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THE STRESS-INTENSITY FACTORS DUE TO THREE GRIFFITH-CRACKS OPENED BYTHERMAL STRESS IN AN INFINITE ORTHOTROPIC MEDIUM

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Abstract : The closed form expressions for stress-intensity factors and of crack opening displacement are obtained by using Fourier Transform Method for three Griffith-cracks opened by thermal stress (Heat source) in orthotropic strip.

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1. Introduction

When there are infinite number of trio's Griffith-cracks and these are equally spaced. We draw a free body diagram as of strip with having 2awith three Griffith-cracks occupying the region $y = 0, 0 \le |x| < b, c < |x| < d < a,$ see figure-1.

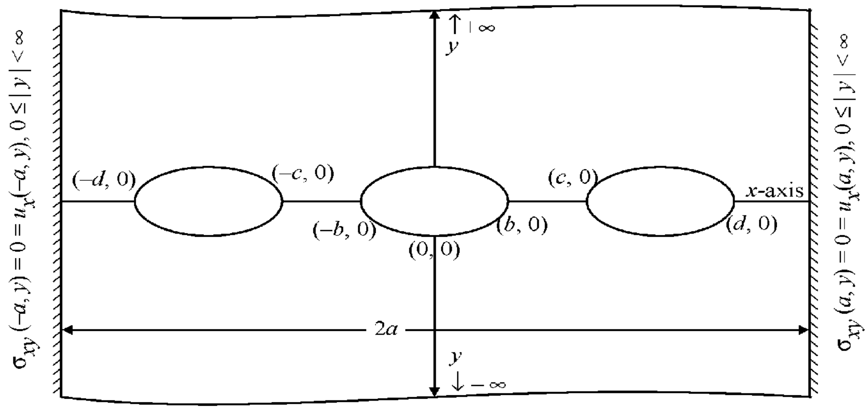

Figure 1. Infinite orthotropic medium having infinite number of crack with respect to three cracks.

The composites are made by putting matrix andfibers alternatively. If the layers of fibers are more than six, then these composites can be treated as orthotropic materials. It have been proved experimentally by Bandopadhya and Murthy.[1] and theoretically by Sharma [9].

The composite materials are very commonly used. They can easily be manipulated to desired weight-to-strength ratios. These can be made with less weight and more strength then steel. Therefore these materials are suitable in aerospace structures.

There are many problems for isotropic medium with one, two or more Griffith cracks opened by mechanical force [11].Parihar and Kushwaha [8]extended to isotropic strip with one or two Griffith cracks. Kushwaha [7] extended to orthotropic strip with one and two Griffith-cracks under body forces. We have solved three Griffith-cracks opened by thermal stress in infinite orthotropic medium [10]. Kardomateas [5] had solved for thermo-elasticity of filament of orthotropic elliptic cylinder. Chen [3] solved for thermo-elastic problem of an anti-symmetrical heat flow disturbed by three coplanar cracks in orthotropic medium.

We have also assumed that the medium is under plane-strain conditions. It is also assumed that elastic properties of medium are not changed due to heat and the crack opening does not alter heat distribution.

The physical problem is reduced to the following mixed-boundaryvalue problem, see figure-2,as

$$\sigma_{xy}(x,0) = 0, 0 \le |x| \le a, \sigma_{xy}(\pm a, y) = 0, u_x(\pm a, y) = 0, 0 \le |y| < \infty \quad .... (1)$$

$$u_y(x,0) = 0, x \in I_2 + I_4 \quad .... (2)$$

$$\sigma_{yy}(x,0) = 0, x \in I_1 + I_3 \quad .... (3)$$

$$\left.\begin{array}{l} \text{with } I_1 = I_1^+ + I_1^-,\, I_2 = I_2^+ + I_2^-,\, I_3 = I_3^+ + I_3^-,\, I_4 = I_5^+ + I_4^- \\ I_1^+ = [0,b),\, I_2^+ = [b,c],\, I_3^+ = (c,d),\, I_4^+ = [d,a], \\ I_1^- = (-b,0],\, I_2^- = [-c,-b],\, I_3^- = (-d,-c),\, I_4^- = [-a,-d] \end{array}\right\} \quad \text{.... (4)}$$

