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NEW INFORMATION INEQUALITIES AND ITS SPECIAL CASES

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Abstract : In this paper, we are introducing new information inequalities corresponding to generalized f- divergence measure $S_f(P,Q)=\sum_{i=1}^{n} q_i\, f\left(\frac{p_i+q_i}{2\,q_i}\right)$, and studying its particular cases by taking different convex and normalized functions and get the different results.

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1. Introduction

Let $\Gamma_n=\left\{P=(p_1,p_2,p_3.........,p_n): p_i>0, \sum_{i=1}^{n} p_i=1\right\}, n\geq 2$ be the set of all complete finite discrete probability distributions. If we take $p_i \geq 0$ for some $i=1,2,3,...,n$, then we have to suppose that $0f(0)=0f(0/0)=0$. (Jain and Saraswat [5]) introduced a generalized measure of information given by

$$S_f(P,Q)=\sum_{i=1}^{n} q_i\, f\left(\frac{p_i+q_i}{2\,q_i}\right) \quad ...(1)$$

where $f:(0,\infty)\to R$ (set of real no.) is a convex function and P, Q $\in \Gamma$n. The new f- divergence is a generalized information divergence measure and it is used in measuring the distance or affinity between two probability distributions. Many known divergences can be obtained from this measure by suitably defining the convex function f.

Some standard divergences which will be used in this paper, are as follows:

Triangular Discrimination (Dacunha- Castelle [1]).

$$\Delta(P,Q) = 2\left[1 - W(P,Q)\right] = \sum_{i=1}^{n} \frac{(p_i - q_i)^2}{p_i + q_i} \qquad \text{.... (2)}$$

where $W(P,Q) = \sum_{i=1}^{n} \frac{2\, p_i\, q_i}{p_i + q_i}$ is Harmonic Mean Divergence Measure.

χ^2 - Divergence Measure (Pearson [8]).

$$\chi^2(P,Q) = \sum_{i=1}^{n} \frac{(p_i - q_i)^2}{q_i} \qquad \text{.... (3)}$$

Relative Information (Kullback and Leibler [7]).

$$K(P,Q) = \sum_{i=1}^{n} p_i \log\left(\frac{p_i}{q_i}\right) \qquad \text{... (4)}$$

Relative J- Divergence (Dragomir, Gluscevic and Pearce [3]).

$$J_R(P,Q) = \sum_{i=1}^{n} (p_i - q_i) \log\left(\frac{p_i + q_i}{2q_i}\right) \qquad \text{... (5)}$$

Symmetric Chi- square Divergence (Dragomir, Sunde and Buse [4]).

$$\Psi(P,Q) = \sum_{i=1}^{n} \frac{(p_i - q_i)^2 (p_i + q_i)}{p_i\, q_i} \qquad \text{... (6)}$$

J- Divergence Measure (Jeffreys and Kullback and Leibler [6, 7]).

$$J(P,Q) = \sum_{i=1}^{n} (p_i - q_i) \log\left(\frac{p_i}{q_i}\right) \qquad \text{... (7)}$$

Relative Arithmetic- Geometric Divergence (Taneja [11]).

$$G(P,Q)=\sum_{i=1}^{n}\left(\frac{p_i+q_i}{2}\right)\log\left(\frac{p_i+q_i}{2\,p_i}\right) \qquad \ldots (8)$$

Relative Jensen- Shannon divergence (Sibson [9]).

$$F(P,Q)=\sum_{i=1}^{n} p_i \log\left(\frac{2\,p_i}{p_i+q_i}\right) \qquad \ldots (9)$$

Unified Relative Jensen-Shannon and Arithmetic-Geometric Divergence of type "s" (Taneja and P. Kumar [12]).

$$FG_s(P,Q)=\left[s(s-1)\right]^{-1}\left[\sum_{i=1}^{n} p_i\left(\frac{p_i+q_i}{2\,p_i}\right)^{s}-1\right],\ s\neq 0,1 \text{ and } s\in R. \qquad \ldots. (10)$$

It's a one parametric generalized divergence measure, by which we can obtain $F(P,Q)$ (at s=0) and $G(P,Q)$ (at s=1) respectively.

