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EVOLUTION OF BIANCHI TYPE-VI_h UNIVERSE WITH POLYTROPIC EQUATION OF STATE IN $f(R,T)$ GRAVITY

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Abstract: The Bianchi type $VI_{h=-1}$ Universe with a variable cosmological parameter Λ has been studied in the framework of $f(R,T)$ theory of gravity, where R is Ricci sclar and T is trace of the stress energy momentum tensor. The general solution of the models is derived by adopting the functional form $f(R,T) = f_1(R) + f_2(T)$ along with polytropic equation of state and a power law approach. By considering these components, a comprehensive analysis of dynamical parameters of the Bianchi type $VI_{h=-1}$ Universe has been performed.

Keywords: The Bianchi type $VI_{h=-1}$ cosmological model, $f(R,T)$ Gravity, Polytropic EoS.

1. Introduction

Data currently gathered from various sources shows that the cosmos is perpetually accelerating and expanding. The study has been done in supernova type Ia [24, 23], there are some lacunas in Einstein's general theory of gravity which needs improvement and modification in it. There are more than one type of different modifications and extensions in the Einstein's general theory of relativity, such as $f(R,T), f(T), f(R)$, etc. theories of gravitation. The $f(R,T)$ gravity is one of the popular type of modified theory of gravitation given by Harko *et al.* [14] in which T is trace of the stress tensor T_{ij} and gravitational lagrangian is determined by arbitrary function of R Ricci scalar. The exciting and the unique feature of this theory is that it might be able to explain the current acceleration of the cosmos without use of dark energy. The transition of deceleration to acceleration was studied by Moraes *et al.* [20] and two sets of cosmological equations where solved by him. Mehmood *et al.* [19], studied the $f(R,T)$ gravity by considering appropriate conformal vector fields of Bianchi type-I space-time with perfect fluid. Several other authors studied Bianchi type cosmological models in $f(R,T)$ gravity in different context [8, 13, 7, 18, 28, 4, 25, 6].

Although considered to be more difficult than the conventional big bang model, the polytropic model with equation of state was previously based around the idea of Newtonian gravity [11] and then extended in the context of general theory of relativity

[29]. It was found to be useful because it could be used to model stars made of realistic matter, such as photon, ideal gas, and in particular quark matter. Mukhopdhyay *et al*. [21] investigated kinematical Λ models by considering the polytropic Equation of state (EoS) of the form $p = \omega\rho^n$, where n is polytropic index. Maeda *et al*. [15] investigated spherically symmetric spacetimes with a perfect fluid and two type of polytropic equation of state admitting a kinematic self-similar vector of the second kind that is neither orthogonal to the fluid flow nor parallel to it. Adhav *et al*. [1] studied Higher dimensional Universe with polytropic EoS of the form $p = K\rho^n$ here K & n are the polytropic constant and its index. The Korkina - Orlyanskii space time has been studied by Singh *et al*. [26] by taking the modified form polytropic EoS for four distinct values of polytropic index n. Bali *et al.* [9] studied the accelerating nature of Bianchi type VI_0 magnetized bulk viscous massive string cosmological model in the theory of gravitation. The polytropic equation of state has been studied in general theory of relativity by several authors [10, 17, 16, 22, 3, 2, 27, 12].

In $f(R,T)$ theory of gravity Agrawal and Nile [5] has studied various space-times with polytropic equation of state. Looking forward this work in $f(R,T)$ gravity here we consider the polytropic EoS in the form as

$$p = \alpha\rho^n - \rho, \tag{1}$$

where $\alpha \neq 0$ is a polytropic constant and n is polytropic index.

In current study we use the form $f(R,T) = f_1(R) + f_2(T)$ of $f(R,T)$ theory of gravity. The basic formalism of the theory is given in section 2, the solution of field equations along with Bianchi type VI_h is given in section 3, the dynamical parameters are discussed in section 4, the graphical representation of dynamical properties of the model are shown in section 5, and the conclusion is discussed in section 6.

