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LRS BIANCHI TYPE-I COSMOLOGY WITH BILINEAR DECELERATION PARAMETER IN SCALE-COVARIANT THEORY OF GRAVITATION

Dinkar Singh Chauhan
Department of Mathematics, Dr. R.M.L.S. College, Muzaffarpur-842001, India
Email: dscmaths9616@gmail.com

Abstract: In this paper we have investigated a spatially homogeneous locally rotationally symmetric (LRS) Bianchi type-I space-time in the presence of perfect fluid source in scale-covariant theory of gravitation formulated by Canuto et al. [7]. The field equations are solved by considering bilinear form of the DP (Deceleration Parameter) q as a function of cosmic time t as: (i) $q = \frac{m(1-t)}{1+t}$, $m \geq 0$, which renders early decelerating and late time accelerating cosmological model and (ii) $q = -\frac{mt}{1+t}$, $m \geq 0$, which renders accelerated expansion model of the universe. The physical and geometrical properties of the derived models are also discussed.

Keywords: Cosmology, Bianchi type-I cosmological model, Deceleration parameter, Accelerating universe, Scale-covariant theory.

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1. Introduction

Alternative theories of gravitation have been extensively studied by many researchers for the last four decades in different physical contexts. Brans-Dicke (BD) theory [6] is one of the noteworthy among all the alternative theories of gravitation. BD theory introduces a dynamical scalar field to account for variable gravitational constant G. Nordtvedt [18] developed a general class of scalar-tensor theory in which the parameter ω of the BD theory is allowed to be an arbitrary function of the scalar field. Saez and Ballester [27] proposed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. Canuto et al. [7] formulated a scale-covariant theory of gravitation in which the gravitational constant G is a variable and where gravitational units are used for Einstein's field equations, but atomic units are used for physical quantities. The metric tensors in the two systems of units are related by a conformal transformation

$$\bar{g}_{ij} = \phi^2(x^k) g_{ij}, \tag{1}$$

where *i*, *j*, *k* = 1, 2, 3, 4, and bars denote gravitational units and unbars denote atomic units. The gauge function ϕ (0 < ϕ < ∞) in its most general formulation is a function of all space-time coordinates. With the above transformation the modified form of the Einstein field equations due to Canuto et al. [7] is

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda\ (\phi)\, g_{ij}, \tag{2}$$

with

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_{,i}\,\phi_{,j} - g_{ij}(\phi\phi^{;k}_{;k} - \phi^{,k}\phi_{,k}). \tag{3}$$

Here R_{ij} is the Ricci tensor, R the Ricci scalar, Λ the cosmological constant, G the gravitational constant, and T_{ij} the energy-momentum tensor. A particular feature of this theory is that no independent equation for ϕ exists. The possibilities that have been considered for gauge function ϕ are (Canuto et al. [8])

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon},\ \varepsilon = \pm 1, \quad \pm\frac{1}{2}, \tag{4}$$

where t_0 is a constant.

In recent years, researchers have shown considerable interest in scale-covariant theory of gravitation. Reddy and Venkateswarlu [22] have investigated Einstein-Rosen universe in scale-covariant theory of gravitation. Reddy et al. [23] have studied axially symmetric Bianchi type-I space-time in the presence of perfect fluid source in scale-covariant theory of gravitation. Belinchon [4] have investigated scale-covariant theory of gravitation under the self-similar hypothesis and applied the obtained results to study the Bianchi I, VII_0, IX and Kantowski-Sachs models. Singh et al. [29] have studied the behaviour and contribution of dark energy to the accelerated expansion of the universe in the scale-covariant theory of gravitation. Recently, Hatkar et al. [14] have investigated Bianchi type-I two fluids cosmological model in scale-covariant theory of gravitation.

Motivated by the above discussions, in this paper, we have investigated LRS Bianchi type-I cosmological models with bilinear deceleration parameter in scale-covariant theory of gravitation. The outline of the paper is as follows: In Section 2, the metric and field equations are described. Section 3, deals with the solutions of the field equations. In Subsection 3.1, we discussed the physical and geometric properties of the model 1. In Subsection 3.2, we discussed the physical and geometric properties of the model 2. Finally conclusions are given in Section 4.

2. The Metric and Field Equations

We consider an axially symmetric and spatially homogeneous LRS Bianchi type-I metric in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2), \tag{5}$$

where the metric potentials A and B are functions of cosmic time t only.