Relative J- Divergence of type "s" (Taneja and P. Kumar [12]).

$$J_{Rs}(P,Q)=(s-1)^{-1}\sum_{i=1}^{n}(p_i-q_i)\left(\frac{p_i+q_i}{2\,q_i}\right)^{s-1},\ s\neq 1 \text{ and } s\in R. \qquad \ldots (11)$$

It's also a one parametric generalized divergence measure, by which we can obtain $\Delta(P,Q)$ (at s=0) and $J_R(P,Q)$ (at s=1) respectively.

$\psi(P,Q)$ is the sum of $\chi^2(P,Q)$ and its adjoint $\chi^2(Q,P)$, $J(P,Q)$ can also be written as the sum of $J_R(P,Q)$ and its adjoint $J_R(Q,P)$ or as the sum of $K(P,Q)$ and its adjoint $K(Q,P)$ and $J_R(P,Q)$ is equal to the two times of the sum of adjoints of $F(P,Q)$ and $G(P,Q)$. where $\psi(P,Q), J(P,Q), \chi^2(P,Q), J_R(P,Q), K(P,Q), F(P,Q)$ *and* $G(P,Q)$ are mentioned from (3) to (9).

In special cases which will studied further, we shall use the p- logarithmic power mean (Stolarski K.B.[10]), given by:

$$L_p^p(\alpha,\beta)=\begin{cases}\dfrac{\beta^{p+1}-\alpha^{p+1}}{(p+1)(\beta-\alpha)}, & p\neq -1,0\\ \dfrac{\log\beta-\log\alpha}{\beta-\alpha}, & p=-1\\ 1, & p=0\end{cases} \quad \text{for all } p\in R, \alpha\neq\beta. \qquad \text{... (12)}$$

2. NEW INFORMATION INEQUALITIES:

The following theorem is well known in the literature [5].

Theorem 2.1 *If the function f is convex and normalized i.e.* $f(1)=0$*, then the new f-divergence* $S_f(P,Q)$ *is non-negative and convex in the pair of probability distribution* $(P,Q)\in\Gamma_n\times\Gamma_n$.

The following lemma is due to Dragomir [2].

Lemma 2.2 *Let* $f: I\subset R_+\to R$ *be a differentiable convex function on the interval I,* $t_i\in I$, $\mu_i\geq 0\ \forall\, i=1,2,\ldots,n$ *with* $\sum_{i=1}^{n}\mu_i=1$. *If* $m,M\in I$ *and* $m\leq t_i\leq M,\ \forall\, i=1,2\ldots,n$ *, then we have the inequalities:*

$$0\leq\sum_{i=1}^{n}\mu_i\, f(t_i)-f\left(\sum_{i=1}^{n}\mu_i\, t_i\right)\leq\sum_{i=1}^{n}\mu_i\, t_i\, f'(t_i)-\left(\sum_{i=1}^{n}\mu_i\, t_i\right)$$

$$\left(\sum_{i=1}^{n}\mu_i\, f'(t_i)\right)\leq\frac{1}{4}(M-m)\left\{f'(M)-f'(m)\right\} \qquad \text{.... (13)}$$

Now, the theorem given below introduces new information inequalities which give bounds on the measure $S_f(P,Q)$.

Theorem 2.3 Let $f: R_+ \to R$ be differentiable convex and normalized i.e. $f(1) = 0$. *If P,Q∈Γn, is such that* $0 < \alpha < \frac{1}{2} \le \frac{p_i + q_i}{2q_i} \le \beta < \infty, \forall \; i = 1, 2, 3..., n$, *for some* α *and* β *with* $0 < \alpha \le 1 \le \beta < \infty, \alpha \ne \beta$, *then we have the following inequalities:*

$$S_f(P,Q) \le T_{S_f}(P,Q) \le Y_{S_f}(\alpha, \beta) \qquad .... (14)$$

where $$T_{S_f}(P,Q) = \frac{1}{2}\sum_{i=1}^{n}(p_i - q_i) f'\left(\frac{p_i + q_i}{2\,q_i}\right). \qquad ..... (15)$$

and $$Y_{S_f}(\alpha, \beta) = \frac{1}{4}(\beta - \alpha)\{f'(\beta) - f'(\alpha)\} \qquad ..... (16)$$