2. Basic Formation of $f(R,T)$ Gravity

The action for $f(R,T)$ theory of gravity is given by [14]

$$S = \frac{1}{16\pi}\int\left(f(R,T) + L_m\right)\sqrt{-g}\,d^4x, \tag{2}$$

Here $f(R,T)$ represents an arbitrary function of Ricci scalar R whereas, the trace T of the stress energy tensor T_{ij}, L_m is the matter Lagrangian density depends on g_{ij} and the stress energy momentum tensor of matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,L_m)}{\partial g^{ij}}, \tag{3}$$

Therefore,

$$T_{ij} = g^{ij}\,L_m - 2\frac{\partial\,L_m}{\partial g^{ij}}, \tag{4}$$

On varying the action S of the gravitational field with respect to the g^{ij}, gives the following $f(R,T)$ field equation as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\nabla^\alpha\nabla_\alpha - \nabla_i\nabla_j\right)f_R(R,T)$$
$$= 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij}\ , \tag{5}$$

where

$$\Theta_{ij} = -\mathrm{T}_{ij} - L_m \mathrm{g}_{ij} - 2g^{\mu\nu}\frac{\partial^2 L_m}{\partial g^{ij} g^{\mu\nu}}, \tag{6}$$

$f_R = \frac{\partial(R,T)}{\partial R}$, $f_T = \frac{\partial(R,T)}{\partial T}$ and T_{ij} is the energy momentum tensor associated with Lagrangian L_m.

For the perfect fluid the energy momentum tensor is given by

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij}, \tag{7}$$

With $u^i = (1,0,0,0)$ being the four velocity, for which $\mathrm{u}^i u_i = 1, \mathrm{u}^i \nabla_j u_i = 0$

From the equation (6) we get

$$\Theta_{ij} = -2\mathrm{T}_{ij} - p\mathrm{g}_{ij}, \tag{8}$$

Three classes of $f(R,T)$ gravity models are summaries as [14]

$$f(R,T) = \begin{cases} R + 2f_1(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}, \tag{9}$$

Here we consider one of the forms of $f(R,T)$gravity as $f(R,T) = f_1(R) + f_2(T)$

Using equation (7) and (8), equation (5) reduces to

$$f'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + \left(g_{ij}\nabla^\alpha\nabla_\alpha - \nabla_i\nabla_j\right)f_1'(R)$$
$$= 8\pi T_{ij} + f_2'(T)T_{ij} + f_2'(T)p\mathrm{g}_{ij} + \frac{1}{2}f_2(T)g_{ij}, \tag{10}$$

3. Field Equation and Dynamical Parameter

On considering $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$ for the function $f(R,T) = f_1(R) + f_2(T)$, the field equation (10) with cosmological parameter Λ reduces to

$$G_{ij} = \left(\frac{8\pi+\lambda}{\lambda}\right)T_{ij} + \left(\frac{\rho-p+2\Lambda}{2}\right)g_{ij}, \tag{11}$$

The Bianchi type VI_h metric is given by

$$ds^2 = dt^2 - J_1^2(t)dx^2 - J_2^2(t)e^{2x}dy^2 - J_3^2(t)e^{2hx}dz^2, \tag{12}$$

Where J_1 , J_2 , J_3 is the functions of cosmic time t. Here the exponent h has the value $-1, 0, 1$. Here to solve the metric in equation (12) with field equation in equation (11) we consider the value of $h = -1$, which gives

$$\frac{\ddot{J_2}}{J_2}+\frac{\ddot{J_3}}{J_3}+\frac{\dot{J_2}\dot{J_3}}{J_2J_3}+\frac{1}{J_1^2}=\left(\frac{16\pi+3\lambda}{2\lambda}\right)p-\frac{\rho}{2}-\Lambda, \tag{13}$$