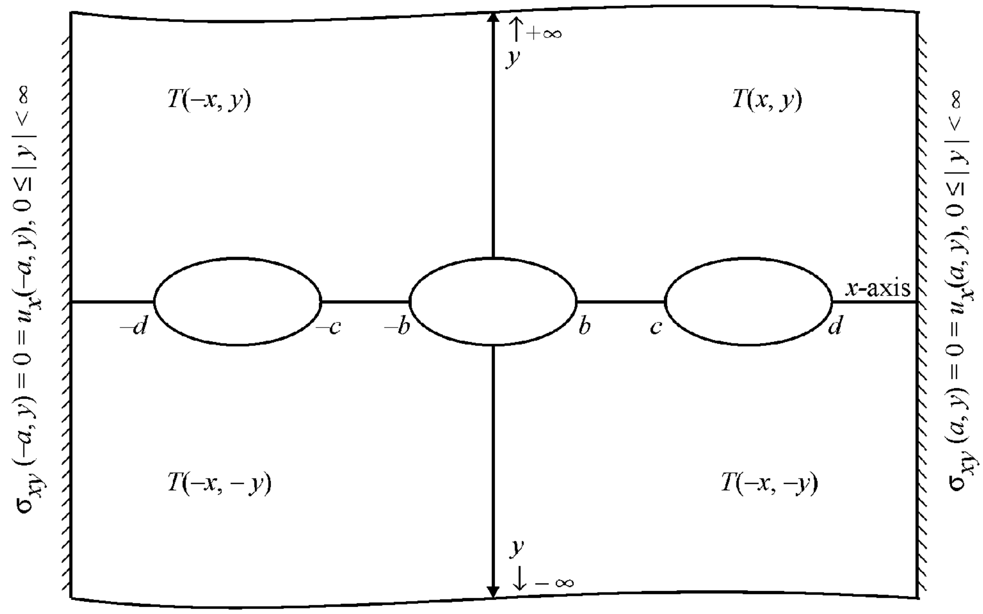

Figure 2. Three Griffith cracks opened by Temperature *T* developed by Heat source *Q*(*x*, *y*) in orthotropic infinite medium.

where σ_{xy}, σ_{yy}, and u_x, u_y are components of stress and of displacement at general point (x, y). We assume that normal stress and component uy of displacement are taken as,

$$\sigma_{ij}(x,y) = \sigma_{ij}^{(e)}(x,y) + \sigma_{ij}^{(h)}(x,y),\ i,j = x,y$$

$$u_i(x,y) = u_i^{(e)}(x,y) + u_i^{(h)}(x,y),\ i = x,y$$

where superscripts (e) and (h) stand for elastic and heat part respectively.

We assumed that co-ordinate axes coincide with axis of symmetry.

The symmetry of geometry reduces the boundary conditions (1) – (3) to the following

$$\sigma_{xy}(x,0) = 0, 0 \le |x| \le a, \sigma_{xy}(a,y) = 0, u_x(a,y) = 0, 0 \le y < \infty \quad ....(5)$$

$$u_y(x,0) = 0, x \in I_2^+ + I_4^+ \quad ....(6)$$

$$\sigma_{yy}(x,0) = 0, x \in I_1^+ + I_3^+ \quad ....(7)$$

where I_i^+, i = 1, 2, 3, 4 are given by (4). The finite Fourier transforms are defined as

$$f_{c,s}(p) = \int_0^a f(x)\{\cos px, \sin px\}dx, p = n\pi / a$$

and the usual inversion formula. We checked throughout that, see [9].

$$u_y^{(e)}(x,0) > 0, x \in I_1 + I_3,$$

which means that cracks, really open out and crack faces do not meet each other except at crack tips. The plan of the paper is as follows :In section 2shall formulate heat problem. Section 3 will solve elasticity problem. The reduction to and solution of quintuple series equation will be done in section 4.The general expressions of physical quantities will be reported in section 5. A special case of point heat source is discussed in section 6. The discussion and conclusion of research endeavour is given in section 7.