Proof : For all P,Q∈Γn , if we take $t_i = \frac{p_i + q_i}{2\,q_i}$ *and* $\mu_i = q_i$ in (13) and sum over all $i = 1, 2, ..., n$, then, we get

$$0 \le \sum_{i=1}^{n} q_i f\left(\frac{p_i + q_i}{2\,q_i}\right) - f\left(\sum_{i=1}^{n} q_i\left(\frac{p_i + q_i}{2\,q_i}\right)\right)$$

$$\le \sum_{i=1}^{n} q_i\left(\frac{p_i + q_i}{2\,q_i}\right) f'\left(\frac{p_i + q_i}{2\,q_i}\right) - \left(\sum_{i=1}^{n} q_i\left(\frac{p_i + q_i}{2\,q_i}\right)\right)\left(\sum_{i=1}^{n} q_i\, f'\left(\frac{p_i + q_i}{2\,q_i}\right)\right)$$

$$\le \frac{1}{4}(\beta - \alpha)\{f'(\beta) - f'(\alpha)\}$$

i.e. $$0 \le S_f(P,Q) - f\left(\sum_{i=1}^{n}\frac{p_i + q_i}{2}\right)$$

$$\le \sum_{i=1}^{n}\left(\frac{p_i + q_i}{2}\right) f'\left(\frac{p_i + q_i}{2\,q_i}\right) - \left(\sum_{i=1}^{n}\frac{p_i + q_i}{2}\right)\left(\sum_{i=1}^{n} q_i\, f'\left(\frac{p_i + q_i}{2\,q_i}\right)\right)$$

$$\le \frac{1}{4}(\beta - \alpha)\{f'(\beta) - f'(\alpha)\}$$

or $0 \le S_f(P,Q) - f(1) \le \sum_{i=1}^{n}\left(\frac{p_i+q_i}{2} - q_i\right) f'\left(\frac{p_i+q_i}{2q_i}\right)$

$\le \frac{1}{4}(\beta-\alpha)\{f'(\beta) - f'(\alpha)\}$

i.e. $S_f(P,Q) \le T_{S_f}(P,Q) \le Y_{S_f}(\alpha,\beta)$

3. SPECIAL CASES OF THE ABOVE THEOREM:

In this section, we will obtain some results (special cases) of above theorem in terms of various standard divergence measures i.e. bounds on $S_f(P,Q)$ by using inequalities (14).

Result 3.*1* *Let* $\Delta(P,Q)$, $\chi^2(P,Q)$ *and* $\psi(P,Q)$ *be defined as in (2), (3) and (6) respectively then for P,Q ∈Γn, we have the result:*

$$\Delta(P,Q) \le \frac{1}{2}\chi^2(P,Q) + \frac{1}{4}\psi(P,Q) \le \frac{1}{2}(\beta-\alpha)^2 L_{-3}^{-3}(\alpha,\beta) \quad \text{... (17)}$$

Proof : let us consider

$$f(t) = \frac{(t-1)^2}{t}, t \in (0,\infty) \text{ and } f(1) = 0$$

and $f'(t) = \frac{(t^2-1)}{t^2}$, $f''(t) = \frac{2}{t^3} > 0 \ \forall \ t > 0$

Since $f''(t) > 0 \ \forall \ t > 0$ and $f(1) = 0$, so $f(t)$ is convex and normalized function respectively.

