frac{e^{ry-\frac{s}{2}} - e^{sy-\frac{r}{2}}}{2sinh\left(\frac{r-s}{2}\right)}\right) e^{i(\pi z-t)}\ . \qquad \text{... (23)}$$

From the velocity field obtained in equation (22) we can get the skin-friction τ_L at the left plate (y = -0.5) in terms of its amplitude |F| and phase angle φ as

$$\tau_L = |F|\ \cos(\pi z - t + \varphi)\ , \text{ with} \qquad \text{... (24)}$$

$$F = F_r + i\,F_i$$
$$= \frac{1}{2sinh\left(\frac{m-n}{2}\right)}\left[\frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{c}\left(\frac{e^{\frac{r-s}{2}}}{C_1}-\frac{e^{-\frac{r-s}{2}}}{C_2}\right)(m-n)e^{-\frac{\lambda}{2}}+\\ \left(\frac{C_1-C_2}{C_1C_2}\right)\left(me^{-\frac{m-n}{2}}-ne^{\frac{m-n}{2}}\right)e^{-\frac{\lambda Pr}{2}}\end{array}\right\} + \frac{2\lambda A}{C_3}\left(me^{-\frac{m}{2}}sinh\frac{n}{2}-ne^{-\frac{n}{2}}sinh\frac{m}{2}\right)\right]$$
$$-\frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left(\frac{r}{C_1}-\frac{s}{C_2}\right)e^{-\frac{\lambda Pr}{2}}\,. \quad ...\ (25)$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle $\varphi = \tan^{-1}\frac{F_i}{F_r}$. ... (26)

Similarity, we can get the Nusselt number, Nu, the heat transfer coefficient in terms of its amplitude $|H|$ and the phase angle ψ from equation (23) for the temperature field as

$$Nu = |H|\ \cos(\pi z - t + \psi), \quad ...\ (27)$$

with $H = H_r + i\,H_i = \frac{(r-s)e^{-\frac{\lambda Pr}{2}}}{2\,sinh\left(\frac{r-s}{2}\right)}$, ... (28)

where the amplitude $|H|$ and the phase angle ψ of the rate of heat transfer are given as

$$|H| = \sqrt{H_r^2 + H_i^2},\quad \psi = \tan^{-1}\frac{H_i}{H_r}\ . \quad ...\ (29)$$

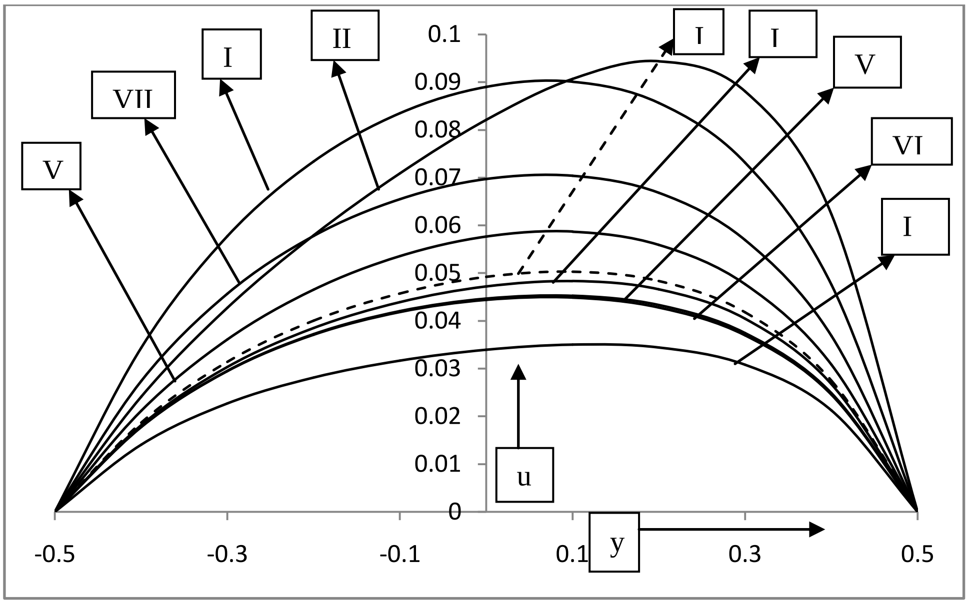

Fig. 2. Velocity profiles for z=0.5 and t=π/2.