The energy-momentum tensor for a perfect fluid distribution is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \tag{6}$$

together with

$$u_i u^i = 1, \qquad u_i u^j = 0, \tag{7}$$

where ρ is the proper energy density, *p* is the isotropic pressure and u^i is the four-velocity of the fluid.

Considering the co-moving coordinate system and taking Λ = 0, the scale-covariant field equations (2) take the forms:

$$\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{\ddot{\phi}}{\phi} - \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{2\dot{B}\dot{\phi}}{B\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G p, \tag{8}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G p, \tag{9}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}\dot{\phi}}{A\phi} + \frac{2\dot{B}\dot{\phi}}{B\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 = 8\pi G \rho, \tag{10}$$

where an overdot denotes ordinary differentiation with respect to cosmic time *t*.

We define the average scale factor *a*, the volume scalar V and the generalized mean Hubble parameter *H* as

$$a = (AB^2)^{\frac{1}{3}}, \tag{11}$$

$$V = a^3 = AB^2, \tag{12}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{13}$$

where $H_1 = \frac{\dot{A}}{A}, H_2 = \quad H_3 = \frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of *x* , *y* and *z* - axes, respectively.

The physical quantities of observational interest in cosmology such as the expansion scalar θ, the shear scalar σ^2 and the average anisotropy parameter A_m are defined as

$$\theta = u^i_{;i} = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 , \tag{15}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 . \tag{16}$$

From equations (11) - (13), we obtain an important relation

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right). \tag{17}$$

The deceleration parameter *q* in a cosmological model is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 . \tag{18}$$

3. Solutions of the Field Equations

Using equations (8) and (9), we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{2\dot{A}\dot{\phi}}{A\phi} + 2\frac{\dot{B}\dot{\phi}}{B\phi} = 0 , \tag{19}$$

which admits as exact solution

$$A = \alpha B, \tag{20}$$

where α is a constant. In this scenario the shear scalar and the anisotropy parameter are zero and the field equations (8) - (10) reduce to

$$\frac{2\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{\dot{B}\dot{\phi}}{B\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi Gp , \tag{21}$$

$$\left(\frac{\dot{B}}{B}\right)^2 + \frac{\dot{B}\dot{\phi}}{B\phi} + \left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{8}{3}\pi G\rho . \tag{22}$$

The field equations (21) and (22) are a system of two independent equations with five unknown parameters, namely, B, ρ, p, G and ϕ. For complete determinacy of the system three extra conditions are required. To obtain the approximated exact solutions of the system the following physical conditions are used.

(i) Following Canuto and Goldman [9], we consider here the gauge function ϕ as

$$\phi = \phi_0 \, t^{\frac{1}{2}}, \tag{23}$$

where ϕ_0 is a constant.

(ii) In scale-covariant theory of gravitation the gravitational term *G* is assumed as variable. Following Hatkar et al. [14], we assume the most useful and simple form of *G* as

$$G = \gamma t, \tag{24}$$

where γ is the proportionality constant. It gives us a physically viable and realistic model of the universe. Such types of relations have already been considered by Levit [15] and Beesham [3] to find pressure and energy density in terms of cosmic time. Cosmological models with variable *G* in C-field cosmology are investigated by Bali and Meghna [2].

(iii) We wish to study the nature of expansion of universe by assuming deceleration parameter (*q*) as a bilinear function of cosmic time *t*. The motivation to choose such type of bilinear time-dependent DP is under the influence of the work published by Mishra and Chand [17] and Singh and Bishi [30]. It is worthwhile to mention here that such type of DP suggest that the universe has an accelerated expansion at present as observed in the recent observations of SNe Ia (Riess et al. [24], Perlmutter et al. [20], Tonry et al. [31], Clocchiatti et al. [10]) and CMB anisotropies (Bennett et al. [5], de Bernardis et al. [11], Hanany et al. [13]) and decelerated expansion in the past. Now for a universe which was decelerating in the past and accelerating at the present time, the DP must show signature flipping (see refs. Padmanabhan and Roy Choudhury [19], Amendola [1], Riess et al. [25]). So, in general the DP is not a constant but time variable.

In the following subsections, we discuss the cosmological models by taking deceleration parameter *q* as a bilinear functions of cosmic time *t* as discussed above.