2. Formulation of heat problem

The formulation of heat problem is done by taking appropriate finite Fourier sine and cosine transforms, with respect to x and y variables, of equations of equilibrium, in the absence of body forces,

$$\frac{\partial\sigma_{xx}^{(h)}}{\partial x} + \frac{\partial\sigma_{xy}^{(h)}}{\partial y} = 0, \frac{\partial\sigma_{xy}^{(h)}}{\partial x} + \frac{\partial\sigma_{yy}^{(h)}}{\partial y} = 0$$

and then using the values of Fourier transforms of $\sigma_{xx}, \sigma_{xy}, \sigma_{yy}$ obtained from the stress-strain relations

$$\left.\begin{aligned}\sigma_{xx}^{(h)} &= \frac{a_{22}e_x^{(h)} - a_{12}e_y^{(h)}}{\alpha} - k_1T\\ \sigma_{xy}^{(h)} &= e_{xy}^{(h)} / a_{66}\\ \sigma_{yy}^{(h)} &= \frac{(a_{22}e_y^{(h)} - a_{12}e_x^{(h)})}{\alpha} - k_2T\end{aligned}\right\} \qquad ....(8)$$

where, $\alpha = a_{11}a_{22} - a_{12}^2$, $k_1 = \alpha_{t_1}(a_{11} - a_{12})a_{22}$, $k_2 = \alpha_{t_2}(a_{22} - a_{12})a_{11}$ and α_{t_1}, α_{t_2} are the coefficients of linear expansion and $a_{11} \sim a_{66}$ are elastic constants of the medium. $e_x^{(h)}, e_y^{(h)}$ and $e_{xy}^{(h)}$ are components of strain. We shall get two linear algebraic equations in $u_{x_{sc}}^{(h)}$, $u_{y_{cs}}^{(h)}$. Solving for $u_{x_{sc}}^{(h)}$ and $u_{y_{cs}}^{(h)}$ respectively and then inverting we get as,

$$u_x^{(h)}(x,y) = \frac{4}{a\pi}\sum_{n=1}^{\infty}\int_0^{\infty} T_{cc}(p,q)\sin(px)\cos(qy)\,W_1 dp\,dq \qquad ....(9)$$

$$u_y^{(h)}(x,y) = \frac{1}{2}u_{yc}(0,y) + \frac{4}{\pi a}\sum_{n=1}^{\infty} u_{yc}(p,y)\cos px \qquad ....(10)$$

$$u_{yc}(p,y) = \int_0^{\infty} T_{cc}(p,q)W_2\sin(qy)dq, \qquad .....(10a)$$

$$\left.\begin{aligned}W_1 &= \left[k_2\left\{\frac{a_{12}pq^2}{\alpha} - \frac{p^2q}{a_{66}}\right\} + k_1\left\{\frac{p^3}{a_{66}} + \frac{q^2p}{\alpha}a_{11}\right\}p\right]/W\\ \text{with,}\quad W_2 &= \left[k_2\left\{\frac{pq^2a_{12}}{\alpha} - \frac{q^3}{a_{66}}\right\} - k_1\left\{\frac{pq^2a_{11}}{\alpha} - \frac{qp^2}{a_{66}}\right\}\right]/W\end{aligned}\right\} \qquad ....(10b)$$

$$W = q^4 + 2B_1p^2q^2 + B_2p^4, B_2 = \frac{a_{22}}{a_{11}}, 2B_1 = \frac{(2a_{12} + a_{66})}{a_{11}}$$

T_{cc} is Fourier cosine transform of T w.r.t., x and y, respectively, while T satisfies the equation

$$\left(k_1 \frac{\partial^2}{\partial x^2} + k_2 \frac{\partial^2}{\partial y^2}\right) T = \alpha Q(x, y) \qquad \text{....(11)}$$

The heat distribution is such that $\sigma_{xy}^{(h)}$ and $u_y^{(h)}$ are zero at y = 0, i.e.,

$$u_y^{(h)}(x, 0) = 0, \quad \sigma_{xy}^{(h)}(x, 0) = 0 \qquad \text{....(12)}$$

Then we shall take Fourier transform of above equation and then substitute in the expressions of $u_x^{(h)}$ and $u_y^{(h)}$ above and then using stress-strain relations to get stress component as,

$$\sigma_{yy}^{(h)}(x, y) = -\frac{4}{a\pi} \sum_{n=1}^{\infty} \int_0^{\infty} q^2 Q_{cc} \cos px \cos qy \frac{\left[k_2(k_3 p^2 - q^2 k_4) + k_1(p^2 k_5 - q^2 k_6)\right]}{W} dq \qquad \text{....(13)}$$

$$where, k_3 = aa_{22} - a_{66}, k_4 = aa_{11}, k_6 = aa_{12}, k_5 = a_1(a - 2_{a_{11}a_{66}})$$

3. Elasticity Problem

We shall follow the method of Kushwaha [5] to obtain the solution of elasticity problem.