Table 1. Sets of parameter values plotted in Figure 2.

λ	Gr	M	K	Pr	N	A	ω	Curve
0.5	1	2	0.2	0.7	1	2	1	I (---)
1.0	1	2	0.2	0.7	1	2	1	II
0.5	5	2	0.2	0.7	1	2	1	III
0.5	1	4	0.2	0.7	1	2	1	IV
0.5	1	2	1.0	0.7	1	2	1	V
0.5	1	2	0.2	7.0	1	2	1	VI
0.5	1	2	0.2	0.7	5	2	1	VII
0.5	1	2	0.2	0.7	1	3	1	VIII
0.5	1	2	0.2	0.7	1	2	5	IX

Table 2. Sets of parameter values plotted in Figs. 3 & 4.

λ	Gr	M	K	Pr	N	A	Curve
0.5	1	2	0.2	0.7	1	2	I (---)
1.0	1	2	0.2	0.7	1	2	II
0.5	5	2	0.2	0.7	1	2	III
0.5	1	4	0.2	0.7	1	2	IV
0.5	1	2	1.0	0.7	1	2	V
0.5	1	2	0.2	7.0	1	2	VI
0.5	1	2	0.2	0.7	5	2	VII
0.5	1	2	0.2	0.7	1	3	VIII

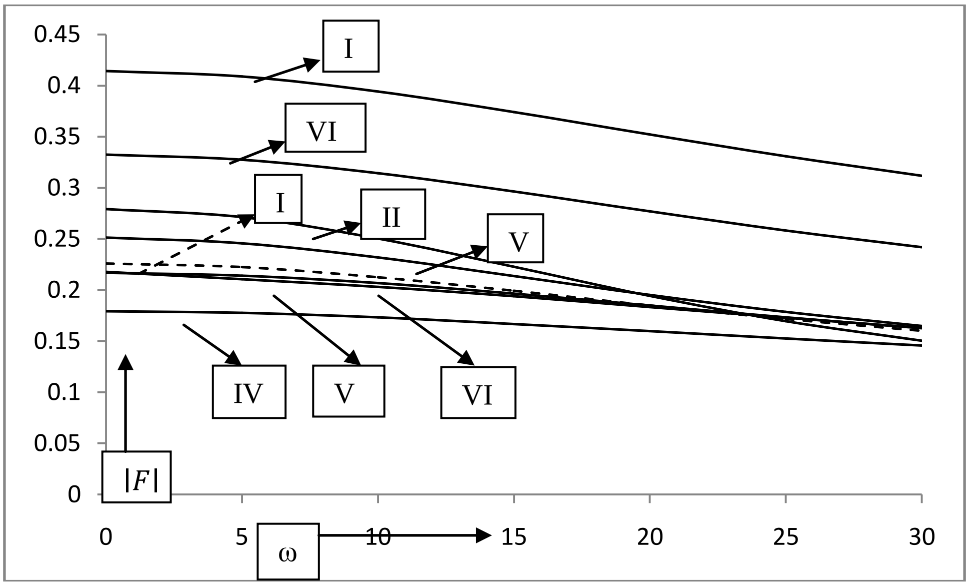

Fig. 3. Amplitude $|F|$of skin-friction.