3.1 Model 1 : If $q(t) = \dfrac{m(1-t)}{1+t}$, $m \geq 0$.

$$q = \frac{m(1-t)}{1+t}. \tag{25}$$

On substituting the value of *q* from equation (25) in equation (18), we get

$$\frac{d}{dt}\left(\frac{1}{H}\right) = 1 + \frac{m(1-t)}{1+t}, \tag{26}$$

which on integration yields

$$H = \frac{1}{(1-m)t + 2m\log(1+t) + c_1}, \tag{27}$$

where c_1 is a constant of integration. The condition $H \to \infty$ when $t \to 0$ gives $c_1 = 0$. Thus equation (27) takes the form

$$H = \frac{1}{(1-m)\,t + 2m\,\log\,(1+t)}\,. \tag{28}$$

Equation (28) is expressed as

$$H = \frac{1}{(1-m)\,t + 2m\left(t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \ldots\right)}$$

$$= \frac{1}{(1+m)\,t\left[1 - \frac{2mt}{(1+m)}\left(\frac{1}{2} - \frac{t}{3} + \frac{t^2}{4} - \ldots\right)\right]}$$

$$= \frac{1}{(1+m)\,t}\left[1 - \frac{2mt}{(1+m)}\left(\frac{1}{2} - \frac{t}{3} + \frac{t^2}{4} - \ldots\right)\right]^{-1}$$

$$= \frac{1}{(1+m)\,t}\left[1 + \frac{2mt}{(1+m)}\left(\frac{1}{2} - \frac{t}{3} + \frac{t^2}{4} - \ldots\right) + \frac{4m^2t^2}{(1+m)^2}\left(\frac{1}{2} - \frac{t}{3} + \frac{t^2}{4} - \ldots\right)^2\right.$$

$$\left. + \frac{8m^3t^3}{(1+m)^3}\left(\frac{1}{2} - \frac{t}{3} + \frac{t^2}{4} - \ldots\right)^3 + \ldots\right]. \tag{29}$$

On simplification of the above expression, we obtain

$$H = \frac{1}{(1+m)\,t} + k_0 + k_1\,t + k_2\,t^2 + k_3 t^3 + O\,(t^4)\,, \tag{30}$$

where

$$k_0 = \frac{m}{(1+m)^2} \quad ; \quad k_1 = \frac{-2m + m^2}{3(1+m)^3} \quad ; \quad k_2 = \frac{3m - 2m^2 + m^3}{6\,(1+m)^4} \quad ;$$

$$k_3 = \frac{-18m + 11m^2 - 14m^3 + 2m^4}{45(1+m)^5}\,.$$

Since $H = \frac{\dot{a}}{a}$, hence integration of equation (30) gives

$$a = c_2\ t^{1/(1+m)}\ e^{F(t)}, \tag{31}$$

where $F(t) = k_0(t) + k_1\left(\frac{t^2}{2}\right) + k_2\left(\frac{t^3}{3}\right) + k_3\left(\frac{t^4}{4}\right) + O(t^5)$, and c_2 is an integration constant.

From equations (11), (20) and (31), we get

$$A = c_2\,\alpha^{2/3}\ t^{1/(1+m)}\ e^{F(t)}, \tag{32}$$

$$B = c_2\,\alpha^{-1/3}\ t^{1/(1+m)}\ e^{F(t)}. \tag{33}$$

Hence the cosmological model (1) can be written as (through a proper choice of integration constant i.e. $c_2 = 1$)

$$ds^2 = dt^2 - t^{2/(1+m)}\ e^{2F(t)}\left[\ \alpha^{4/3}\ dx^2 + \alpha^{-2/3}(dy^2 + dz^2)\ \right]. \tag{34}$$

It is observed from equation (25) that $q > 0$ for $0 < t < 1$, $q = 0$ at $t = 1$ and $q < 0$ for $t > 1$, which means that universe expansion is decelerating at early time $t < 1$, constant expansion for a moment at $t = 1$ and for $t > 1$ expanding with an accelerating rate. It is clear that the DP has shown transitional nature of the universe (i.e. from the early decelerating phase to the recent accelerating phase) for chosen value of m. It is also observed that such type of nature of the universe is confirmed by recent Supernovae Ia observations.