$$\frac{\ddot{J_1}}{J_1}+\frac{\ddot{J_3}}{J_3}+\frac{\dot{J_1}\dot{J_3}}{J_1J_3}+\frac{1}{J_1^2}=\left(\frac{16\pi+3\lambda}{2\lambda}\right)p-\frac{\rho}{2}-\Lambda, \tag{14}$$

$$\frac{\ddot{J_1}}{J_1}+\frac{\ddot{J_2}}{J_2}+\frac{\dot{J_2}\dot{J_1}}{J_2J_1}+\frac{1}{J_1^2}=\left(\frac{16\pi+3\lambda}{2\lambda}\right)p-\frac{\rho}{2}-\Lambda, \tag{15}$$

$$\frac{\dot{J_2}\dot{J_1}}{J_2J_1}+\frac{\dot{J_2}\dot{J_3}}{J_2J_3}+\frac{\dot{J_1}\dot{J_3}}{J_1J_3}-\frac{1}{J_1^2}=-\left(\frac{16\pi+3\lambda}{2\lambda}\right)\rho+\frac{p}{2}-\Lambda, \tag{16}$$

$$\frac{\dot{J_2}}{J_2}-\frac{\dot{J_3}}{J_3}=0, \tag{17}$$

where overheated dot $(\,\dot{}\,)$ represent derivative with respect to time t.

Integrating equation (17), we get

$$J_2=kJ_3, \tag{18}$$

where k is an integrating constant.

Assuming constant of integration $k=1$ in equation (18), gives

$$J_2=J_3, \tag{19}$$

Using equation (19) the system of equation (13)-(17) reduces to

$$2\left(\frac{\ddot{J_3}}{J_3}\right)+\left(\frac{\dot{J_3}}{J_3}\right)^2+\frac{1}{J_1^2}=\left(\frac{16\pi+3\lambda}{2\lambda}\right)p-\frac{\rho}{2}-\Lambda, \tag{20}$$

$$\frac{\ddot{J_1}}{J_1}+\frac{\ddot{J_3}}{J_3}+\frac{\dot{J_1}\dot{J_3}}{J_1J_3}+\frac{1}{J_1^2}=\left(\frac{16\pi+3\lambda}{2\lambda}\right)p-\frac{\rho}{2}-\Lambda, \tag{21}$$

$$2\frac{\dot{J_2}\dot{J_1}}{J_2J_1}+\left(\frac{\dot{J_3}}{J_3}\right)^2-\frac{1}{J_1^2}=-\left(\frac{16\pi+3\lambda}{2\lambda}\right)\rho+\frac{p}{2}-\Lambda, \tag{22}$$

On solving above system of equation (20)-(22) we get,

$$J_1(t)=d_1V^{1/3}exp\left(c_1\int\frac{dt}{V}\right), \tag{23}$$

$$J_2(t)=J_3(t)=d_2V^{1/3}exp\left(c_2\int\frac{dt}{V}\right), \tag{24}$$

Here d_1, d_2 and c_1, c_2 are integrating constant satisfying the relation $d_1d_2^2=1$ and $c_1+2c_2=0$.

From equation (1), (16)-(13), we obtained

$$\rho^n=\frac{\lambda}{\alpha(8\pi+\lambda)}\left[\frac{\ddot{J_3}}{J_3}+\frac{\ddot{J_1}}{J_1}+\frac{2}{J_1^2}-\frac{\dot{J_3}}{J_3}\left(\frac{\dot{J_3}}{J_3}-\frac{\dot{J_1}}{J_1}\right)\right], \tag{25}$$

The deceleration parameter $q(t)$ is defined as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1, \tag{26}$$

The directional Hubble parameters are given by

$$H_x = \frac{\dot{J_1}}{J_1}, H_y = H_z = \frac{\dot{J_2}}{J_2}, \tag{27}$$

The mean Hubble parameter, is given by

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right), \tag{28}$$

Here H_x, H_y, H_z refers the directional Hubble parameters in the direction of x, y, z axes respectively.