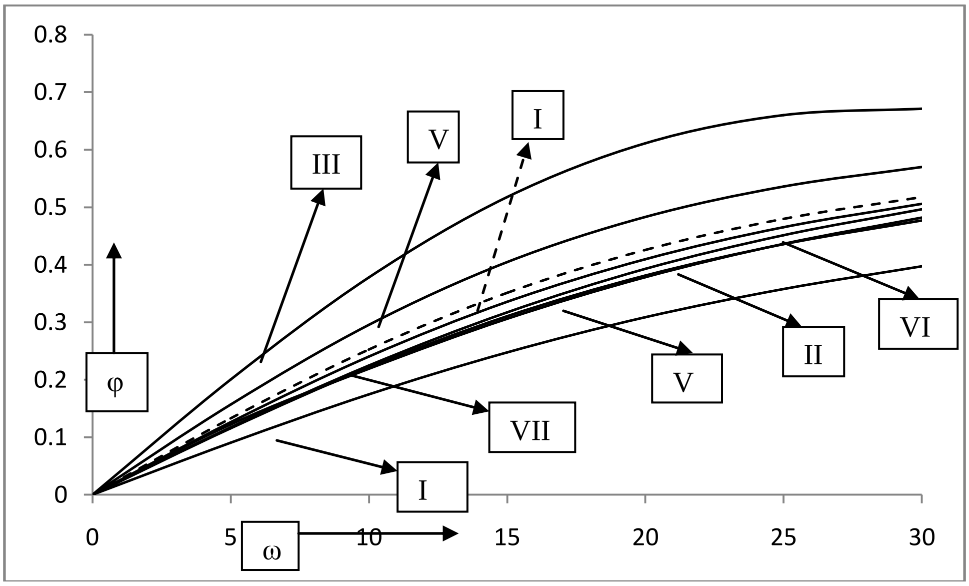

Fig. 4. Phase angle φ of the skin-friction.

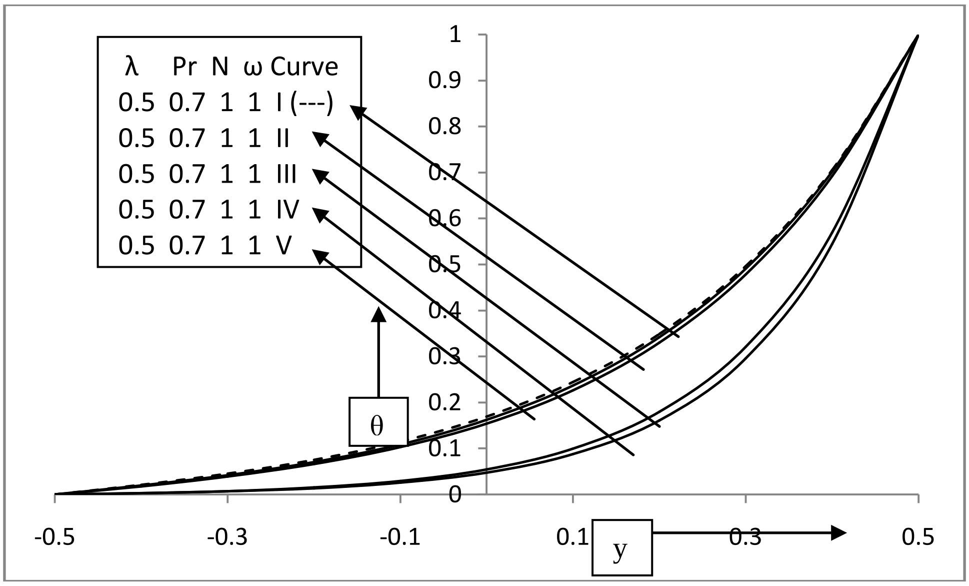

Fig. 5. The temperature field for z=0.5 and t=π/2.