Put $f(t)$ in (1) and $f'(t)$ in (15) and (16), we get the followings

$$S_f(P,Q) = \frac{1}{2}\sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i+q_i} = \frac{1}{2}\Delta(P,Q) \quad \text{.... (18)}$$

$$T_{S_f}(P,Q)=\frac{1}{2}\sum_{i=1}^{n}(p_i-q_i)\left[1-\frac{4\,q_i^2}{(p_i+q_i)^2}\right]$$

$$=\frac{1}{2}\sum_{i=1}^{n}(p_i-q_i)\left[\frac{(p_i+q_i)^2-4\,q_i^2}{(p_i+q_i)^2}\right]=\frac{1}{2}\sum_{i=1}^{n}\frac{(p_i-q_i)^2\,(p_i+3\,q_i)}{(p_i+q_i)^2} \quad \text{..... (19)}$$

$$Y_{S_f}(\alpha,\beta)=\frac{1}{4}(\beta-\alpha)\left[\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right]=\frac{1}{2}(\beta-\alpha)^2\,L_{-3}^{-3}(\alpha,\beta) \quad \text{.... (20)}$$

Now from (18) and (19), we get

$$S_f(P,Q)\le T_s,(P,Q)\Rightarrow\frac{1}{2}\Delta(P,Q)\le\frac{1}{2}\sum_{i=1}^{n}\frac{(p_i-q_i)^2(p_i+3q_i)}{(p_i+q_i)^2}$$

$$\Rightarrow\ \Delta(P,Q)\le\left[\sum_{i=1}^{n}\frac{(p_i-q_i)^2\,(p_i+q_i)}{(p_i+q_i)^2}+\sum_{i=1}^{n}\frac{2\,q_i\,(p_i-q_i)^2}{(p_i+q_i)^2}\right]$$

Since $\frac{p_i+q_i}{2}\ge\sqrt{p_i\,q_i}$ (A.M $\ge$ G.M) with equality if $p_i=q_i\ \forall\ i$.

Therefore $\Delta(P,Q)\le\left[\frac{1}{4}\sum_{i=1}^{n}\frac{(p_i-q_i)^2\,(p_i+q_i)}{p_i\,q_i}+\frac{1}{2}\sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i}\right]$

After interchanging P and Q, we get

$$S_f(P,Q)\le T_s,(P,Q)\Rightarrow\Delta(P,Q)\le\frac{1}{2}\chi^2(P,Q)+\frac{1}{4}\psi(P,Q) \quad \text{.... (21)}$$

Put the values from (20) and (21) in (14) and get the result (17).

Result 3.2 *Let* $K(P,Q)$ *and* $J(P,Q)$ *be defined as in (4) and (7) respectively then for P,Q∈Γn, we have the result:*

$$K(P,Q)\le J(P,Q)\le\frac{1}{2}(\beta-\alpha)\log\left(\frac{2\beta-1}{2\alpha-1}\right)\ \forall\ \frac{1}{2}<\alpha\le1 \quad \text{... (22)}$$

Proof : let us consider

$$f(t)=(2t-1)\log(2t-1),\ t\in\left(\frac{1}{2},\infty\right) and\ f(1)=0$$

and $f'(t)=2\left[1+\log(2t-1)\right]$, $f''(t)=\frac{4}{2t-1}>0\ \forall\ t>\frac{1}{2}$

Since $f''(t)>0\ \forall\ t>1/2$ and $f(1)=0$, so $f(t)$ is convex and normalized function respectively. Put $f(t)$ in (1) and $f'(t)$ in (15) and (16), we get the following

$$S_f(P,Q)=\sum_{i=1}^{n} p_i \log\left(\frac{p_i}{q_i}\right)=K(P,Q) \qquad \text{..... (23)}$$

$$T_{S_f}(P,Q)=\sum_{i=1}^{n}(p_i-q_i)\log\left(\frac{p_i}{q_i}\right)=K(P,Q)+K(Q,P)=J(P,Q) \qquad \text{..... (24)}$$

$$Y_{S_f}(\alpha,\beta)=\frac{1}{2}(\beta-\alpha)\log\left(\frac{2\beta-1}{2\alpha-1}\right) \qquad \text{.... (25)}$$

Put the values from (23), (24) and (25) in (14) and get the result (22).