The anisotropic parameter Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{29}$$

The special volume of model is given by

$$V = a^3, \tag{30}$$

4. Solution of Field Equations

In the system of equation (13)-(17) the total numbers of unknowns are six. These are given as $J_1(t), J_2(t), J_3(t), p, \rho$ and Λ. To solve the system of equations here we consider the power law of cosmology in the form

$$a(t) = \kappa t^{\gamma} \tag{31}$$

where $\kappa > 0$ and $\gamma > 0$ are constant.

From equation (22)-(24), (31) we get

$$J_1(t) = d_1(\kappa t^{\gamma}) exp\left(-\frac{c_1\kappa^{-3}}{3\gamma - 1}\right) t^{-3\gamma+1}, \tag{32}$$

$$J_2(t) = d_2(\kappa t^{\gamma}) exp\left(-\frac{c_2\kappa^{-3}}{3\gamma - 1}\right) t^{-3\gamma+1}, \tag{33}$$

$$J_3(t) = d_2(\kappa t^{\gamma}) exp\left(-\frac{c_2\kappa^{-3}}{3\gamma - 1}\right) t^{-3\gamma+1}, \tag{34}$$

where d_1, d_2 and c_1, c_2 are integrating constant satisfying the relation $d_1 d_2^2 = 1$ and

$c_1 + 2c_2 = 0$.

By using equations (25)-(30), the dynamical parameters of the model are obtained as follows:

Using equation (27) the directional Hubble parameter is given by

$$H_x = \frac{-c_1(-3\gamma+1)}{(3\gamma-1)\kappa^3 t^{3\gamma}}, H_y = H_z = \frac{-c_2(-3\gamma+1)}{(3\gamma-1)\kappa^3 t^{3\gamma}} + \frac{\gamma}{t}, \tag{35}$$

Using equation (28) the mean Hubble parameter is given by

$$H = \frac{\gamma}{t}, \tag{36}$$

The spatial volume is given by

$$V = (\kappa t^{\gamma})^3, \tag{37}$$

Using equation (26) the deceleration parameter is given by

$$q = -\left(\frac{\gamma-1}{\gamma}\right), \tag{38}$$

Using equation (29) the mean anisotropic parameter Δ is given by

$$\Delta = \frac{1}{2}\left[\frac{c_1(-3\gamma+1)}{\gamma(3\gamma-1)\kappa^3 t^{(3\gamma-1)}}\right]^2, \tag{39}$$

The energy density ρ, by using equation (25) and (32)-(34) is obtained as

$$\rho = \left[\frac{-\lambda}{(8\pi+\lambda)}\left[\begin{array}{l}\frac{c_2(c_1+c_2)(1-3\gamma)^2}{\kappa^6 t^{6\gamma}(3\gamma-1)^2} - \frac{3\gamma c_2(1-3\gamma)}{\kappa^3 t^{(3\gamma-1)}(3\gamma-1)} - (c_1^2+c_2^2)(1-3\gamma)^2 \\ -\frac{2\gamma}{t}\left(1+\frac{(1+3\gamma)}{t^{3\gamma}}-\frac{1}{t}\right) - \left(d_1^2\kappa^2 t^{2\gamma} exp\left(\frac{-3c_1 t^{(1-3\gamma)^2}}{\kappa^3(3\gamma-1)}\right)^{-1}\right)\end{array}\right]\right]^{1/n}, \tag{40}$$