We take displacement components as,

$$u_x^{(e)}(x, y) = \frac{2}{a} \sum_{n=1}^{\infty} p^{-1} \left[a_{11} H_{,yy} - p^2 a_{12} H\right] \sin px, p = \frac{n\pi}{a} \qquad \text{....(14)}$$

$$u_y^{(e)}(x, y) = \frac{2}{a} \sum_{n=1}^{n=1} p^{-2} \left[a_{11} H_{,yyy} - p^2(a_{12} + a_{66}) H_{,y}\right] \cos px + \frac{1}{2} A(0) \text{....(15)}$$

with (,) is differentiation w.r.t. y and H is given as

$$(\gamma_1 - \gamma_1) H(p, y) = \{(\gamma_1 - \gamma_2) A(p) - B(p)\} e^{-p\gamma_1 y} + B(p) e^{-r_2 py}, \qquad \text{....(16)}$$

with A(p) and B(p) as two arbitrary constants and γ_1^2 and γ_2^2 are two roots of *equation*, $\gamma^4 - 2B_1\gamma^2 + B_2 = 0$

Now making use of stress-strain relations (8) for $\sigma_{xy}^{(e)}$ and $\sigma_{yy}^{(e)}$with (14) – (16), we get as,

$$\sigma_{xy}^{(e)}(x,y) = \frac{2}{a(\gamma_1-\gamma_2)}\sum_{n-1}^{\infty} p \sin px\Big[\gamma_1\left\{(\gamma_1-\gamma_2)A(p)-B(p)\right\}e^{-\gamma_1 py}$$

$$+B(p)e^{-\gamma_2 py}\gamma_2\Big] \qquad ....(17)$$

$$\sigma_{yy}^{(e)}(x,y) = \frac{2}{a\alpha}\sum_{n=1}^{\infty} p^2 \cos px\Big[\alpha_1\Big\langle\left\{(\gamma_1-\gamma_2)A(p)-B(p)\right\}e^{-r_1 py}\Big\rangle$$

$$+B(p)e^{-r_2 py}\Big\rangle-\alpha_2\Big\{\big\langle(\gamma_1-\gamma_2)A(p)-B(p)\big\rangle e^{-r_1 py}+B(p)e^{-r_2 py}\Big\} ....(18)$$

$$\alpha_1 = \frac{(a_{11}-a_{12})}{\gamma_1-\gamma_2}, \quad \alpha_2 = \frac{a_{12}}{r_1-\gamma_2}$$

Now, after making use of (12) in boundary conditions (5) - (6), they are reduced to

$$\sigma_{xy}^{(e)}(x,0) = 0, 0 \le x \le a \qquad ....(19)$$

$$u_y^{(e)}(x,0) = 0, x \in I_2^+ \cup I_4^+ \qquad ....(20)$$

$$\sigma_{yy}^{(e)}(x,0) = -\sigma_{yy}^{(h)}(x,0),\ x \in I_1^+ \cup I_3^+ \qquad ....(21)$$

4. Reduction to and Solution of Qintuple Series Equation

Reduction

The boundary condition (19) along with (17) gives

$$\gamma_1 A(p) = B(p) \qquad ....(21)$$

The boundary condition (20) and relation (15)-(16) and then using (21) we get,

$$\frac{A_{(0)}}{2} + \alpha_3 \sum_{n=1}^{\infty} p\,B(p)\cos px = 0, x \in I_2^+ + I_4^+, \qquad \text{....(22)}$$

The condition (21) and relation (18)give,

$$\sum_{n=1}^{\infty} p^2 \cos(px) B(p) = -\frac{\pi\alpha}{2\alpha_4} \sigma_{yy}^{(h)}(x,0), x \in I_1^+ + I_3^+ \qquad \text{....(23)}$$

with, $\alpha_4 = \dfrac{\left(\gamma_1^2\alpha_1 - \alpha_2\gamma_2^2\right)}{\gamma_1} + \left(\gamma_2^2 - \gamma_1^2\right)\alpha_1$

$$\alpha_3 = \frac{2}{\pi}\left[(\gamma_1 + \gamma_2)^2 - \gamma_1\gamma_2 + 2(a_{12} + a_{66})\right]$$

