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AN INTERIOR GRIFFITH-CRACK OPENED BY THERMAL STRESS IN AN ORTHOTROPIC INFINITE STRIP

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Abstract : The stress-intensity factors and the crack shape are obtained in closed form by using Fourier transform method. The Griffith-crack is opened by thermal stress in an infinite orthotropic strip whose edges are parallel to crack axis.

Keywords & Phrases : Interior, Griffith-Crack, Thermal Stress, Orthotropic, Strip.

1. Introduction

A considerable progress, in the field of air-craft and machine structures, mainly with gas and steam turbines, have given rise to numbers of problems in which thermal stresses play an important and frequently even a primary role. The advent of modern composite-material, which are easily amenable to weight-to-strength ratio has facilitated the aerospace industry. The composite materials are made by layers of matrix and of fibres.

Bandopadhya and Murthy [5] have proved experimentally that if the number of layers in a composite material exceeds six, then the medium of composite material can be considered as orthotropic medium. The same was proved analytically by Sharma [8]. Therefore, the problem is of practical importance.

Singh [9] has solved a Griffith-crack opened by thermal stress in an anisotropic medium. Author [1-4] had solved for three and four Griffith-cracks opened by thermal stress is an infinite orthotropic medium and in an infinite strip whose edges as are perpendicular to crack-axis.

The title problem can be thought of that where there is an infinite row of Griffith-cracks of length 2c and equispaced with distance 2δ, see figure 1. Then we choose one Griffith-crack (any one) and take *x*-axis parallel to crack axis and *y*-axis passing through middle of the crack and perpendicular to *x*-axis, see figure 2, with the boundary conditions prescribed in figure.

It is assumed that the medium is such that elastic properties are not effected by temperature distribution. It is also assumed that the work done by elastic movement, too, does not effect the temperature distribution. Elastic symmetry coincides with co-ordinate axes. The medium is in plane-strain condition.

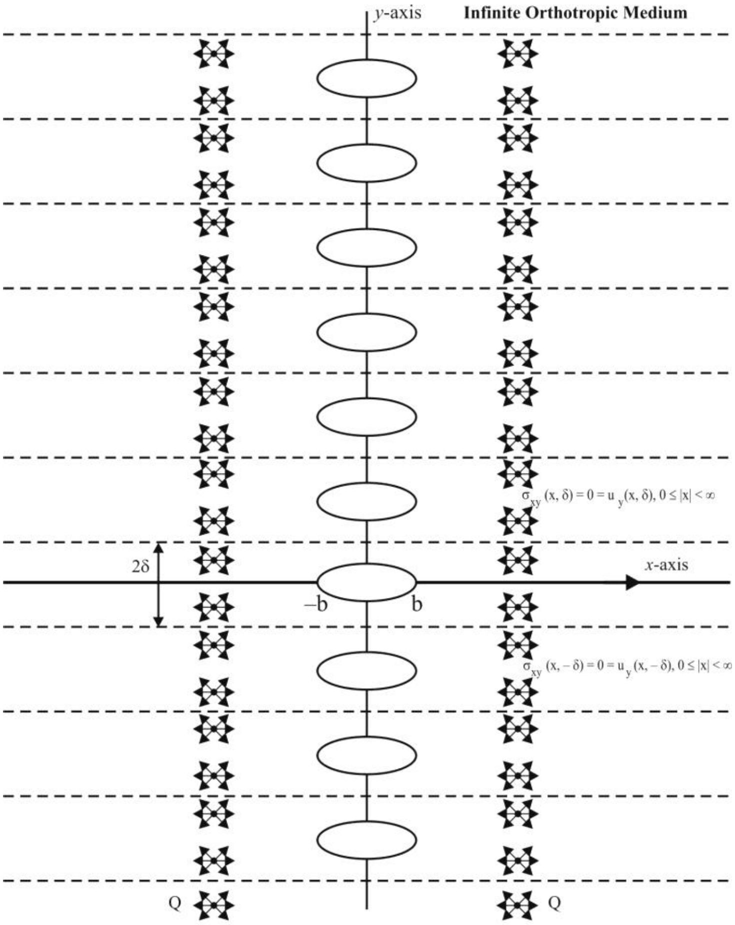

Figure 1 : VERTICAL ROW OF GRIFFITH CRACKS OPENED BY HEAT SOURCE WHICH ARE REDUCED TO AN INFINITE ORTHOTROPIC STRIP.