Result 3.3. *Let* $\Delta(P,Q),G(P,Q),F(P,Q)$ *and* $J_R(P,Q)$ *be defined as in (2), (8), (9) and (5) respectively then for* $P,Q\in\Gamma n$, *we have the result*

$$\frac{1}{2}J_R(P,Q)\le\left[F(Q,P)+G(Q,P)\right]+\frac{1}{2}\Delta(P,Q)\le\frac{1}{4}(\beta-\alpha)^2\left[L_{-1}^{-1}(\alpha,\beta)+L_{-2}^{-2}(\alpha,\beta)\right] \qquad \text{.... (26)}$$

Proof : let us consider

$$f(t)=(t-1)\log t,\ t\in(0,\infty)\ and\ f(1)=0$$

and $f'(t)=\frac{(t-1)}{t}+\log t$, $f''(t)=\frac{1+t}{t^2}>0\ \forall\ t>0$

Since $f''(t)>0\ \forall\ t>0$ and $f(1)=0$, so $f(t)$ is convex and normalized function respectively. Put $f(t)$ in (1) and $f'(t)$ in (15) and (16), we get the following :

$$S_f(P,Q)=\frac{1}{2}\sum_{i=1}^{n}(p_i-q_i)\log\left(\frac{p_i+q_i}{2\,q_i}\right)=\frac{1}{2}J_R(P,Q) \qquad \text{... (27)}$$

$$T_{S_f}(P,Q)=\sum q_i\log\left(\frac{2\,q_i}{p_i+q_i}\right)+\sum_{i=1}^{n}\left(\frac{p_i+q_i}{2}\right)\log\left(\frac{p_i+q_i}{2\,q_i}\right)+\frac{1}{2}\sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i+q_i}$$

$$=\left[F(Q,P)+G(Q,P)\right]+\frac{1}{2}\Delta(P,Q) \qquad \text{.... (28)}$$

$$Y_{S_f}(\alpha,\beta)=\frac{1}{4}(\beta-\alpha)\left[\log\frac{\beta}{\alpha}+\frac{1}{\alpha}-\frac{1}{\beta}\right]$$

$$=\frac{1}{4}(\beta-\alpha)^2\left[L_{-1}^{-1}(\alpha,\beta)+L_{-2}^{-2}(\alpha,\beta)\right] \qquad \text{.... (29)}$$

Put the values from (27), (28) and (29) in (14) and get the result (26).

Result 3.4. *Let* $FG_s(P,Q)$ *and* $J_{Rs}(P,Q)$ *be defined as in (10) and (11) respectively then for P,Q ∈Γn and* $s\in R$, *we have the result:*

$$FG_s(Q,P)\le\frac{1}{2}J_{Rs}(P,Q)\le\frac{1}{4}(\beta-\alpha)^2 L_{s-2}^{s-2}(\alpha,\beta),\quad s\ne 0,1 \qquad \text{.... (30)}$$

Proof : let us consider

$$f_s(t)=\begin{cases}\left[s(s-1)\right]^{-1}\left[t^s-s(t-1)-1\right], & s\ne 0,1\\ t-1-\log t, & s=0\\ 1-t+t\log t, & s=1\end{cases}\quad s\in R,\ t\in(0,\infty)\ \text{and}\ f_s(1)=0$$

$$f_s'(t)=\begin{cases}(s-1)^{-1}\left[t^{s-1}-1\right], & s\ne 1\\ 1-\dfrac{1}{t}, & s=0\\ \log t, & s=1\end{cases}\quad s\in R$$

and $f_s''(t) = t^{s-2} > 0 \ \forall \ t > 0 \ and \ s \in R$

Since $f_s''(t) > 0 \ \forall \ t > 0$ and $f_s(1) = 0$, so $f_s(t)$ are convex and normalized functions respectively $\forall \ s \in R$. Put $f_s(t)$ in (1) and $f_s'(t)$ in (15) and (16), we get:

$$S_f(P,Q) = \left[s(s-1)\right]^{-1}\left[\sum_{i=1}^{n} q_i \left(\frac{p_i + q_i}{2\,q_i}\right)^s - 1\right] = FG_s(Q,P), \ s \neq 0, 1, s \in R \quad \text{... (31)}$$

