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# INHOMOGENEOUS BIANCHI TYPE $VI_0$ STRING DUST COSMOLOGICAL MODEL OF PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

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**Abstract** : We have investigated inhomogeneous Bianchi type $VI_0$ string dust cosmological model in general relativity . To obtain the deterministic solution of Einstein’s field equations, we assume that string tension density $\lambda$ is equal to rest density $\rho$ i.e. $\lambda = \rho$. The model obtained is expanding, shearing and non-rotating universe. Some physical and geometrical features of the model are also discussed.

**Keywords** : Bianchi –$VI_0$ space times, cosmic string and general relativity.

**2010 Mathematics Subject Classification** : 83CXX.83FXX

## 1. Introduction

In recent year, there has been lot of interest in string cosmology because cosmic strings play an important role in study of the early universe. Cosmic string may have been created during phase transitions in the early era [7] and they act as a source of gravitational field [10]. It is also believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures of the universe. So far a considerable amount of work has been done on cosmic strings and string cosmological models by Krori et al. [9, 8], Tikekar and Patel [16], Bali et al.[1]. Ellis and MacCallum [6] obtained solutions of Einstein's field equations for a Bianchi type $VI_0$ space-time in the case of a stiff-fluid. Collins [5] and Ruban [14] presented some exact solutions of Bianchi type $VI_0$ for perfect fluid distributions satisfying specific equations of state.

Wainwright et al.[18] have obtained some exact solutions which generalize Bianchi type-III, V, and $VI_h$ models for vacuum and for stiff perfect fluid. Carmeli et al.[3] have constructed new inhomogeneous generalisations of Bianchitype-III, V, and $VI_h$ models for vacuum and for the case in which mass less scalar fieldis present. They have also generalized certain Bianchi-type models making use of the description that inhomogeneity propagates in the form of pulses considered by Matzner and Centrella[4]. Roy and Narain[12] have derived solutions generalizing theBianchi type-I and V models for perfect fluid distribution. Roy and Narain[13] have also derived inhomogeneous generalization of Bianchi type $VI_0$ cosmological model of perfect fluid distribution. Pradhan and Bali[11] have investigated Bianchi type $VI_0$ string cosmological models in presence and absence of magnetic field. Bali et al.[2] have investigated some LRS Bianchi Type $VI_0$Cosmological Models with Special Free Gravitational Fields. Tyagi et al.[17] have obtained Bianchi type IX string cosmological models for perfect fluid distribution. Verma and Shri Ram[15]have investigated Bianchi-Type $VI_0$ Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constants.

In this paper, we have investigated inhomogeneous Bianchi type $VI_0$ string dust cosmological model in general relativity. For the complete deterministic solution of Einstein's field equations, we assume that string tension density $\lambda$ is equal to rest density $\rho$. Some physical and geometrical features of the model are also discussed.

**2. Solution of Field Equations**

We consider inhomogeneous Bianchi type $VI_0$ metric in the form:

$$ds^2 = A(x,t)\{dt^2 - dx^2\} - e^{2x}B^2(t)dy^2 - e^{-2x}C^2(t)dz^2 \qquad \text{....(1)}$$

The Einstein's field equation for a cloud string takes the form

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j = -\left(\rho v_i v^j - \lambda x_i x^j\right) \qquad \text{....(2)}$$

as given by Letelier [10].

Where $v_i$ is unit flow vector and $x_i$ satisfy conditions

$v^i v_i = -x^i x_i = 1$, and $v^i x_i = 0$

Here, $\rho$ is the rest energy of the cloud of strings with massive particles attached to them. $\rho = \rho_p + \lambda$, $\rho_p$ being the rest energy density of particles attached to the strings and $\lambda$ the density of tension that characterizes the strings. The unit space-like vector $x^i$represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector $v^i$ describes the four-velocity vector of the matter satisfying the following conditions