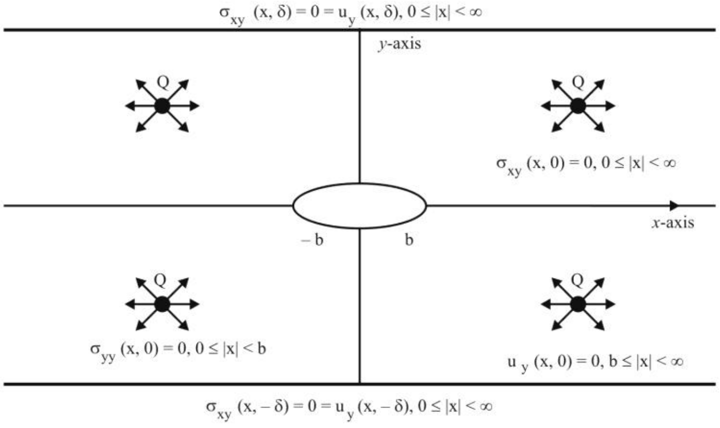

Figure 2 : GEOMETRY OF CRACK OPENING WITH THERMAL STRESS AND BOUNDARY CONDITIONS IN AN INFINITE STRIP.

The physical problem, as shown in figure 1 & 2, is translated into mathematical model of mixed-boundary value problem, as

$$\sigma_{xy}(x,\pm\delta) = 0, u_y(x,\pm\delta) = 0,\ \sigma_{xy}(x,0) = 0, 0 \le |x| < \infty \qquad (1)\text{-}(2)$$

$$u_y(x,0) = 0, b \le |x| < \infty\ ,\ \sigma_{yy}(x,0) = 0, 0 \le |x| < b \qquad (3)\text{-}(4)$$

Thus we are to solve the problem of mixed-boundary value given by (1)-(4) over the domain $[-\delta,\delta] \cup (-\infty,\infty)$. But the symmetry of geometry and the symmetry of orthotropy coinciding with co-ordinate axes in the problem above is reduced to the following :

$$\sigma_{xy}(x,\delta) = 0, u_y(x,\delta) = 0,\ \sigma_{xy}(x,0) = 0, 0 \le x < \infty \qquad (5)\text{-}(6)$$

$$u_y(x,0) = 0, b \le x < \infty\ ,\ \sigma_{yy}(x,0) = 0, 0 \le x < b \qquad (7)\text{-}(8)$$

The domain of solution is now $[0,\delta] \cup [0,\infty)$. We checked throughout as given by Burniston [6], That

$$u_y(x,0) > 0, \quad 0 \le |x| < b \qquad (9)$$

Which means that crack faces do not meet each other otherthan at crack tips. We have divided the physical quantities at a general point (*x*, *y*) of the medium into two namely [A] Heat Problem [B] Elasticity Problem.

$$\sigma_{ij}(x,y) = \sigma_{ij}^{(b)}(x,y) + \sigma_{ij}^{(c)}(x,y),\ i, j = x, y \qquad (10)$$

$$u_i(x,y) = u_i^{(h)}(x,y) + u_i^{(e)}(x,y),\ i = x, y \qquad (11)$$

The boundary conditions for heat distribution are given as

$$\sigma_{xy}{}^{h}(x,\delta) = 0, u_y^{(h)}(x,\delta) = 0, 0 \le x < \infty \qquad (12)$$

$$\sigma_{xy}{}^{h}(x,0) = 0, 0 \le x < \infty\ ,\ u_y{}^{(h)}(x,0) = 0, 0 \le x < \infty \qquad (13)\text{-}(14)$$

Thus the boundary conditions (5)-(8), after using (10)-(11) and (12)-(14) in (5)-(8), reduce to the following conditions for elasticity problem.

$$\sigma_{xy}{}^{(e)}(x,\delta) = 0,\ u_y{}^{(e)}(x,0) = 0,\ \ \sigma_{xy}{}^{(e)}(x,0) = 0, \quad 0 \le x < \infty \qquad (15)\text{-}(16)$$

$$u_y{}^{(e)}(x,0) = 0, \quad b \le x < \infty\ ,\ \ \sigma_{yy}{}^{(e)}(x,0) = -\sigma_{yy}{}^{(h)}(x,0) \quad 0 \le x < b \qquad (17)\text{-}(18)$$

The plan of the paper is as follows : in section 2, the problem is formulated. The mixed-boundary value problem is reduced to triple integral equation whose solution is also given in section 3. The solution of Fredholm integral equation is given in section 4.

The physical quantities are given in section 5. An example of special heat source is given in section 6, section 7 presents discussion and conclusion.