The expressions for the isotropic pressure (p) and the proper energy density (ρ) for the derived model are obtained as

$$p = \frac{(5m-3)t - 3(1+m)t^2 - 2m(1+t)\log(1+t)}{16\pi\gamma t^2(1+t)[(1-m)t + 2m\log(1+t)]^2} + \frac{1}{16\pi\gamma t^3}, \tag{35}$$

$$\rho = \frac{3[(3-m)t + 2m\log(1+t)]}{16\pi\gamma t^2\,[(1-m)t + 2m\log(1+t)]^2} + \frac{1}{8\pi\gamma t^3}. \tag{36}$$

The directional Hubble parameters H_i (i = 1, 2, 3), the average Hubble parameter H, the expansion scalar θ and the spatial volume V of the model are, respectively given by

$$H_1 = H_2 = H_3 = H = \frac{1}{(1-m)t + 2m\log(1+t)}, \tag{37}$$

$$\theta = \frac{3}{(1-m)t + 2m\log(1+t)}, \tag{38}$$

$$V = t^{3/(1+m)}\ e^{3F(t)}. \tag{39}$$

From the above set of solutions we observe that the expansion scalar θ is infinite and the spatial volume V is zero at $t = 0$, which shows that the universe starts evolving with zero volume at $t = 0$, which is a Big-Bang scenario. The energy density and pressure become infinite at $t = 0$. It is observed that the scale factors A and B are positive and increasing function of cosmic time t, and the model is expanding (i.e. $\theta > 0$). The Hubble parameters H_i (($i = 1, 2, 3$) in the direction of coordinate axes (x, y, z) are the same. In this scenario the average anisotropy parameter A_m and shear scalar σ^2 are zero. When $t \to \infty$, $p \to 0$ and $\rho \to 0$ and hence the model would give essentially an empty universe at large time. The spatial volume tends to infinity as $t \to \infty$. The rate of expansion in the model tends to zero when $t \to \infty$. It is also observed that $\lim_{t\to\infty} \sigma/\theta^2 = 0$ and $\lim_{t\to\infty} \rho/\theta^2 = 0$, which shows that the universe remains isotropic and homogeneous with cosmic time. Thus our model represents a non-shearing, non-rotating, isotropic, homogeneous and expanding universe, which was decelerating in the past and accelerating at present time.

3.2 Model 2 : If $q(t) = -\frac{mt}{1+t}, \quad m \geq 0$.

$$q = -\frac{mt}{1+t}. \tag{40}$$

Using equation (40) in equation (18), we get

$$\frac{d}{dt}\left(\frac{1}{H}\right) = 1 - \frac{mt}{1+t}, \tag{41}$$

which on integration gives

$$H = \frac{1}{(1-m)t + m\log(1+t) + d_1}, \tag{42}$$

where d_1 is an integration constant. The condition $H \to \infty$ when $t \to 0$ gives $d_1 = 0$. Thus equation (42) takes the form

$$H = \frac{1}{(1-m)t + m\log(1+t)}. \tag{43}$$

The above equation is expressed as

$$H = \frac{1}{t}\left[1 - mt\left(\frac{1}{2} - \frac{t}{3} + \frac{t^2}{4} - \ldots\right)\right]^{-1}, \tag{44}$$

which on simplification takes the form

$$H = \frac{\dot{a}}{a} = \frac{1}{t} + k_0 + k_1 t + k_2 t^2 + k_3 t^3 + O(t^4), \tag{45}$$

where

$$k_0 = \frac{m}{2} \; ; \quad k_1 = \frac{-4m + 3m^2}{12} \; ; \quad k_2 = \frac{6m - 8m^2 + 3m^3}{24} \; ;$$

$$k_3 = \frac{-144m + 260m^2 - 180m^3 + 45m^4}{720}.$$

Integration of equation (45) yields

$$a = d_2 \, t \; e^{G(t)}, \tag{46}$$

where

$G(t) = k_0 (t) + k_1 \left(\frac{t^2}{2}\right) + k_2 \left(\frac{t^3}{3}\right) + k_3 \left(\frac{t^4}{4}\right) + O\left(t^5\right)$, and d_2 is an integration constant.

