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STEADY STATE DISPERSION OF NON-BUOYANT AIR POLLUTANTS WITH VARIABLE WIND VELOCITY AND VARIABLE REMOVAL RATE

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Abstract: The purpose of this paper is not only to construct a mathematical model for the steady-state dispersion of non-buoyant air pollutants emitted from a continuous point source but also to find the analytical solution to it and then to explain graphically the effect of various parameters on the concentration of air pollutants. The wind velocity and removal rate are both assumed to be variable and are assumed to follow the power law profile. The concentration profile of non-buoyant air pollutants is analyzed for different parametric values in relation to the downwind, crosswind, and vertical distances. The results are analyzed graphically and conclusions are drawn from the various graphs.

Keywords: Non-buoyant air pollutants, Variable wind velocity, Variable removal rate, Power law profile.

1. Introduction

With the rapid expansion in the global population, transportation, urbanization, and rapid industrialization, air pollution has become a serious problem for living species. With the development of the economy and people's awareness of environmental protection, people are paying increasing attention to indoor and outdoor air quality. The air pollutants are harmful airborne particles and substances that cause air pollution [20]. The air pollutants present in high enough concentrations threaten human health, harm vegetation, and affect visibility, including many more adverse effects on the atmosphere. The standard definition of air pollution is the state of dis-equilibrium in which the air becomes harmful to biological populations due to the entrance of foreign materials from both natural and man-made sources. Hence, an imbalance in the quality of air that has a negative impact on the earth's living beings might be defined as air pollution.

In addition to contributing to climate change, air pollution is one of the main problems that impacts public and individual health today. It increases rates of sickness and mortality. Many different types of air pollution have negative effects on human health. One type of air pollution is particulate matter (PM), which is inhaled by humans and can cause respiratory diseases due to its small size. Other diseases like cancer and issues with the reproductive, cardiovascular, and central nervous systems can also be brought on by PM exposure. There is a high correlation between short-term exposure to air pollution and respiratory problems, wheezing, and asthma. [9]. The World Health Organization (WHO) monitors lead, sulfur oxides, nitrogen oxides, carbon monoxide, particle pollution, and ground-level ozone as its six main air pollutants. Air pollution can have catastrophic impacts on soil, groundwater, and other natural elements. Additionally, it seriously endangers living creatures. Air pollution has detrimental ecological repercussions that result in acid rain, global warming, the greenhouse effect, and climate change [19].

Various air dispersion models have been used in a great deal of research on the complex relationship between emission sources and air quality. The analytical solutions are advantageous in several ways because they provide a closed-form mathematical expression for each impacting parameter. Investigating the effects of each parameter on the models separately is easy. Analytical solutions are useful for assessing the accuracy and performance of the numerical models as well [6,7]. By analyzing the analytical solutions, significant insights into the behavior of a system can be explored. For investigators to forecast how a particular source emission would affect the air quality, dispersion modelling plays a very significant role.

The attention of researchers is growing as the air quality outside the world deteriorates. An increasing number of strategies for controlling air pollution have been developed that significantly enhance the quality of outside air in various urban settings [8].

Mathematical modelling is widely recognized as an important and necessary approach in the scientific community. Numerous modelling approaches have been effectively used in the past to address the dispersion of air pollution. Furthermore, continuous attempts are being made to improve forecast accuracy by the application of the latest advancements in computational technology in addition to improvements to the modelling and observational frameworks. [11]. An attempt has been made to summarize the important literature on dispersion modelling by Sharan et al. [11, 12]. The three-dimensional advection-diffusion equations have been analytically solved by Essa et al. [5]. They have assumed a Gaussian shape for the concentration distribution of air pollutants in the crosswind direction and have used the vertical eddy diffusivity and wind speed as dependent variables for the vertical height. Srivastava et al. [13], Nirmaladevi [10] have introduced a three-dimensional atmospheric diffusion model with variable wind velocity and removal rate, utilizing a power law profile. Using the variable separable approach, Bhandari and Verma [3] have provided an analytical model for the dispersion of air contaminants in a finite atmospheric boundary layer. A mathematical analysis of the dispersion of non-buoyant air pollutants released from a point source with variable wind velocity has been studied by Bhandari et al. [4].Verma et al. [15] have given an analytical approach to a problem concerning the dispersion of air pollutants by taking constant eddy diffusivities into account. Verma [18] has also offered an analytical solution to the issue of air pollution dispersion with constant wind velocity and constant removal rate. Agarwal et al. [1] have studied an unsteady-state three-dimensional atmospheric diffusion equation for air pollutants emitting from a point source by considering the constant removal rate and the wind velocity in the form of a wave function.