2. Formulation of Problem

HEAT PROBLEM

The solution of heat problem is obtained by taking appropriate finite Fourier transform with respect to variable *y* and Fourier transforms with respect to variable *x* of equation of equilibrium, in absence of body force along with transform of stress-strain relations, too, as

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}=0,\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}=0 \tag{19}$$

With stress-strain relations as,

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}=\frac{1}{b}\begin{bmatrix}a_{22} & -a_{12} & 0\\ -a_{12} & a_{11} & 0\\ 00 & 0 & \frac{b_0}{a_{66}}\end{bmatrix}\begin{bmatrix}e_x\\ e_y\\ e_{xy}\end{bmatrix}-\begin{bmatrix}k_1\\ k_2\\ 0\end{bmatrix}T \tag{20}$$

Where T satisfies the equation

$$\left(k_1\frac{\partial^2}{\partial x^2}+k_2\frac{\partial^2}{\partial y^2}\right)T=bQ(x,y) \tag{21}$$

$a_{11}\sim a_{66}$ are elastic constants

$$k_1=\alpha_{t_1}(a_{11}-a_{12}),k_2=\alpha_{t_2}(a_{22}-a_{22}),b_0=a_{11}a_{22}-a_{12}^2 \tag{22}$$

α_{t_1} and α_{t_2} are the coefficients of linear expansions along *x* and *y* axes, respectively. $Q(x,y)$ is known function of temperature.

$$u_x^{(h)}(x,y)=\frac{1}{2}u_{x_c}^{(h)}(x,0)+\frac{4}{\pi\delta}\sum_{n=1}^{\infty}\int_0^{\infty}\sin(\xi x)\cos(p_n y)u_{x_{sc}}^{(h)}(\xi,p_n)d\xi \tag{23}$$

$$u_y^{(h)}(x,y)=\frac{4}{\pi\delta}\sum_{n=1}^{\infty}\int_0^{\infty}\cos(\xi x)\sin(p_n y)u_{y_{sc}}^{(h)}(\xi,p_n)d\xi, \tag{24}$$

$$u_{x_{sc}}^{(h)}(\xi,p_n)=Q_{cc}(\xi,p_n)\xi^2[k_3p_n^2+k_1b\xi^2a_{11}a_{66}p_n^2]W_1 \tag{25}$$

$$u_{y_{cs}}^{(h)}(\xi,p_n)=Q_{cc}(\xi,p_n)p_n^2[k_2(\xi^2a_{22}a_{66}+bp_n^2)+k_4\xi^2]W_1 \tag{26}$$

Making use of stress-strain relations (20) and the displacement components (23) – (26). We get stress components corresponding to heat problem as,

$$\sigma_{xx}^{(h)}(x,y)=\frac{1}{2}\sigma_{xxcc}^{(h)}(\xi,0)+\frac{4}{\pi h}\sum_{n=1}^{\infty}Q_{cc}(\xi,p_n)\cos(\xi x)\cos(p_n y)\sigma_{xxcc}^{(h)}(\xi,p_n)d\xi \tag{27}$$

$$\sigma_{yy}^{(h)}(x,y)=\frac{1}{2}\sigma_{yycc}^{(h)}(\xi,0)+\frac{4}{\pi h}\sum_{n=1}^{\infty}\int_0^{\infty}Q_{cc}(\xi,p_n)\cos(\xi x)\cos(p_n y)\sigma_{yycc}^{(h)}(\xi,p_n)d\xi \tag{28}$$

$$\sigma_{xxcc}^{(h)}(\xi,p_n)=\left[k_1p_n^2(\xi^2k_6+p_n^2)ba_{12}+k_2p_n^2(\xi^2k_{11}-p_n^2k_{12})\right]/W$$

$$\sigma_{yycc}^{(h)}(\xi,p_n)=\left[k_2p_n^2(k,\xi^2k_{10})+k_1p_n^2(\xi^2k_{11}-p_n^2k_{12})\right]/W$$

$$\sigma_{xy}^{(h)}(x,y)=\frac{-4}{\pi\delta}\sum_{n=1}^{\infty}\int_0^{\infty}Q_{cc}(\xi,p_n)\sin(\xi x)\sin(p_n y)\sigma_{xy_{ss}}^{(h)}(\xi,p_n) \tag{29}$$

$$\sigma_{xy}^{(h)}(\xi,p_4)=\frac{Q_{cc}(\xi,p_n)}{W}\left[(k_3+k_4)\xi p_n^2+k_1\xi(\xi^2b_0+p_n^2a_{11}a_{66})+k_2p_n(p_n^2b_0+\xi a_{22}^2a_{66})\right]$$