The pressure p can be obtained by using the equation (1) and equation (40) as

$$p = \left[\frac{-\lambda}{(8\pi+\lambda)}\left[\begin{array}{l}\frac{c_2(c_1+c_2)(1-3\gamma)^2}{\kappa^6 t^{6\gamma}(3\gamma-1)^2} - \frac{3\gamma c_2(1-3\gamma)}{\kappa^3 t^{(3\gamma-1)}(3\gamma-1)} - (c_1^2+c_2^2)(1-3\gamma)^2 \\ -\frac{2\gamma}{t}\left(1+\frac{(1+3\gamma)}{t^{3\gamma}}-\frac{1}{t}\right) - \left(d_1^2\kappa^2 t^{2\gamma} exp\left(\frac{-3c_1 t^{(1-3\gamma)^2}}{\kappa^3(3\gamma-1)}\right)^{-1}\right)\end{array}\right]\right]$$

$$-\left[\frac{-\lambda}{\alpha(8\pi+\lambda)}\left[\begin{array}{l}\frac{c_2(c_1+c_2)(1-3\gamma)^2}{\kappa^6 t^{6\gamma}(3\gamma-1)^2} - \frac{3\gamma c_2(1-3\gamma)}{\kappa^3 t^{(3\gamma-1)}(3\gamma-1)} - (c_1^2+c_2^2)(1-3\gamma)^2 \\ -\frac{2\gamma}{t}\left(1+\frac{(1+3\gamma)}{t^{3\gamma}}-\frac{1}{t}\right) - \left(d_1^2\kappa^2 t^{2\gamma} exp\left(\frac{-3c_1 t^{(1-3\gamma)^2}}{\kappa^3(3\gamma-1)}\right)^{-1}\right)\end{array}\right]\right]^{1/n}, \tag{41}$$

Using the equations (20), (40) and (41) the cosmological constant Λ is given by

$$\Lambda = \left[-\left(\frac{16\pi+3\lambda}{4(8\pi+\lambda)}\right)\left[\begin{array}{l}\frac{c_2(c_1+c_2)(1-3\gamma)^2}{\kappa^6 t^{6\gamma}(3\gamma-1)^2} - \frac{3\gamma c_2(1-3\gamma)}{\kappa^3 t^{(3\gamma-1)}(3\gamma-1)} - (c_1^2+c_2^2)(1-3\gamma)^2 \\ -\frac{2\gamma}{t}\left(1+\frac{(1+3\gamma)}{t^{3\gamma}}-\frac{1}{t}\right) - \left(d_1^2\kappa^2 t^{2\gamma} exp\left(\frac{-3c_1 t^{(1-3\gamma)^2}}{\kappa^3(3\gamma-1)}\right)^{-1}\right)\end{array}\right]\right]$$

$$-\left(\frac{8\pi+\lambda}{\lambda}\right)\left[\frac{-\lambda}{\alpha(8\pi+\lambda)}\left[\begin{array}{l}\frac{c_2(c_1+c_2)(1-3\gamma)^2}{\kappa^6 t^{6\gamma}(3\gamma-1)^2} - \frac{3\gamma c_2(1-3\gamma)}{\kappa^3 t^{(3\gamma-1)}(3\gamma-1)} - (c_1^2+c_2^2)(1-3\gamma)^2 \\ -\frac{2\gamma}{t}\left(1+\frac{(1+3\gamma)}{t^{3\gamma}}-\frac{1}{t}\right) - \left(d_1^2\kappa^2 t^{2\gamma} exp\left(\frac{-3c_1 t^{(1-3\gamma)^2}}{\kappa^3(3\gamma-1)}\right)^{-1}\right)\end{array}\right]\right]^{1/n}$$

$$-\left[\begin{array}{l}\frac{c_2(c_1^2+c_{32}^2)(1-3\gamma)^2}{\kappa^6 t^{3\gamma}(3\gamma-1)^2}+\frac{3\gamma}{t}+\frac{3\gamma^2}{t^2}-\frac{3\gamma}{t^2}+\frac{2\gamma(1-3\gamma)}{t^{(3\gamma-1)}}+\frac{(c_1c_2(1-3\gamma)^2)}{\kappa^6 t^{6\gamma}(3\gamma-1)^2}+\frac{c_2\gamma(1-3\gamma)}{\kappa^3 t^{(3\gamma+1)}(3\gamma-1)}\\ +\left(d_1^2\kappa^2 t^{2\gamma} exp\left(\frac{-3c_1 t^{(1-3\gamma)^2}}{\kappa^3(3\gamma-1)}\right)^{-1}\right)\end{array}\right], \quad (42)$$