$g_{ij}v^i v^j = 1.$

In the present scenario, the commoving coordinates are taken as

$$v^i = \left(0,0,0,\frac{1}{A}\right)$$

and choose $x^i$ parallel to x-axis so that

$$x^i = \left(\frac{1}{A},0,0,0\right)$$

The Einstein's field equations (2.2) for the line-element (2.1) lead to the following system of equations:

$$\frac{1}{A^2}\left[\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{B_4C_4}{BC}+1\right]=\lambda \qquad \text{.... (3)}$$

$$\left(\frac{A_4}{A}\right)_4-\left(\frac{A_1}{A}\right)_1+\frac{B_{44}}{B}-1=0 \qquad \text{.... (4)}$$

$$\left(\frac{A_4}{A}\right)_4-\left(\frac{A_1}{A}\right)_1+\frac{C_{44}}{C}-1=0 \qquad \text{....(5)}$$

$$\frac{1}{A^2}\left[1-\frac{B_4C_4}{BC}-\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\right]=-\rho \qquad \text{....(6)}$$

$$\left(\frac{B_4}{B}-\frac{C_4}{C}\right)-\frac{A_1}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \qquad \text{.... (7)}$$

The suffices 1 to 4 after A, B and C denote partial differentiation with respect to x and t respectively.

From equation (4) and (5) we have

$$\frac{B_{44}}{B}=\frac{C_{44}}{C} \qquad \text{.... (8)}$$

Equation (7) leads to

$$\frac{A_1}{A} = \frac{\left(\frac{B_4}{B} - \frac{C_4}{C}\right)}{\left(\frac{B_4}{B} + \frac{C_4}{C}\right)} \quad \text{.... (9)}$$

which leads to

$$\log A = xF(t) + G(t) \quad \text{....(10)}$$

Where G(t) is a function of the time.

From equation (8)

$$CB_4 - BC_4 = b \quad \text{.... (11)}$$

Where b is a constant.

Equations (9) and (11) give

$$F = \frac{b}{(BC)_4} \quad \text{.... (12)}$$

From equations (3), (4), (6), (10), and (11) and string tension density $\lambda$ is equal to rest density $\rho$ i.e. $\lambda = \rho$, we have

$$x\left[F_{44} + F_4\left(\frac{B_4}{B} + \frac{C_4}{C}\right)\right] + \left[G_{44} + G_4\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - 2\right] = 0 \quad \text{....(13)}$$

from which we conclude that

$$F_{44} + F_4\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0 \quad \text{....(14)}$$

and

$$G_{44} + G_4\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - 2 = 0 \quad \text{.... (15)}$$

Equation (14) on integration yields

$$F_4 = \frac{a}{BC} \quad \text{.... (16)}$$

Where a is a constant.

From equations (12) and (16), we get

$$F = \beta [BC]^{\frac{a}{b}} \qquad \text{…. (17)}$$

Where $\beta$ is a constant.

From equations (12) and (17), we get

$$BC = LT^{\frac{b}{a+b}} \qquad \text{…. (18)}$$

Where $L = \left\{ \frac{a+b}{\beta} \right\}^{\frac{b}{a+b}}, T = t + t_0, t_0$ being a constant.

Equations (11) and (18) give

$$\frac{B}{C} = \alpha \exp\left[ \frac{b(a+b)}{aL} T^{\frac{a}{a+b}} \right] \qquad \text{…. (19)}$$

Where $\alpha$ is a constant.

Equation (15) on integration yields

$$G = \frac{a+b}{a+2b} T^2 + \frac{l(a+b)}{aL} T^{\frac{a}{a+b}} + k \qquad \text{…. (20)}$$

Where l and k are integration constant.

From equations (10), (17), (18) and (20), we have

$$A = \exp\left[ (Xm + S) T^{1-n} + \frac{1}{n+1} T^2 + k \right] \qquad \text{…. (21)}$$

Where

$$X = x,\ m = \beta L^{\frac{a}{b}},\ S = \frac{l(a+b)}{aL},\ n = \frac{b}{a+b}.$$

From equations (18) and (19), we have

$$B = (\alpha L)^{1/2} T^{n/2} \exp\left(\frac{b}{2L(1-n)} T^{1-n}\right) \quad \text{.... (22)}$$

and

$$C = \left(\frac{L}{\alpha}\right)^{1/2} T^{n/2} \exp\left(-\frac{b}{2L(1-n)} T^{1-n}\right) \quad \text{.... (23)}$$

By suitable transformation of coordinates and remaining constants the line element (1) reduces to the form

$$ds^2 = \exp\left[2\left\{(Xm+S)T^{1-n} + \frac{1}{n+1}T^2 + k\right\}\right][dT^2 - dX^2]$$
$$-T^n \exp\left(2X + \frac{b}{L(1-n)}T^{1-n}\right)dY^2$$
$$-T^n \exp\left\{-\left(2X + \frac{b}{L(1-n)}T^{1-n}\right)\right\}dZ^2 \quad \text{.... (24)}$$

Which may be considered as an inhomogeneous generalization of Bianchi type-$VI_0$ string cosmological model of perfect fluid distribution.