$$\left.\begin{array}{l} k_3=k_5k_2,k_4=K_5k_1,k_5=a_{12}a_{66}-b_0 \\ k_6=2a_{12}a_{22}a_{66}-ba_{22},k_7=b_0a_{22}a_{66}, \\ k_8=a_{66}-ba_{22}=-k,k_{10}=b_0a_{11} \\ k_{11}=k_{10}-2a_{11}a_{12}a_{66},k_{12}=b_0a_{12} \end{array}\right\} \tag{30}$$

$$W(\xi,p_n)=p_n^4+2B_1p_n^2\xi^2+B_2\xi^4, \tag{31}$$

$$2B_1=(2a_{12}+a_{66})/a_{11},B_2=\frac{a_{22}}{a_{11}},p_n=\frac{n\pi}{\delta} \tag{32}$$

We can obtain stress components by using (23)-(28) along with stress-strain relation (20).

ELASTICITY PROBLEM

The solution of elasticity problem is obtained by the method of Kushwaha [7]. The displacement components are assumed as

$$u_x^{(e)}(x,y)=\frac{2}{\pi}\int_0^{\infty}\xi^{-1}[a_{11}H,yy-a_{12}\xi^2H]\sin(\xi x)d\xi \tag{33}$$

$$u_y^{(e)}(x,y)=\frac{2}{\pi}\int_0^{\infty}\xi^{-1}[a_{11}H,yyy-\xi^2(a_{12}+a_{66})H,y\cos(\xi x)d\xi \tag{34}$$

With H as

$$(r_1+r_2)+H(\xi,y)=A\cos(r_1\xi y)+B\sinh(r_1\xi y)+c\cosh(r_2\xi y)+D\sinh(r_2\xi y) \tag{35}$$

Where A, B, C and D are four arbitrary constants to be determined and r_1^2 and r_2^2 are two real roots of

$$r^4 - 2B, r^2 + B_2 = 0 \tag{36}$$

The stress-components are given as below

$$\sigma_{yy}^{(e)}(x, y) = -\frac{2}{\pi}\int_0^\infty \xi^2 \cos(\xi x) H(\xi, y) d\xi \tag{37}$$

$$\sigma_{xy}^{(e)}(x, y) = -\frac{2}{\pi}\int_0^\infty \xi^2 \sin(\xi x)[Ar_1 \sin(r, \xi y) + r_1 B \cosh(\xi r_1 y) + r_2 C \sinh(\xi r_2 y) + r_2 D \cosh(\xi r_2 y)] d\xi \tag{38}$$

$H(\xi, y)$ is given by (35).

3. Reduction to and Solution of Dual Integral Equation Reduction

The boundary conditions for heat problem given by (12)-(14) and relations in (24) and (29) given that there satisfied identically. The boundary conditions given by (15)-(16) for elasticity problem and relations (34), (37) and (38) lead to

$$Ab_1 + Bb_2 + b_3 C + Db_4 = 0, Ab_5 + Bb_6 + b_7 C + Db_8 = 0, B = -\frac{r_2}{r_1} D \tag{39}$$

Thus we get three arbitrary constants in terms of fourth. We will solve A, B, C in terms of D. And b$_i$s' is are given as,

$$b_1 = \alpha_1' b_5, b_2 = \alpha_1' b_6, b_3 = \alpha_2 b_7, b_4 = \alpha_2' b_8$$

$$b_5 = r_1 \sinh(\xi r_1 \delta), b_6 = r_1 \cosh(\xi r_1 \delta), b_7 = r_2 \sinh(r_2 \xi \delta), b_8 = r_2 \cosh(\xi r_2 \delta)$$

$$\alpha_i' = \alpha_i - a_{66}, \alpha_i = a_{11} r_i^2 - a_{12}, i = 1, 2 \tag{40}$$

The mixed-boundary conditions (17) –(18) and using (39)-(40) for constants A, B and C in terms of D.

$$\int_0^\infty D(\xi) \cos(\xi x) d\xi = 0, b \le x < \infty \tag{41}$$

$$\int_0^\infty \xi^2 D(\xi) M_1(\xi) \cos(\xi x) d\xi = +\frac{\pi}{2}(r_1 - r_2)\sigma_{yy}^{(h)}(x, 0) \tag{42}$$