5. Graphical Representation of Dynamical Parameters

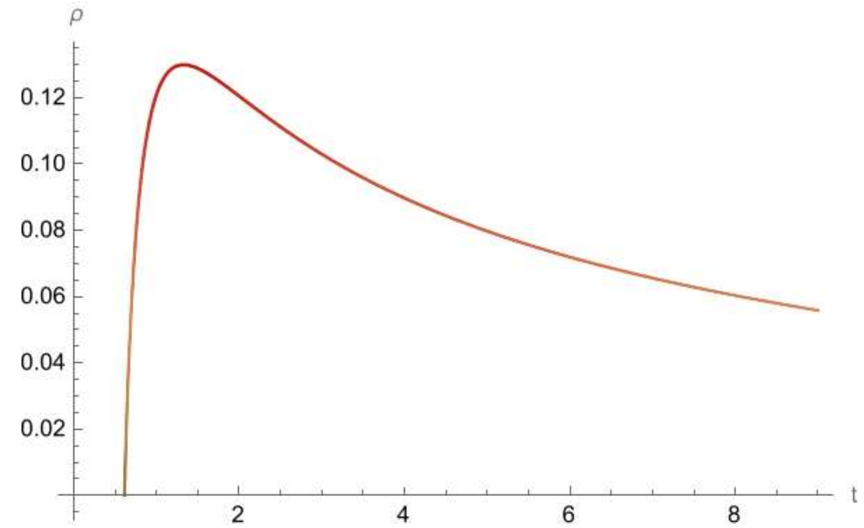

Figure-1: Energy density verses cosmic time plotted by considering
$\lambda = -1, c_1 = -0.5, c_2 = 0.5, \gamma = 0.5, \kappa = 0.5.$

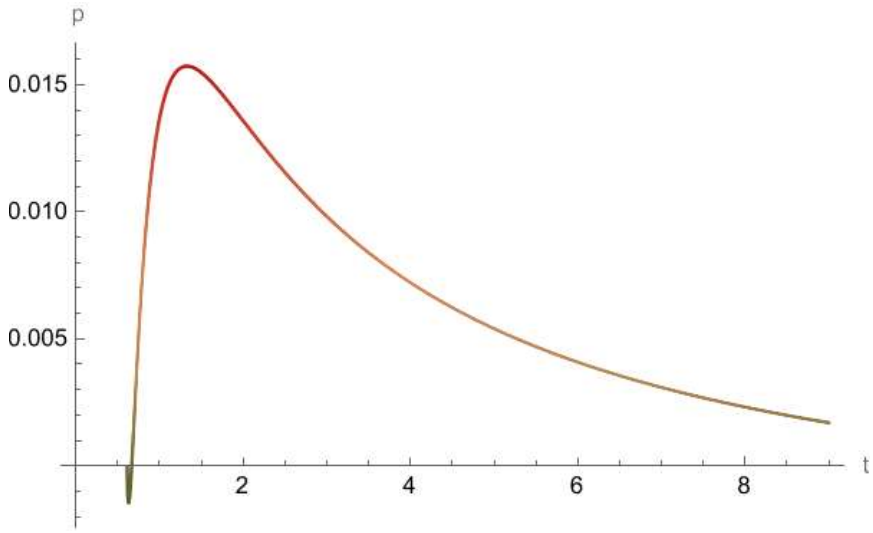

Figure-2: Pressure verses cosmic time plotted by considering
$\lambda = -1, c_1 = -0.5, c_2 = 0.5, \gamma = 0.5, \kappa = 0.5.$