**3. Some Geometrical and Physical Properties of the Model**

The physical and geometrical properties of the model are given as fallows.

String tension density $\lambda$ and rest density $\rho$ are

$$\rho = \frac{\left[\frac{n-1}{n+1} + \frac{n^2}{4T^2} + \frac{(Xm+S)n(1-n)}{T^{n+1}} - \frac{b^2}{4L^2T^{2n}}\right]}{\exp\left[2\left\{(Xm+S)T^{1-n} + \frac{1}{n+1}T^2 + k\right\}\right]} = \lambda \quad \text{.... (25)}$$

Magnitude of rotation $\omega$ is zero i.e.

$$\omega = 0 \quad \text{.... (26)}$$

Scalar expansion ($\theta$) is given by

$$\theta = \frac{\dfrac{(Xm+S)(1-n)}{T^n} + \dfrac{2T}{n+1} + \dfrac{n}{T}}{\exp\left[(Xm+S)T^{1-n} + \dfrac{1}{n+1}T^2 + k\right]} \quad \text{.... (27)}$$

Shear ($\sigma$) is given by

$$\sigma = \frac{\left[\dfrac{4T^2}{(n+1)^2} + \left\{(n-1)^2(Xm+S)^2 + \dfrac{3b^2}{4L^2}\right\}\dfrac{1}{T^{2n}}\right.}{\sqrt{3}\exp\left[(Xm+S)T^{1-n} + \dfrac{1}{n+1}T^2 + k\right]}$$

$$\frac{\left. - \dfrac{4(n-1)(Xm+S)}{(n+1)T^{n-1}} + \dfrac{n(n-1)(Xm+S)}{T^{n+1}} + \dfrac{n^2}{4T^2} - \dfrac{2n}{n+1}\right]^{\frac{1}{2}}}{} \quad \text{.... (28)}$$

Deceleration parameter (q) is given by

$$q = -1 + \frac{3\left\{\dfrac{2n(1-n)(Xm+S)}{T^{n+1}} + \dfrac{2(n-1)}{n+1} + \dfrac{n}{T^2} + \dfrac{(n-1)^2(Xm+S)^2}{T^{2n}}\right.}{\left[(Xm+S)(1-n)T^{-n} + \dfrac{2T}{n+1} + \dfrac{n}{T}\right]^2}$$

$$\frac{\left. + \dfrac{4T^2}{(n+1)^2} + \dfrac{4(1-n)(Xm+S)}{T^{n-1}}\right\} \exp\left\{(Xm+S)T^{1-n} + \dfrac{1}{n+1}T^2 + k\right\}}{} \quad \text{.... (29)}$$

**4. Conclusion**

The model (24) starts with a big bang at T = 0 and goes on expanding till $T = \infty$ when $\theta$ becomes zero. It is clear that as T increases, the ratio of the shear scalar $\sigma$ and expansion $\theta$ tends to finite value i.e.

$$\frac{\sigma}{\theta} \to \frac{1}{\sqrt{3}} \text{ as } T \to \infty$$

Hence model does not approach isotropy for large value of T. However, we can make $\frac{\sigma}{\theta}$ less than any small arbitrary positive number by suitably choosing constants appearing in the models and for a finite period of time the models will be approximately FRW. Since the sign of the deceleration parameter q is positive for n = 1, T = 0 and $X = -\left(\frac{S+k}{m}\right)$ that yields the decelerating phase of the universe.It is observed that forsufficiently large time T, $\lambda$ tends to zero. Therefore, the strings disappear from theUniverse at a later time (i.e. present epoch).In general the model represents expanding, shearing and non-rotating universe.

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