And thus writing

$\xi D(\xi) = \phi(\xi)$ in (41) - (42) and a little manipulation, we get

$$\int_0^\infty \phi(\xi) \cos(\xi x) d(\xi) = 0, b \le x < \infty \tag{43}$$

$$\int_0^\infty \xi\phi(\xi)\cos(\xi x)d\xi = F_1(x), 0 \le x < b \tag{44}$$

$$F_1(x) + \frac{\pi}{2}(r_1 - r_2)\sigma_{yy}^{(h)}(x,0) - \int_0^\infty \xi\phi(\xi)M_1(\xi)\cos(\xi x)d\xi \tag{45}$$

$$M_1(\xi) = \frac{b_{10}}{b_9} + \frac{b_{12}}{b_{11}} - 1 \tag{46}$$

$$\left.\begin{array}{l} b_9 = b_1b_7 - b_3b_5, b_{10} = r_{21}(b_2b_7 - b_3b_6) - (b_4b_7 - b_3b_8) \\ b_{11} = b_3b_5 - b_1b_7, b_{12} = r_{21}(b_2b_5 = b_1b_6) - (b_5b_4 - b_1b_8) \\ r_{21} = r_2 / r_1 \end{array}\right\} \tag{47}$$

SOLUTION OF INTEGRAL EQUATION

The solution of dual integral equation (43) – (44) is obtained by the method of Srivastava and Lowengrub [10]. We assume the trial solution as

$$\pi\phi(\xi) = 2\int_0^b g(t)\sin(\xi t)dt \tag{48}$$

With no loss of generality g(0) = 0

Now we substitute (48) in (43) and use the integral

$$\int_0^\infty \frac{\sin(\xi x)\cos(\xi t)d\xi}{\xi} = \begin{cases} \pi/2, & x > t \\ \pi/4, & x = t \\ 0 & x < t \end{cases}$$

The equation is satisfied identically. Then we use (48) in (44) then use the method of Srivastava and Lowengrub, we get,

$$g(t) = \frac{2t}{\pi^2\sqrt{b^2 - t^2}}\left[\Delta_0(t) + \int_0^b K(\alpha,t)g(\alpha)d\alpha\right] \tag{49}$$

$$\left.\begin{array}{l} \Delta_0(t) = \dfrac{\pi}{2}\displaystyle\int_0^b \frac{\sqrt{b^2 - x^2}\sigma_{yy}^{(h)}(x,0)}{x^2 - t^2} \\ K(\alpha,t) = \displaystyle\int_0^b \frac{\sqrt{b^2 - x^2}}{x^2 - t^2}.H_1(\alpha,x)dx \\ H_1(\alpha,x) = \displaystyle\int_0^\infty M(\xi)\sin(\xi\alpha)\cos\xi(\xi x)d\xi \end{array}\right\} \tag{50}$$

Thus, the physical problem is reduced to Fredholm integral equation of second kind given by (49). In next section we will solve this Fredholm integral equation by the Method of approximation.

4. Solution of Fredholm Integral Equation of Second Kind

First we approximate $M(\xi)$ in terms of $\{\delta^{-1}\}$ from (46), then use this in third of (50) and evaluating the integral we get $H_1(\alpha, x)$ which will be substituted in second of (50) and evaluating the integrals $K(\alpha, t)$ will be evaluated.

$$H_1(\alpha, x) = \sum_{k=0}^{\infty} \psi(\alpha, x; k) P_6(k) \tag{51}$$

$$P_6(k) = \alpha_3^{-}(p_2 - p_3) + \alpha_3^{+} P_4 - r_2(\alpha_1 - r_2) P_5$$

$$P_2 = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} \frac{1}{(lr_1 + mr_2)^{2k+2}}, P_3 = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} \frac{1}{[(l+1)r_1 + mr_2]^{2k+2}}$$

$$P_4 = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} \frac{1}{[lr_1 + (m+1)r_2]^{2k+2}}, P_5 = \sum_{m=0}^{\infty}\sum_{l=0}^{\infty} \frac{1}{[(l+1)r_1 + (m+1)r_2]^{2k+2}}$$

$$\psi(\alpha_1 x; k) = 2\sum_{r=0}^{k} \alpha^{2k+1-2r}\ {}^{2R+1}C_{2r} x^{2k}$$

$$\alpha_3^{\pm} = r_2(\alpha_1 - \alpha_2) \pm 2r_1\alpha_1,$$

$\alpha_1, \alpha_2, \alpha_1'$ are given in (40). And then

$$K(\alpha, t) = \sum_{k=0}^{\infty} \frac{(-1)^k P_6(k)}{(2\delta)^{2k+2}} \sum_{r=0}^{k} \alpha^{2k+1-2r}\ {}^{2k+1}C_{2r}^{M(k,t)} \tag{52}$$