Thus, the physical problem is reduced to the solution of quintuple series equations (22) – (23).We assume

$$p\,B(p) = \phi(p)$$

Then using above in (22) – (23) we get

$$\frac{A_1}{2} + \sum_{n=1}^{\infty} \phi(p)\cos px = 0, x \in I_2^+ + I_4^+, A_1 = \frac{A_0}{\alpha_3}, \qquad \text{....(24)}$$

and $\displaystyle\sum_{n=1}^{\infty} p\,\phi(p)\cos px = -\alpha_5\sigma_{yy}^{(h)}(x,0), \alpha_5 = \frac{a\alpha}{2\alpha_4}$ ....(25)

Solution

We take trial solution of (24) – (25) as, see Kushwaha [10],

$$a\phi(p) = 2\left[\left\langle \int_0^b g_1(t) + \int_c^d g_2(t)\right\rangle \frac{\sin pt\,dt}{p}\right. \qquad \text{....(26)}$$

Now we use (26) in (24), then use the series

$$\frac{x}{2}+\sum_{n=1}^{\infty}\frac{\sin px\cos qx}{x}=\begin{cases}\frac{\pi}{2}, p>q\\ \frac{\pi}{4}, p=q\\ 0, p<q\end{cases}$$

and taking $g_1(0)=0$, with no loss of generality, the equation (24) is satisfied identically if,

$$\int_c^d g_2(t)dt=0 \qquad \text{....(27)}$$

The substitution of (26) in (25) and using,

$$\sum_{n=1}^{\infty}\frac{\sin px\sin qx}{x}dx=\frac{1}{2}\log\left|\frac{\sin\frac{\pi x}{2a}+\sin\frac{\pi q}{2}}{\sin\frac{\pi x}{2a}-\sin\frac{\pi q}{2}}\right|$$

We get

$$g_1(t)=\frac{2t}{a^2\psi(t)}[\Delta(t)], t\in I_1^+ \qquad \text{....(28)}$$

$$g_2(t)=-\frac{2t}{a^2\psi(t)}\Delta(t), t\in I_3^+ \qquad \text{....(29)}$$

$$\Delta(t)=\left\langle\int_0^b-\int_c^d\right\rangle\cos\left(\frac{qx}{2a}\right)\frac{p(x)\psi(x)}{G(x,t)}dx+D_0, \qquad \text{....(30)}$$

$$p(x)=\alpha_5\sigma_{yy}^{(h)}(x,0) \qquad \text{....(30a)}$$

where Do To an arbitrary constant to be evaluated through (27). and

$$\psi(t)=\{|G(b,t)|,|G(c,t)|,|G(t,d)|\}^{\frac{1}{2}}, \qquad \text{....(31)}$$

5. Physical Quantities

The quantities of physical importance are the components of stress and of displacement in the vicinity of crack tips.

Crack shape

The crack opening displacement is evaluated through the left hand side of (22) for $x \in I_1^+ + I_3^+$. We evaluate the integral with the help of (26) and is given as,

$$u_y^{(e)}(x,0) = \frac{\alpha_3 a}{2}\begin{cases} \int_x^b g_1(t)dt, x \in I_1^+ \\ -\int_x^d g_2(t)dt, x \in I_3^+ \end{cases} \qquad \text{....(32)}$$

Stress component

Normal stress component is given through

$$\sigma_{yy}(x,0) = \sigma_{yy}^{(h)}(x,0) + \sigma_{yy}^{(e)}(x,0),\ x \in I_2^+ + I_4^+$$

$\sigma_{yy}^{(h)}(x,0)$ will be evaluated through (13) and is given as

$$\sigma_{yy}^{(e)}(x,0) = \frac{2}{a}\frac{\alpha_4}{\alpha}\sum_{n=1}^{\infty} p\phi(p)\cos px,\ x \in I_2^+ + I_4^+$$

Now, using (26) in above and evaluating integral we get

$$\sigma_{yy}^{(e)}(x,0) = \frac{2}{a}\cdot\frac{\alpha_4}{\alpha}\begin{cases} \dfrac{\sin(qx/2)\Delta(x)}{\psi(x)}, x \in I_2^+ \\ -\dfrac{\sin(qx/2)\Delta(x)}{\psi(x)}, x \in I_4^+ \end{cases} \qquad \text{....(33)}$$

where $\Delta(x)$ and $\psi(x)$ are defined in (30) –(31),respectively.