$$T_{S_f}(P,Q) = \frac{(s-1)^{-1}}{2}\sum_{i=1}^{n}(p_i - q_i)\left(\frac{p_i + q_i}{2\,q_i}\right)^{s-1} = \frac{1}{2}J_{Rs}(P,Q), \ s \neq 1, s \in R \quad \text{....(32)}$$

$$Y_{S_f}(\alpha,\beta) = \frac{1}{4}(\beta-\alpha)(s-1)^{-1}\left[\beta^{s-1} - \alpha^{s-1}\right] = \frac{1}{4}(\beta-\alpha)^2\left[L_{s-2}^{s-2}(\alpha,\beta)\right], s \in R \quad \text{.... (33)}$$

Put the values from (31), (32) and (33) in (14) and get the result (30).
Now we are considering two special cases of results (30) at s = 0 and s=1.

Case 3.(i) (at s = 0). *For P,Q ∈Γn, we have the following inequalities:*

$$F(P,Q) \leq \frac{1}{2}\Delta(P,Q) \leq \frac{1}{4}(\beta-\alpha)^2 L_{-2}^{-2}(\alpha,\beta) \quad \text{... (34)}$$

where $\Delta(P,Q)$ *and* $F(P,Q)$ are defined as in (2) and (9) respectively.

Proof : At s = 0, we get the following

$$FG_0(Q,P) = F(Q,P) = \sum_{i=1}^{n} q_i \log\left(\frac{2\,q_i}{p_i + q_i}\right) \quad \text{... (35)}$$

$$J_{R0}(P,Q) = \sum_{i=1}^{n}(q_i - p_i)\left(\frac{p_i + q_i}{2\,q_i}\right)^{-1} = \sum_{i=1}^{n}(q_i - p_i)\left(\frac{2\,q_i}{p_i + q_i}\right)$$

$$= 2\sum_{i=1}^{n} \frac{q_i^2 - 2\,p_i q_i + p_i q_i}{p_i + q_i} = \sum_{i=1}^{n} 2\frac{q_i^2 + p_i q_i}{p_i + q_i} - \frac{4\,p_i q_i}{p_i + q_i} = 2\left[\sum_{i=1}^{n} q_i - \frac{2 p_i q_i}{p_i + q_i}\right]$$

$$= 2\left[1 - W(P,Q)\right] = \Delta(P,Q) \qquad \text{... (36)}$$

$$Y_{S_f}(\alpha,\beta) = \frac{1}{4}(\beta-\alpha)^2 L_{-2}^{-2}(\alpha,\beta) \qquad \text{...(37)}$$

By putting (35), (36) and (37) in (30) at s = 0 and get the result (after interchanging P and Q) (34).

Case 3.(ii) *(at s=1). For P,Q ∈Γn, we have the following inequality*

$$G(P,Q) \le \frac{1}{2} J_R(Q,P) \le \frac{1}{4}(\beta-\alpha)^2 L_{-1}^{-1}(\alpha,\beta) \qquad \text{.... (38)}$$

where $J_R(P,Q)$ *and* $G(P,Q)$ are defined as in (5) and (8) respectively.

Proof : At s=1, we get the following

$$FG_1(Q,P) = G(Q,P) = \sum_{i=1}^{n}\left(\frac{p_i + q_i}{2}\right)\log\left(\frac{p_i + q_i}{2\,q_i}\right) \qquad \text{... (39)}$$

$$J_{R1}(P,Q) = J_R(P,Q) = \sum_{i=1}^{n}(p_i - q_i)\log\left(\frac{p_i + q_i}{2\,q_i}\right) \qquad \text{... (40)}$$

$$Y_{S_f}(\alpha,\beta) = \frac{1}{4}(\beta-\alpha)^2 L_{-1}^{-1}(\alpha,\beta) \qquad \text{... (41)}$$

By putting (39), (40) and (41) in (C.14) at s = 1 and get the result (after interchanging P and Q) (38).

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