$$\left.\begin{aligned}
M_2(k,t) &= \left[\frac{\sqrt{\pi} b^{2k}\sqrt{\dfrac{2k+1}{2}}}{2(k!)} + (b^2 - t^2) I_{2k}(t)\right] \\
I_{2k}(t) &= \frac{\sqrt{\pi} b^{2k-2}\sqrt{\dfrac{2k-1}{2}}}{2(k-1)!} + t^2 I_{2k-2}(t) \\
I_0(t) &= -\left\{\begin{array}{c} 0, 0 \le t < b \\ \dfrac{\pi}{2\sqrt{t^2 - b^2}}, t > b \end{array}\right\}
\end{aligned}\right\} \tag{53}$$

Thus the Fredholm integral equation (49) is given as

$$g(t) = \frac{t}{\pi\sqrt{b^2 - t^2}}\left[\Delta_0(t) + \sum_{k=0}^{\infty}\sum_{r=0}^{k}\frac{(-1)^k\, p_6(k)}{(2\delta)^{2k+2}}\Delta_1 t\right] \tag{54}$$

$$\Delta_1(t) = \int_0^b g(\alpha)K_1(\alpha, t; k)\,d\alpha$$

$$K(\alpha, t; k) = \alpha^{2k+1-2r2k+1}C_{2r}M_2(k, t)$$

$M_2(k, t)$ is given by first of (53).

Now we shall assume that

$$g(t) = \sum_{i=0}^{\infty} g_i(t)\delta^{-i} \tag{55}$$

Substituting (55) in (54) and then comparing the coefficients of $\{\delta^{-i}\}$ on both sides of equation, we get

$$\left.\begin{aligned} &g_0(t) = \frac{t\Delta_0(t)}{\pi\sqrt{b^2 - t^2}},\ g_1(t) = 0,\ g_2(t) = \frac{t\beta_2}{\pi\sqrt{b^2 - t^2}} \\ &g_3(t) = 0,\ g_4(t) = \frac{t\beta_4}{\pi\sqrt{b^2 - t^2}},\ g_5(t) = 0 \end{aligned}\right\}$$

With

$$\left.\begin{aligned} \beta_2 &= \frac{\pi}{6}b^2 P_6(0)\beta_1,\ \beta_1 = \int_0^b \sqrt{b^2 - x^2}\,\sigma_{yy}^{(h)}(x, 0)\,dx \\ \beta_4 &= \frac{\pi}{2}\left[(3b^2 - 2t^2)\left\langle \beta_2\frac{b^3}{3} + \beta_3\right\rangle + P_6(0) + \frac{b^6}{3}P_6^2(0)\right] \\ \beta_3 &= \int_0^b x^2\sqrt{b^2 - x^2}\,\sigma_{yy}^{(b)}(x, 0)\,dx \end{aligned}\right\} \tag{56}$$

Now using the values of $g_i(t)$ is (55), we get

$$g(t) = \frac{t}{\pi\sqrt{b^2 - t^2}}\left[\Delta_0(t) + \frac{\beta_2}{\delta^2} + \frac{\beta_4^{(t)}}{\delta^4}\right] \tag{57}$$

We retained the terms upto δ^{-5} only.

5. Physical Quantities

The physical quantities, which are of interest in fracture design parameters, are crack opening displacement and components of stress in the neighbourhood of crack-tips.

CRACK OPENING DISPLACEMENT

The crack opening displacement is obtained from the value of integral (43) for $0 \le x < b,$ and it is given as

$$u_y^{(e)}(x,0) = [a_{11} r_2 (r_1^2 - r_2^2)^{-1}] \int_x^b g(t)\,dt, 0 \le x < b$$

Using the value of $g(t)$ from (57) and then evaluating the integrals we get

$$u_y^{(e)}(x,0) = [\pi a_{11} r_2 (r_1^2 - r_2^2)]^{-1} \left[\int_x^b P(x) T_1(x,\alpha)\,d\alpha + \left(\beta_2 + \frac{\beta_{41}}{\sigma^2} \right) \frac{F_1(x)}{\delta^2} - \frac{2\beta_{42}}{\delta^4} F_3(x) \right], \tag{58}$$

$T_n(x,\alpha) = b^{n-2} F_{n-2}(x) + \alpha^2 T_{n-2}(x,\alpha),$ n is odd.