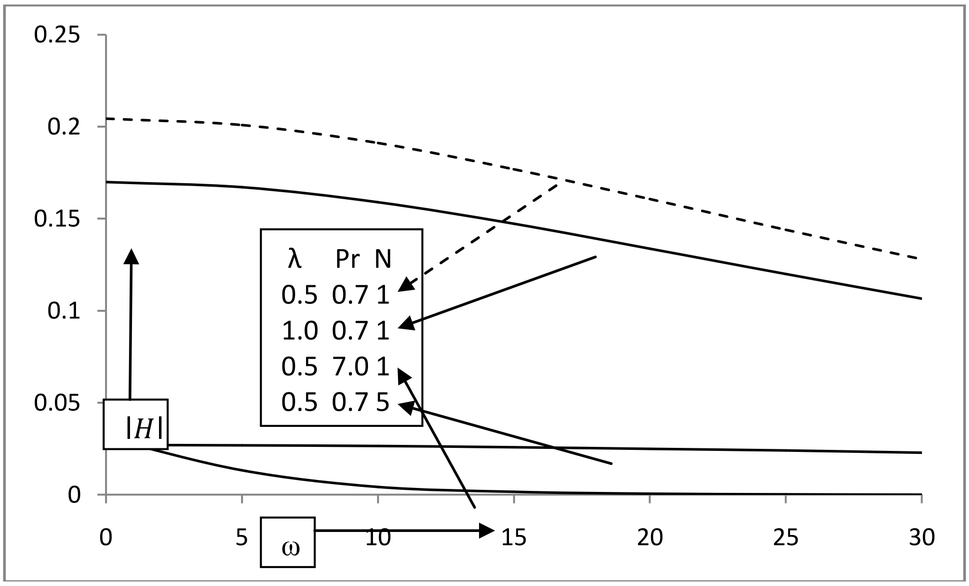

Fig. 6. Amplitude $|H|$ of the rate of heat transfer.

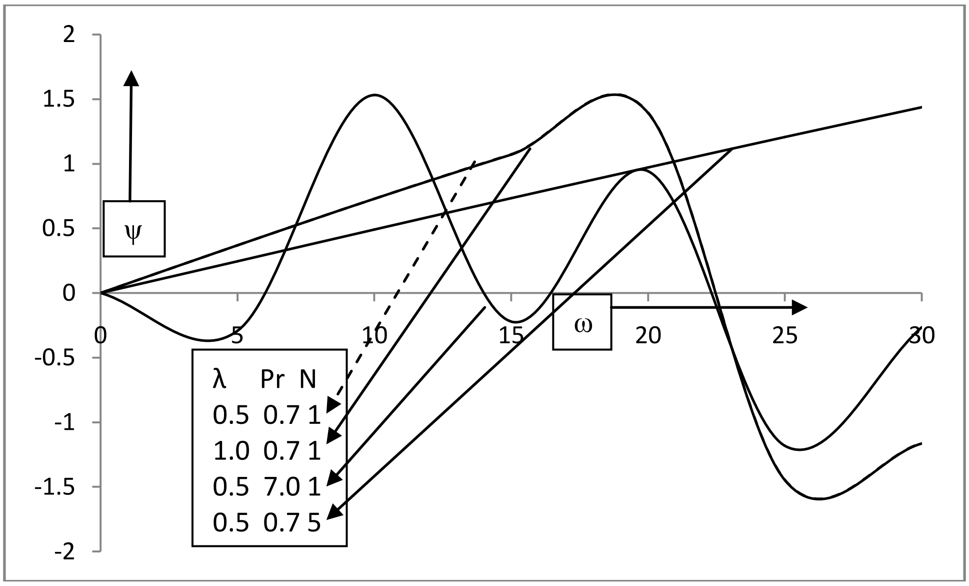

Fig. 7. Phase angle ψ of the rate of heat transfer.

4. Results and Discussion

An exact analytical solution of the problem of MHD free convection flow through a porous medium bounded by two infinite vertical porous plates is obtained when the plate temperature of the channel varies spanwise cosinusoidally. The two porous plates are subjected to constant injection and suction. It is also assumed that the conducting fluid is optically-thin gray gas, absorbing/ emitting radiation and non-scattering. The analytical results obtained in the previous section are evaluated numerically for different sets of values of the parameters involved in the flow field. In order to have a better insight of the influence of the parameters on the velocity and temperature fields these numerical values are then illustrated through figures. The influence of each of the parameters on the physical quantities like the velocity, the temperature, the amplitude and the phase of the skin-friction are depicted through these figures.

The effect on the velocity field u(y, z, t) with the increase of different parameters over the width of the channel are shown in Figure 2. This figure clearly shows that the velocity is maximum in the middle of the channel which leads to parabolic velocity profiles in the channel as expected. To assess the influence of each of the parameter on the velocity every curve is compared with the dotted curve I (---). This figure clearly shows that curves II, III, V and VIII lie above the dotted curve I (---) which means that the velocity increases with the increase of injection/suction parameter λ, Grashof number Gr, permeability of the porous medium K and favorable pressure gradient A respectively. There is a sharp rise in the velocity with the increase of the injection/suction parameter λ. The increase of velocity with the increase of the Grashof number Gr physically means that the enhancement of the buoyancy force leads to increase of the velocity component u(y, z, t) of the velocity. The increase of velocity with the increase of permeability of the porous medium indicates that the resistance posed by the porous medium reduces as the permeability of the medium increases because of which the velocity increases. As expected the larger favorable pressure gradient in the channel leads to faster flow, hence, velocity increases.