Stress-intensity factors

The stress-intensity factors are very important in fracture design parameter. These are defined at crack tipsas,

$$K_b = \lim_{x\to b^-} \sqrt{x-b}\ \sigma_{yy}(x,0), K_c = \lim_{x\to c^+} \sqrt{c-x}\,\sigma_{yy}(x,0), K_d = \lim_{x\to d^+} \sqrt{d-x}\,\sigma_{yy}(x,0)$$

Thus,

$$\left.\begin{aligned} K_b &= \lim_{x\to b^-} \sqrt{x-b}\ \sigma_{yy}^{(h)}(x,0) + \frac{1}{a}\frac{\alpha_4}{\alpha}\frac{\sqrt{2\sin(qb/2)}}{\psi_1(b)}\Delta_1(b) \\ K_c &= \lim_{x\to c^+} \sqrt{c-x}\ \sigma_{yy}^{(h)}(x,0) + \frac{\alpha_4}{a\alpha}\frac{\sqrt{2\sin(qc/2)}}{\psi_2(c)}\Delta_1(c) \\ K_d &= \lim_{x\to d^-} \sqrt{x-d}\ \sigma_{yy}^{(h)}(x,0) + \frac{\alpha_4}{a\alpha}\frac{\sqrt{2\sin(qd/2)}}{\psi_3(d)}\Delta_1(d) \end{aligned}\right\} \quad \text{....(34)}$$

with $\psi_1(b) = [(\cos qb - \cos qc)(\cos qb - \cos qd)].\psi_2(c)$

$[(\cos qb - \cos qc)(\cos qc - \cos qd)]^{1/2}$

$$\psi_3(d) = \left[(\cos qc - \cos qd)(\cos qb - \cos qd)\right]^{\frac{1}{2}}, q = \frac{\pi}{a}$$

The heat distribution is due to heat source therefore, $\sigma_{yy}^{(h)}(x,0)$, is not singular at crack tips.

6. Special Case of Point Heat Source

Since the rivets and stiffeners are simulated by point forces in the medium, therefore, heated rivets/stiffeners can be simulated by point heat source. Point heat source is defined as, see figure-3,

$$Q(x,y) = Q_0 \frac{\delta(x)}{2}\{\delta(y-h) + \delta(y+h)\} \quad \text{....(35)}$$

which means that the point heat sources of strength Q_0 and is acting at points $(0,\pm h)$. Taking Finite Fourier cosine transform of above with respect to x and Fourier cosine transform w.r.t.y, we get

$$Q_{cc}(p,q) = Q_0 \cos(qh)$$

Substituting the value of Q_{cc}, in (13) and evaluating the integrals,

$$\sigma_{yy}^{(h)}(x,0) = \frac{Q_0 \sin(qh)}{a\alpha a_{66}}\left[\frac{C+\alpha k_2}{\psi_4(k_2)} + \frac{D}{\psi_4(\gamma_1)} + \frac{G}{\psi_4(\gamma_2)}\right] \qquad \text{....(36)}$$

$$\psi_4(t) = \cosh qh + t^2 \cos qx, t = k_2, \gamma_1, \gamma_2, q = \frac{\pi}{a}$$