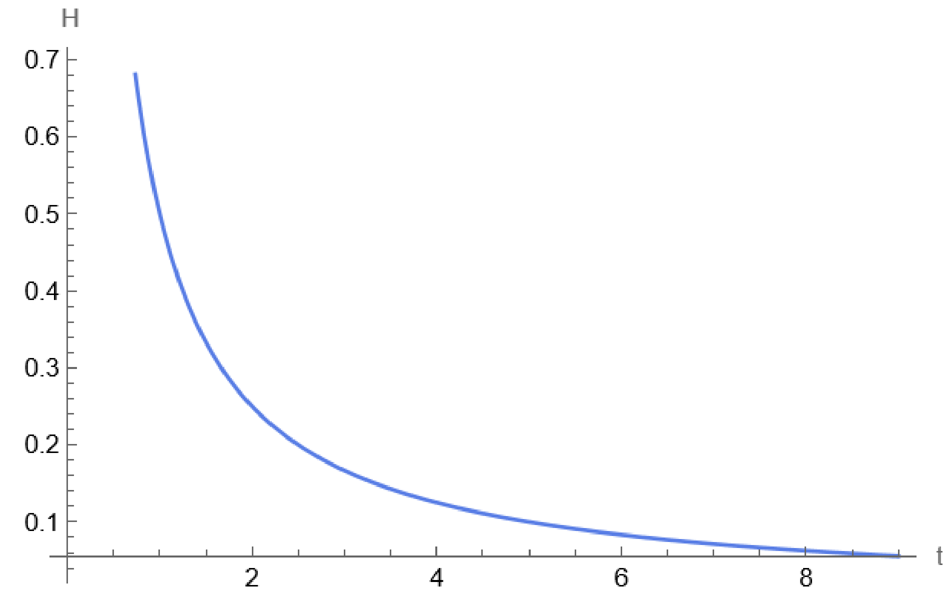

Figure-3: Mean Hubble parameter verses cosmic time plotted by considering $\lambda = -1, c_1 = -0.5, c_2 = 0.5, \gamma = 0.5, \kappa = 0.5$.

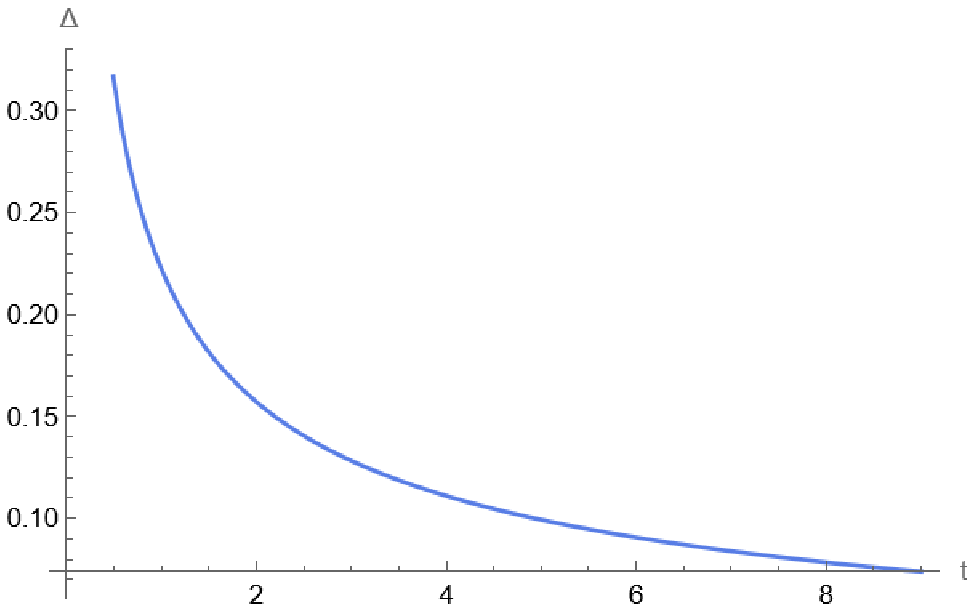

Figure-4: Mean Anisotropic parameter verses cosmic time plotted by considering $\lambda = -1, c_1 = -0.5, c_2 = 0.5, \gamma = 0.5, \kappa = 0.5$.