$$P(x) = \frac{\pi}{2} \sigma_{yy}^{(h)}(x,0)$$

$$F_{n(x)} = \frac{\sqrt{b^2 - x^2}}{b^n} \left(\frac{x}{b} \right)^{n-1} + \frac{n-1}{n} F_{n-2}(x)$$

$$T_1(x,\alpha) = -\frac{1}{2\alpha\sqrt{b^2 - \alpha^2}} \log \beta_1(x,\alpha), F_1(x) = \frac{\sqrt{b^2 - x^2}}{b}$$

$$\beta_{41} = \frac{\pi}{2} \left[\left(b^5 \beta_2 + \beta_3 b^2 \right) P_6(1) + \frac{3b^6}{32} P_6^2(0) \right]$$

$$\beta_{42} = \frac{\pi}{2} \left[\left(\frac{b^3}{3} \beta_2 + \beta_3 \right) P_6(1) \right]$$

$$\beta_1(x,\alpha) = [\alpha\sqrt{b^2 - x^2} + x\sqrt{b^2 - \alpha^2}]^2 / (\alpha^2 - x^2) b^2$$

Thus knowing $u_y^{(e)}(x,0)$, after evaluating integrals numerically, we can polot this which will show crack shape

STRESS COMPONENTS

In the viscinity of crack tips $\sigma_{xy}(x,0)$ is zero, therefore, $\sigma_{yy}^{(e)}(x,0)$ will be evaluated. As we know that

$$\sigma_{yy}(x,0) = \sigma_{yy}^{(e)}(x,0) + \sigma_{yy}^{(n)}(x,0)$$

The second term in above experession is due to heat. This term will be evaluated when heat distribution is taken as an example. We will evaluate only $\sigma_{yy}^{(e)}(x,0)$.

It is given as

$$\sigma_{yy}^{(e)}(x,0) = \frac{2}{\pi}\int_0^\infty \xi\phi(\xi)M(\xi)\cos\xi x d\xi$$

$$= \frac{2}{\pi}\int_0^\infty \xi\phi(\xi)\cos(\xi x) + \frac{2}{\pi}\int_0^\infty \xi\phi(\xi)M_1(\xi)\cos\xi xd \tag{59}$$

$$M_1(\xi) = M(\xi) - 1 = \frac{b_{10}}{b_9} + \frac{b_{12}}{b_{11}} - 1 \tag{60}$$

Now using (48) is (64) we get,

$$\sigma_{yy}^{(e)}(x,0) = \int_0^b t\frac{g(t)}{t^2 - x^2}dt$$

$$M_3(x) = \int_0^\infty\int_0^\infty g(t)M_1(\xi)\sin(\xi t)\cos(\xi x)d\xi dt$$

Using $g(t)$ from (57) in above and observing that singularity will appear in first term only, therefore,

$$\sigma_{yy}^{(e)}(x,0) = -\frac{x}{\pi\sqrt{x^2 - c^2}}\left[\Delta_0(x) + \frac{\beta_2}{\delta^2} + \frac{\beta_4(x)}{\delta^4}\right] + M_3(x), b < x < \infty, \tag{61}$$

STRESS-INTENSITY FACTORS

The Stress-intensity factors at crack tips are defined as,

$$K_b = \lim_{x\to b}\sqrt{x-b}\,\sigma_{yy}(x,0)$$

$$= \lim_{x\to b}\sqrt{x-b}\,\sigma_{yy}^{(e)}(x,0) = -\sqrt{\frac{b}{2\pi}}\left[\Delta_0(b) + \frac{\beta_2}{\delta^2} + \frac{\beta_4(b)}{\delta^4}\right], \tag{62}$$

AN EXAMPLE OF HEAT DISTRIBUTION

We consider the point heat source of equal strenfths Q_0 and acting at $(0, \pm\delta_1)$, see figure 3,