From equations (11), (20) and (46), we get

$$A = d_2 \, \alpha^{2/3} \; t e^{G(t)}, \tag{47}$$

$$B = d_2 \; \alpha^{-1/3} \; t \; e^{G(t)}. \tag{48}$$

Using equations (47) and (48), the metric (1) takes the form (through a proper choice of integration constant i.e. $d_2 = 1$)

$$ds^2 = dt^2 - t^2 \; e^{2G(t)} \left[\alpha^{4/3} \; dx^2 + \alpha^{-2/3} \; \left(dy^2 + dz^2\right)\right]. \tag{49}$$

From equation (40), it is observed that $q < 0$ for $m > 0$. Hence our universe is expanding at an accelerating rate with the evolution of time, which is supported by recent Supernovae Ia observations.

The expressions for the isotropic pressure (p) and the proper energy density (ρ) for the above model are obtained as

$$p = \frac{(m-3)t - 3(m+1)t^2 - m(1+t)\log(1+t)}{16\pi \gamma t^2 (1+t)[(1-m)t + m\log(1+t)]^2} + \frac{1}{16\pi \gamma t^3}, \tag{50}$$

$$\rho = \frac{3\left[(3-m)t + m\log(1+t)\right]}{16\pi \gamma t^2 \left[(1-m)t + m\log(1+t)\right]^2} + \frac{1}{8\pi \gamma t^3}. \tag{51}$$

The directional Hubble parameters H_i (i = 1, 2, 3), the average Hubble parameter H, the expansion scalar θ and the spatial volume V of the model are, respectively given by

$$H_1 = H_2 = H_3 = H = \frac{1}{(1-m)t + m\log(1+t)}, \tag{52}$$

$$\theta = \frac{3}{(1-m)t + m\log(1+t)}, \tag{53}$$

$$V = t^3\, e^{3G(t)}. \tag{54}$$

From the above set of solutions we observe that the cosmic scale factors A, B and the spatial volume V are increasing function of cosmic time t. Initially, when $t \to 0$, the scale factors A, B and spatial volume V attain zero value and finally, when $t \to \infty$, they attain infinite values. The model starts expanding with Big-Bang at $t = 0$, which is a point type singularity (MacCallum [16]) in the model. Expansion parameters H_i ($i = 1, 2, 3$) in x, y and z directions are the same, which means that universe is expanding uniformly in all directions with time. In this scenario the average anisotropy parameter A_m and shear scalar σ^2 are zero and hence $\sigma/\theta = 0$ for all values of t. Also, we have $\lim_{t\to\infty} \rho/\theta^2 = 0$, which shows that our universe remains isotropic and homogeneous with cosmic time. It is also observed that isotropic pressure p and proper energy density ρ are decreasing function of cosmic time t and tend to zero when t tends to infinity. Therefore, the model would give essentially an empty universe at late times. The model represents a non-shearing, non-rotating, isotropic, homogeneous and expanding universe, which is expanding at an accelerating rate with evolution of time.

4. Conclusion

In present work, we have studied LRS Bianchi type-I cosmological models based on approximated exact solutions of modified Einstein's field equations with the help of bilinear DP (linear in both q an t) in scale-covariant theory of gravitation.

In model 1, the solution of the field equations has obtained by choosing the time dependent bilinear DP $q = \frac{m(1-t)}{1+t}$, which yields time dependent scale factor $a = t^{1/(1+m)}\, e^{F(t)}$. The model has an initial point type singularity [16]. It is observed that in early phase of universe (i.e. when $t < 1$), the value of deceleration parameter q is positive while for $t > 1$, the value of deceleration parameter q is negative. Hence the universe had a decelerated expansion in the past and has accelerated expansion at late time which is in good agreement with the recent observations of SNe Ia and CMB anisotropies.

In model 2, the solution of the field equations has obtained by choosing the time dependent bilinear DP $q = -\frac{mt}{1+t}$, which again yields time dependent scale factor $a = t\,e^{G(t)}$. The model has an initial point type singularity [16]. It is observed that the value of deceleration parameter *q* is negative for all the values of *t*. Hence the universe is in accelerated expansion mode since its beginning after the big-bang.

It is observed that in our derived models, the model 1 is more appropriate than the model 2, to understand the dynamics of the universe. It is worth mentioning that both the derived models are expanding, non-shearing, non-rotating, isotropic and homogeneous for large value of *t*. Thus, derived models are in good agreement with recent cosmological observations (Perlmutter et al. [20,21], Riess et al. [24,26], Schmidt et al. [28], Garnavich et al. [12]).

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