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A STUDY OF FINITE INTEGRAL INVOLVING GENERALIZED FORM OF THE ASTROPHYSICAL THERMONUCLEAR FUNCTION

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Abstract : In this paper, we evaluate a finite integral whose integrand involves the generalized form of the astrophysical thermonuclear function I_3 (z, t, ν, ρ, μ, a, α) introduced by Saxena [2]. On account of the general nature of the function I_3 occurring in the integrand, several new results can be obtained as its special cases. For the sake of illustration, we present here one special case of our main integral, which is also believed to be new.

Keywords and Phrases : Generalized form of the astrophysical thermonuclear function, Multivariable H-function.

1. Introduction

The multivariable H-function

The multivariable H-function used in the paper is defined and represented in the following form [3, p.251-252, Eq.(C.1-C.3)]

$$H[z_1,......z_r] = H^{0,B:A_1,B_1;\cdots;A_r,B_r}_{C,D:C_1,D_1;\cdots;C_r,D_r}\left[\begin{array}{c} z_1 \\ \vdots \\ z_r \end{array}\middle| \begin{array}{l} \left(a_j;\alpha_j^{(1)},...,\alpha_j^{(r)}\right)_{1,C} : \left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,C_1};...;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,C_r} \\ \left(b_j;\beta_j^{(1)},...,\beta_j^{(r)}\right)_{1,D} : \left(d_j^{(1)},\delta_j^{(1)}\right)_{1,D_1};...;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,D_r} \end{array}\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(\xi_1\cdots\xi_r)\prod_{i=1}^{r}\left\{\theta_i(\xi_i)\,z_i^{(\xi_i)}\right\}d\xi_1\ldots d\xi_r \tag{1}$$

Where $\omega = \sqrt{-1}$

$$\phi(\xi_1,...,\xi_r) = \frac{\prod_{j=1}^{B}\Gamma(1-a_j+\sum_{j=1}^{r}\alpha_j^{(i)}\xi_i)}{\prod_{j=1}^{D}\Gamma(1-b_j+\sum_{j=1}^{r}\beta_j^{(i)}\xi_i)\prod_{j=n+1}^{C}\Gamma(a_j-\sum_{j=1}^{r}\alpha_j^{(i)}\xi_i))} \quad (2)$$

$$\theta_i(\xi_i) = \frac{\prod_{j=1}^{A_i}\Gamma(d_j^{(i)}-\delta_j^{(i)}\xi_i)\prod_{j=1}^{B_i}\Gamma(1-c_j^{(i)}+\gamma_j^{(i)}\xi_i)}{\prod_{j=n+1}^{C_i}\Gamma(c_j^{(i)}-\gamma_j^{(i)}\xi)\prod_{j=m+1}^{D_i}\Gamma(1-d_j^{(i)}+\delta_j^{(i)}\xi_i)}, (i=1,2,....r) \quad (3)$$

All the Greek letters occurring on the left hand side of (1) are assumed to be positive real numbers for standardization purpose. It is assumed that multivariable H-function occurring at various places in the present paper satisfy the condition of existence corresponding appropriately to those mentioned in the book [3.p.252-253, Eq.(C.4-C.6)].

GENERALIZED FORM OF THE ASTROPHYSICAL THERMONUCLEAR FUNCTION

In the present paper, we present the generalized form of the astrophysical thermonuclear function introduced by Saxena [2] in the following form

$$I_3(z,t,\nu,\rho,\mu,a,\alpha) = \int_0^\infty y^{\nu-1}\left[1+a(\alpha-1)y^\rho\right]^{\frac{-1}{(\alpha-1)}} e^{-z(y+t)^{-\mu}}dy$$

$$= \frac{t^\upsilon}{\Gamma\left(\frac{1}{\alpha-1}\right)} H^{0,1:1,0:2,1}_{1,0:0,2:1,2}\left[\begin{array}{c}\frac{z}{t^\mu}\\ \frac{1}{a(\alpha-1)t^\rho}\end{array}\middle| \begin{array}{c}(1+\upsilon;\mu,\rho):-;(1,1)\\ -:(0,1),(1,\mu);\left(\frac{1}{\alpha-1},1\right),(\upsilon,\rho)\end{array}\right] \quad (4)$$

where $\min\{\nu,z,\rho,\mu,a\}>0, \alpha>1, |\arg(z)|<\left(\frac{2\mu}{\rho}+1\right)\frac{\pi}{2}$; t is the electron screening parameter.

2. Main Integral

$$\int_0^1 t^{\lambda-1}(1-t)^{\sigma-1}\left(1-ut^\ell\right)^{-\gamma}\left(1+vt^m\right)^{-\beta} I_3\left[z,t^{-\lambda_1}(1-t)^{-\sigma_1},\upsilon,\rho,\mu,a,\alpha\right]dt =$$

$$\frac{1}{\Gamma\left(\frac{1}{\alpha-1}\right)\Gamma(\gamma)\Gamma(\beta)} H^{0,3:1,0:2,1:1,1:1,1}_{3,1:0,2:1,2:1,1:1,1}\left[\begin{array}{c|c} z & \\ \frac{1}{a(\alpha-1)} & 0 \\ -u & \\ v & \end{array} \begin{array}{l} \left(1-\lambda+\lambda_1\upsilon;\mu\lambda_1,\rho\lambda_1,l,m\right),\left(1+\upsilon;\mu,\rho,0,0\right),(1-\sigma+\sigma_1\upsilon;\mu\sigma_1,\rho\sigma_1,0,0): \\ \left(1-\sigma-\lambda+(\sigma_1+\lambda_1)\upsilon;\mu(\sigma_1+\lambda_1),\rho(\sigma_1+\lambda_1),l,m\right): \end{array}\right.$$

$$\left.\begin{array}{llll} -; & (1,1); & (1-\gamma,1);(1-\beta,1) \\ (0,1),(1,\mu); & \left(\frac{1}{\alpha-1},1\right),(\upsilon,\rho); & (0,1);(0,1) \end{array}\right]$$

(5)

provided that the following conditions are satisfied: Re(β, γ) > 0 , $\mathrm{Re}(\lambda - \lambda_1\upsilon + \lambda_1\rho/(\alpha - 1)) > 0$, $\mathrm{Re}(\sigma - \sigma_1\upsilon + \sigma_1\rho/(\alpha - 1)) > 0$, min{υ, z, ρ , μ, a} > 0, α > 1 and | arg (z) | < (2μ/ρ + 1) < π/2 ; t is the electron screening parameter.