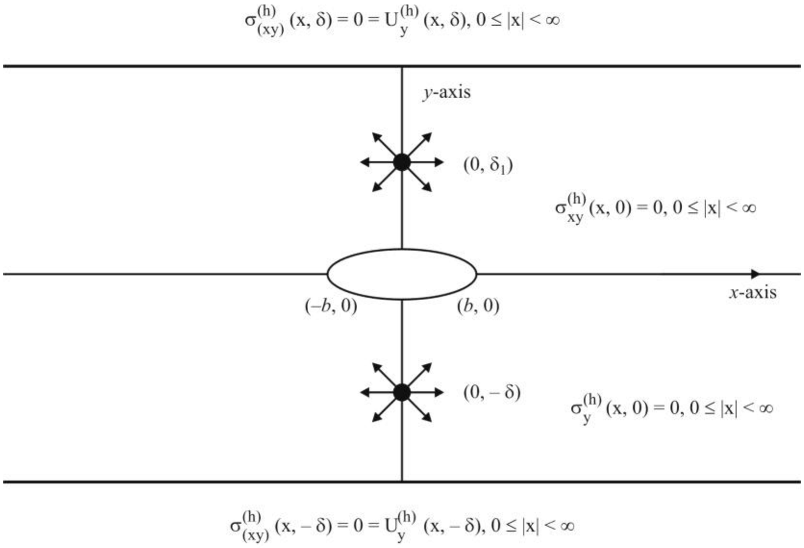

Figure 3 : Boundary Conditions For Special Point Heat Source.

$$Q(x, y) = Q_0 \frac{\delta(x)}{2}[\delta(y - \delta_1) - (\delta(y + \delta_1)], 0 \le |\delta_1| < \delta \tag{63}$$

Then, from above

$$Q_{cc}(\xi, p_n) = Q_0 \cos(p_n \delta_1) \tag{64}$$

Thus $\sigma^{(h)}_{yy}(x, 0)$ is evaluated through (28), (64) and thus evaluating the integrals, and given as

$$\sigma^{(h)}_{yy}(x, 0) = Q_0 b_0 \left[b_1 \sqrt{\frac{k_1}{k_2}} R_1(\beta_1' x, q\delta_1) + b_2 r_1^{-1/2} R_1(\beta_2 x, q\delta_1) + b_3 r_2^{-3/2} R_1(\beta_3' x, q\delta_1) \right], b < x < \infty \tag{65}$$

$$R_1(x, y) = \cosh x - \cos y$$

$$\beta_1' = q\sqrt{\frac{k_2}{k_1}}, \beta_2' = q\sqrt{r_1}, \beta_3' = q\sqrt{r_2}, b_0 = \frac{4}{\pi a_{66} \delta}$$

$$k_2 b_1 = k_9 k_2, B_1 - k_2 k_{10} B_2 - k_1 k_{12} B_3$$

$$b_2 = k_2 k_9 D_1 - k_2 k_{10} D_2 - k_1 k_{12} D_3$$

$$b_3 = k_2 k_9 F_1 - k_2 k_{10} F_2 - k_1 k_{12} F_3$$

$$B_1 = \frac{1}{(k_1 + r_1)}\frac{}{(r_1 - k_1)}, B_2 = \frac{K_1 (k_1 + r_1)}{(k_1 - r_2)(r_1 - k_1)}, B_3 = \frac{k_1^2}{r_2 (r_1 - k_1)}$$

$$D_1 = B_1, D_2 = \frac{r_1 (k_1 + r_1)}{(r_1 - r_2)(k_1 - r_1)}, D_3 = \frac{k_1^2 (r_2 - r_1) + (k_1^2 - r_1^2)}{r_2 (r_2 - r_1)(r_1 - k_1)}$$

$$F_1 = \frac{k_1 - r_1}{(k_1 - r_1)(k_1 - r_2)(r_1 - r_2)}, F_2 = \frac{r_2 (k_1 - r_1)}{(k_1 - r_2)(k_1 - r_1)}$$

$$F_3 = \frac{r_2 (r_2 - r_1)(r_1 - k_1) + (r_1^2 - k_1^2) - 2k_1^2 (r_2 - r_1)}{r_2 (r_2 - r_1)(r_1 - k_1)}$$

r_1^2, r_2^2 are two real roots of (36). β_2 and $\beta_4(x)$ are to be evaluated through (61) and (56).

6. Discussion and Conclusion

DISCUSSION

Dividing the physical problem into two parts, at a general point (x, y), namely (A) Heat Problem (B) Elasticity Problem then using Fourier transform which reduces partial differential equation into ordinary differential equations. The use of boundary conditions reduce the problem to dual integral equation. And solution of dual integral equation reduces to Fredholm integral equation of second type.

Fredholm integral equation is solved by the method of approximation interms of $\{\delta^{-m}\}$; δ is half width of the strip.

The effect of orthotropy over physical quantities is through thermal stresses. Thermal stresses are developed by heat sources/sinks. In the present research endeavour we considered heat sources only.

CONCLUSION

(1) This method can be applied to anisotropic elastic strip also.

(2) We can solve the problem of n-Griffith cracks, too, in orthotropic or anisotropic medium.

(3) If heat is prescribed over crack faces only ie., heat sources/sinks are absent, then solve after making the right hand side of (21) as zero and different boundary. Conditions in place of (12)–(14).

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