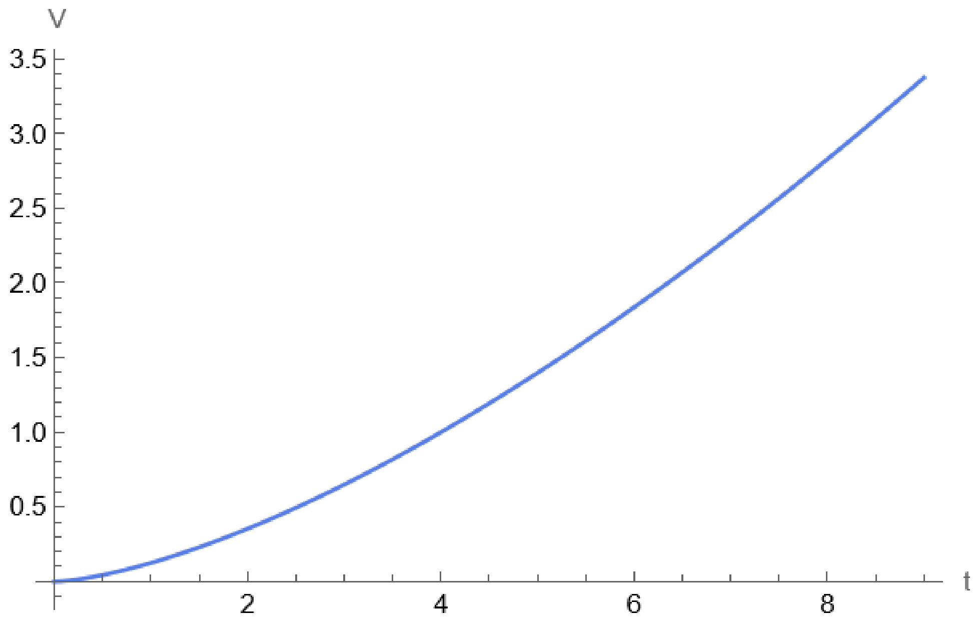

Figure-5: Spatial Volume verses cosmic time plotted by considering

$\lambda = -1, c_1 = -0.5, c_2 = 0.5, \gamma = 0.5, \kappa = 0.5.$

6. Discussion and Conclusion

In this work we have focussed on the dynamics of Bianchi type -VI_h cosmological model with a time varying cosmological constant Λ in the context of $f(R,T)$ theory of gravity. We have considered the functional form $f(R,T) = f_1(R) + f_2(T)$, which allows separating the contributions from the Ricci scalar and energy-momentum tensor in gravitational action. Additionally, the model incorporates the polytropic equation of state and is constructed using time dependent power law approach.

The energy density and pressure both are the decreasing function of cosmic time t and tends to zero for late time as depicted in figure-1 & figure-2 respectively. Figure-3 and figure-4 demonstrates that the mean Hubble parameter and the mean anisotropic parameter also decrease with time, indicating that the Universe evolves towards isotropy after a finite cosmic time. This implies that the anisotropies present in the early universe gradually diminish, leading to a more uniform and symmetric distribution of matter and energy.

As seen from figure-5, the Spatial volume V of the universe is zero at origin and expands continuously throughout its evolution. Moreover, the cosmological constant Λ is observed to be decreasing function of cosmic time, tending to zero at late times. The physical parameters, such as expansion scalar (θ) and the shear scalar (σ^2), also approach zero as time increases. Furthermore, the deceleration parameter remains constantly negative for all values of $\gamma > 1$, indicating an accelerating phase of Universe's expansion.

The results are consistent with the present observational data [1, 2], confirming that the constructed model effectively aligns with the current cosmological observations.

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