Derivation of the Main Integral Formula

To prove the main integral we first express generalized form of the astrophysical thermonuclear function $I_3\left[z, t^{-\lambda_1}(1-t)^{-\sigma_1}, \upsilon, \rho, \mu, a, \alpha\right]$ in the form of multivariable H-function with the help of equations (4) then write H-function in their respective contour form with the help of (1). Next, we change the order of ξ_1, ξ_2-integral with t-integral (which is permissible under the conditions stated).Thus the left hand side of (5) takes the following form (say Δ) after a little simplification:

$$\Delta = \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2} \frac{\Gamma(-\upsilon+\mu\xi_1+\rho\xi_2)\Gamma(-\xi_1)\Gamma(\xi_2)\Gamma\left(\frac{1}{\alpha-1}-\xi_2\right)\Gamma(\upsilon-\rho\xi_2)z^{\xi_1}}{\Gamma\left(\frac{1}{\alpha-1}\right)\Gamma(\mu\xi_1)\left[a(\alpha-1)\right]^{\xi 2}} \times$$

$$\left[\int_0^1 x^{\lambda-\lambda_1\nu+\lambda_1(\mu\xi_1+\rho\xi_2)-1}(1-x)^{\sigma-\sigma_1\upsilon+\sigma_1(\mu\xi_1+\rho\xi_2)-1}\left(1-ux^l\right)^{-\gamma}\left(1+vx^m\right)^{-\beta}dx\right]d\xi_1 d\xi_2$$

(6)

Now we evaluate the x-integral involved in (6) with the help of result [1, p.287, Eq. (3.211)] and write the F_1 function in the contour form, we have

$$\Delta = \frac{1}{(2\pi\omega)^4}\int_{L_1}\int_{L_2}\int_{L_3}\int_{L_4} \frac{\Gamma\left(-\upsilon+\mu\xi_1+\rho\xi_2\right)\Gamma\left(-\xi_1\right)\Gamma\left(\xi_2\right)\Gamma\left(\frac{1}{\alpha-1}-\xi_2\right)\Gamma\left(\upsilon-\rho\xi_2\right)z^{\xi_1}\Gamma\left(\sigma-\sigma_1\upsilon+\mu\sigma_1\xi_1+\rho\sigma_1\xi_2\right)}{\Gamma\left(\frac{1}{\alpha-1}\right)\Gamma\left(\mu\xi_1\right)\left[a\left(\alpha-1\right)\right]^{\xi_2}\Gamma\left(\gamma\right)\Gamma\left(\beta\right)}$$

$$\frac{\Gamma\left(\lambda-\lambda_1\upsilon+\mu\lambda_1\xi_1+\rho\lambda_1\xi_2+l\xi_3+m\xi_4\right)\Gamma\left(\gamma+\xi_3\right)\Gamma\left(\beta+\xi_4\right)\Gamma\left(-\xi_3\right)\Gamma\left(-\xi_4\right)}{\Gamma\left(\sigma+\lambda-(\sigma_1+\lambda_1)\upsilon+\mu(\sigma_1+\lambda_1)\xi_1+\rho(\sigma_1+\lambda_1)\xi_2+l\xi_3+m\xi_4\right)}\left(-u\right)^{\xi_3}\left(v\right)^{\xi_4}\, d\xi_1 d\xi_2 d\xi_3 d\xi_4 \tag{7}$$

Finally, we get the right hand side of (5) by re-interpreting the result in terms of H-function of four variables.

3. Special cases of the main integral

In the main integral (5) , if we take a = 1 and ρ = 1 , $\sigma_1 = 0$ and reduce generalized form of the astrophysical thermonuclear function (4) to astrophysical thermonuclear function studied by Anderson et al. [4].

$$\int_0^1 t^{\lambda-1}(1-t)^{\sigma-1}\left(1-ut^{\ell}\right)^{-\gamma}\left(1+vt^{m}\right)^{-\beta} I_3\left[z, t^{-\lambda_1}, \upsilon, \mu, \alpha\right]dt = \frac{1}{\Gamma\left(\frac{1}{\alpha-1}\right)\Gamma(\gamma)\Gamma(\beta)}$$

$$H^{0,3:1,0:2,1:1,1:1,1}_{3,1:0,2:1,2:1,1:1,1}\left[\begin{array}{c} z \\ \frac{1}{(\alpha-1)} \\ -u \\ v \end{array}\middle| \begin{array}{l} (1-\lambda+\lambda_1\upsilon;\mu\lambda_1,\lambda_1,l,m),(1+\upsilon;\mu,1,0,0),(1-\sigma;0,0,0,0): \quad -; \quad (1,1); \quad (1-\gamma,1);(1-\beta,1) \\ (1-\sigma-\lambda+\lambda_1\upsilon;\mu\lambda_1,\lambda_1,l,m): \quad (0,1),(1,\mu); \quad \left(\frac{1}{\alpha-1},1\right),(\upsilon,1); \quad (0,1);(0,1) \end{array}\right]$$

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