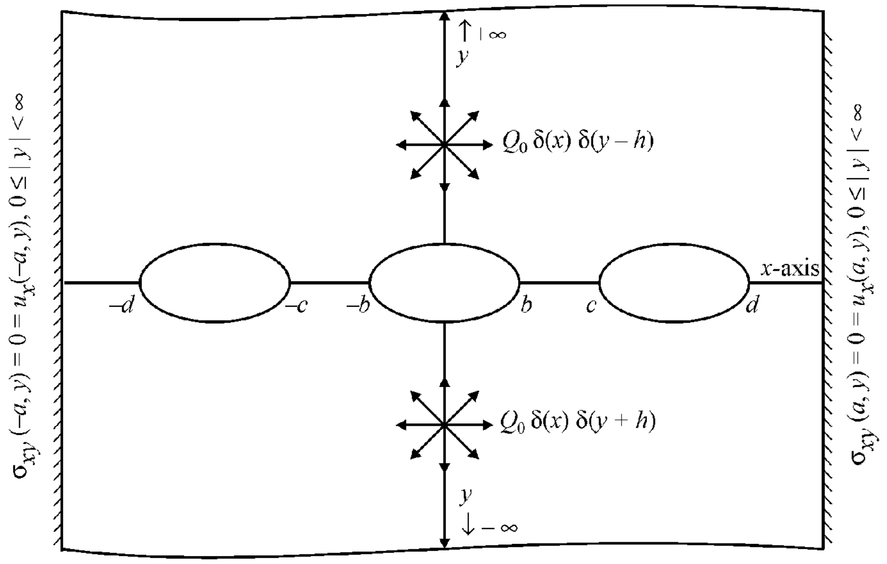

Figure 3. $Q(x,y) = Q_0 \frac{\delta(x)}{2}\{\delta(y-h) + \delta(y+h)\}$. **The Heat source is of strength** Q_0 **acting at** $(0, \pm h)$**.**

where C, D and G are to be evaluated through,

$$C + D = 1,$$

$$C\gamma_1\gamma_4 + Dk_2\gamma_2 + Gk_3\gamma_1 = \gamma_3 + \gamma_4$$

$$\gamma_1 = B_1 + \sqrt{B_1^2 - B_2}, \gamma_2 = B_1 - \sqrt{B_1^2 - B_2},$$

$$\gamma_3 = D_1 + \sqrt{D_1^2 - D_2}, \gamma_4 = D_1 - \sqrt{D_1^2 - D_2},$$

$$D_1 = k_7k_8, D_2 = k_7k_9, k_7 = k_1a_{12}, \alpha$$

$$k_8 = k_2k_3 + k_1k_5, k_9 = k_2a_{11}\alpha$$

Thus we see that $\sigma_{yy}^{(h)}(x,0)$ is not singular.

Stress-Component

$\sigma_{yy}^{(e)}(x,0)$, $x \in I_2^+ + I_4^+$ is to be evaluated through (33) and $\Delta(x)$ from (30) for $x \in I_2^+ + I_4^+$, which is given as below

$$\Delta(x) = \alpha_5 \frac{Q_0 h \sin\left(\frac{qx}{a}\right)}{a\alpha a_{66}}\left[\delta_5(h_1,x)B_0 + D_0\delta_5(h_2,x) + F_0\delta_5(h_3,x)\right] + M_0, \quad \text{....(37)}$$

$$\delta_5(y,x) = \frac{\left[(\cosh qy - \cos qc)E\left(\frac{\pi}{2},m_0\right) + \left\langle \psi_1(x) + \psi_{11}(y)\right\rangle F\left(\frac{\pi}{2},m_0\right) + \psi_{12}(y)\right]}{(\cosh qy - \cos qx)}$$

$$m_0^2 = \frac{\cos ab - \cos qc}{\cos qb - \cos qd}, \; \psi_1(x) = \left[\left|(\cos qx - \cos qc)(\cos qx - \cos qd)\right|\right]^{\frac{1}{2}}$$

$$\psi_{12}(y) = \left[\left|(\cos qb - \cos qy)(\cos qc - \cos qy)(\cos qy - \cos qd)\right|\right]^{\frac{1}{2}}$$

$$\psi_{11}(y) = \left[\left(\sin^2 qd + \sinh qy\right)\left(\sin^2 qb + \sinh^2 qy\right)\right]^{\frac{1}{2}},$$

$$h_1 = \frac{h}{\sqrt{k_1 k_2}}, h_2 = \frac{h}{\sqrt{\gamma_1}}, h_3 = \frac{h}{\sqrt{\gamma_2}}$$

$$M_0 = \frac{\left[B_0\Pi_1(h_1) + D_0\Pi_1(h_2) + F_0\Pi_1(h_3)\right]}{F_2}$$

$$F_2 = F\left(\frac{\pi}{2},\mu_3\right), \Pi_1(y) = \frac{\psi_{12}(y)}{\sinh^2 ay + \sin^2 qc}\Pi\left(\frac{\pi}{2},m_2 y,m_3\right), m_3^2 = \frac{\cos qc - \cos qd}{\cos qb - \cos qc}$$

$$m_2^2 y^2 = \frac{\sin^2 qd - \sin^2 qc}{\sinh^2 ay + \sin^2 qc}, B_0 = \frac{bk_2}{\sqrt{k_1 k_2}}, D_0 = \frac{D}{\sqrt{r}}, F_0 = \frac{G}{\sqrt{r}}$$

where F, E and Π are complete elliptic integrals, see Gradstyen and Ryzhik [11], of first, second and third types respectively.