The effects of other parameters like Hartmann number M, Prandtl number Pr, radiation parameter N and frequency of oscillations ω are represented by curves IV, VI, VII and IX respectively. These curves show smaller values than those of dotted curve I (---). This means that the flow velocity decreases with the increase of these parameters. Lorentz force which is introduced due to the application of the transverse magnetic field retards the velocity. This force gives a dragging effect on the flow. The two values of the Prandtl number chosen are Pr=0.7 and Pr=7 which represent air and water respectively, most commonly found fluids on the planet earth. It is evident that the velocity is less in water than in air. Since the Prandtl number gives the relative importance of viscous dissipation to the thermal dissipation so for larger Prandtl number viscous dissipation is predominant and due to this velocity decreases. The increase of radiation N and the frequency ω leads to a decrease in velocity.

The variation of the amplitude |F| of the skin-friction with the increase of different parameters like the injection/suction parameter λ, Grashof number Gr, Hartmann number M, permeability of the porous medium K, Prandtl number Pr, radiation parameter N, and the pressure gradient is presented in Fig.3. It is obvious from this figure that for any set of parameters the amplitude goes on decreasing with the increasing frequency of oscillationsω. The skin-friction amplitude increases with the increase of injection/suction parameter λ, Grashof number Gr, permeability of the porous medium K and the pressure gradient A. It is true physically also because the increase in these parameters results into velocity increase which consequently leads to the enhancement of shear stress. However, the increase in Hartmann number M, Prandtl number Pr or the radiation parameter N attribute towards the decrease in the amplitude of the skin-friction.

The behavior of the phase angle φ of the skin-friction τ_L is shown in Figure 4 for different values of various flow parameters. From this figure it is evident that there is always a phase lead because its values computed numerically remain positive throughout for any set of values of the flow parameters. We notice from this figure that the phase lead goes on increasing with increasing frequency of oscillations ω. The phase angle increases when the value of the Grashof number Gr is increased from 1 to 5 and that of the permeability of the porous medium K from 0.2 to 1 keeping the values of all other parameters fixed. However, the phase lead reduces when the values of other parameters increased i.e., the injection/suction parameter increased from λ=0.5 to λ=1, Hartmann number increased from M=2 to M=4, Prandtl number is increased from Pr = 0.7 to Pr = 7.0, the radiation parameter is increased from N = 1 to N = 5 or the pressure gradient A=2 to 3. This variation with the Prandtl number indicates that the phase lead is less in water (Pr = 7.0) than in air (Pr = 0.7).

The variation of the temperature with the injection/suction parameter λ, Prandtl number Pr, radiation parameter N and the frequency of oscillations ω are shown in Fig. 5. The comparison of different curves with the dotted curve reveals that the temperature decreases with the increase of all these parameters. The amplitude $|H|$ and the phase angle of the rate of heat transfer against the frequency of oscillations ω are illustrated in Figures 6 and 7 respectively. It is evident from Fig. 6 that the amplitude $|H|$ decreases with the increase of injection/suction parameter λ, Prandtl number Pr and the radiation parameter N. The amplitude in the case of water (Pr =7) becomes negligible for larger values of oscillations ω. The increase of radiation parameter stabilizes the amplitude with increasing frequency of oscillations ω. Figure 7 shows that with increasing oscillations ω the phase angle ψ of the rate of heat transfer oscillates between the phase lag and the phase lead as the injection/suction parameter or the Prandtl number are increased. For the values of injection/suction parameter the amplitude remains linear initially for smaller oscillations but oscillates thereafter for larger ω. It is interesting to note that the phase lead with increasing radiation becomes linear with the increasing frequency of oscillations ω.

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