Thus knowing $\Delta(x)$, $\sigma_{yy}^{(e)}(x,0)$, for $x \in I_2^+ + I_4^+$ is evaluated through (33) and then used the definition of stress-intensity factors which are given in (34)

$$\left.\begin{array}{l} K_b = K_b^{(e)} = \dfrac{\alpha_4 \sqrt{2\sin(qb/2)}}{\pi\alpha\ \psi_1(b)} \Delta_1(b), K_c = K_c^{(e)} = \dfrac{\alpha_4 \sqrt{2\sin(qc/2)}}{\pi\alpha\ \psi_2(c)} \Delta_1(c) \\ K_d = K_d^{(e)} = \dfrac{\alpha_4 \sqrt{2\sin(qd/2)}}{a\alpha\ \psi_3(d)} \Delta_1(d), \Delta_1(x) = \dfrac{\Delta(x)}{\sin(qx/2)}, \end{array}\right\} \quad ....(38)$$

Crack Opening Displacement

The crack opening displace $u_y^{(e)}(x,0)$ is given through (32), where $g_1(t)$ and $g_2(t)$ are given in (28) – (29) and $\Delta(t)$ is evaluated through (30, (30)a and (36) as

$$\Delta(t) = \frac{Q_0 \sin(qh)\alpha_5}{a\alpha a_{66}} \left[\sum_{i=1}^{3} \frac{E_i}{P_4(h_i,t)} \{a_2 P_4(h_i,t) + a_3 \left\langle \left|(\cos qt - \cos qb)(\cos qt - \cos qc)\right| \right\rangle \right.$$

$$\left. P_5 h_i, b, c + a_4 \prod_{i=1}^{3} P_4(h_i,b) \right] + M_{0,} t \in I_1^+ + I_3^+ \quad ....(39)$$

$$P_4(h_i,t) = \cosh qh_i + \cos qt$$

$$P_5(h_i,b,c) = P_4(h_i,b) P_4(h_i,c)$$

Thus displacement will be evaluated through numerical method for integration. And,

$$a_2 = -a\left[1 + \frac{\sqrt{\alpha^2 + \alpha P_6 = (2\sin^2(qd) - \sin^2 ab) + 1}}{2\alpha\sqrt{P_7(2\sin^2 qc - \sin^2 qb) - 4}}\right]$$

$$\alpha = P_7 + \sqrt{P_7 - 4}$$

$$a_3 = -\frac{a}{2} + \sqrt{P_6} \cdot F\left(\frac{\pi}{2}, m_1\right), m_1^2 = \frac{\sin^2 qd - \sin^2 qc}{\sin^2 qd - \sin^2 qb}$$

$$a_4 = 2(\sin^2 qd - \sin^2 qb)F\left(\frac{\pi}{2}, m_1\right) - E\left(\frac{\pi}{2}, m_1\right) - \frac{1}{(\sin^2 qd - \sin^2 qb)^{3/2}}\Pi\left(\frac{\pi}{2}, m_1, m_1\right)$$

where F, E and Π are complete elliptic integrals of first, second and third type.

7. Discussion and Conclusion

Discussion

Heat flow is through conduction and is governed by equation (11). Heat flow is because of heat source, $Q(x, y)$. If there is no heat source/sink, then equation (11) will reduce to second order homogeneous partial different equation. Then we are to solve by another method. The boundary conditions (12) over heat introduces the symmetry. If these conditions are removed, then problem becomes difficult but not impossible.

Conclusion

Present method can be extended to the problem of three Griffith-cracks in an orthotropic rectangle. This problem will be reduced to quadruple series equations. The solution can be obtained by the method of Kushwaha [6]. There will be a closed form solution and will be reported in next research paper.

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