By assuming different parameters on wind velocity and eddy diffusivity, different outcomes for the advection-diffusion model have been observed by various investigators. [16, 17, 18].

Van Ulden [14] has investigated the non-Gaussian diffusion model and made comparisons with the experimental data on the Gaussian diffusion model [14]. The effects of air pollution on the ecosystem and human health have been reviewed by Manisalidis et al. [9].

Air pollutants have the property to settle on the atmosphere's surface; therefore, it is important to take into account that they are non-buoyant [2]. It is also required to use a negative sink velocity in the upward direction to account for the impact of buoyancy on the air pollutants in light of the previously mentioned research on how changing wind velocity affects the dispersion of non-buoyant contaminants from a single source that is situated at the ground's origin. Therefore, by keeping previous studies in view, the dispersion of non- buoyant air pollutants emitted from a point source in the atmosphere with Dirichlet-type boundary conditions has been studied. While developing the model, the variable wind velocity and the variable removal rate wind velocity have been taken into account that follow the power law profile.

2. Mathematical Model

Consider the steady state dispersion of non-buoyant air pollutants emitted from a point source of emission strength Q located at height h_s from the ground. The wind velocity and removal rate are both assumed to be variables that follow the power law profile and vary with the vertical height. Also, the turbulent diffusion in the wind direction is neglected in comparison to advection (Pasquill and Smith, 1981). The effect of buoyancy in the problem is incorporated by prescribing a negative sink velocity ($-w_s$) in the z-direction, where $w_s = |w_s|$.

The partial differential equation describing the steady state dispersion of non-buoyant air pollutants can be written as follows:

$$U(z)\frac{\partial C}{\partial x} - w_s\frac{\partial C}{\partial z} = K_y\frac{\partial^2 C}{\partial y^2} + K_z\frac{\partial^2 C}{\partial z^2} - \alpha(z)C \qquad (1)$$

where $C(x, y, z)$ is the concentration of the non-buoyant air pollutants and the downwind diffusion is negligible in comparison to advection. *U(z)* is the variable wind velocity which follows power law profile and is taken as $U(z) = \frac{U_H}{H^p}z^p$.$K_y$ and K_z are eddy diffusivities in y- and z-directions, respectively,w_s is the sink velocity and $\alpha(z)$ is the removal rate of air pollutants varying with the vertical height, which also follows the power law profile and is taken as $\alpha(z) = \frac{\alpha_H}{H^q} z^q$.Here, U_H and α_H are respectively the wind velocity and removal rate at a reference height H.

For the sake of convenience of the problem, we take $p = q = 1/2$

The initial and boundary conditions for equation (1) are taken as follows:

$$C(x, y, z) = \frac{Q\delta(y)\delta(z - h_s)}{U(z)}, x = 0 \quad (2)$$

$$C(x, y, z) = 0, \ \ y \to \pm\infty \quad (3)$$

$$C(x, y, z) = 0, \ \ z=0 \quad (4)$$

$$C(x, y, z) = 0, \ \ z = H \quad (5)$$

where δ is the Dirac-delta function.

Taking $U(z) = \frac{U_H z^p}{H^p}$ and $\alpha(z) = \frac{\alpha_H z^q}{H^q}$ with $p = q = 1/2$, equation (1) can be rewritten as:

$$\frac{U_H}{H^{1/2}} z^{1/2} \frac{\partial C}{\partial x} - w_s \frac{\partial C}{\partial z} = K_y \frac{\partial^2 C}{\partial y^2} + K_z \frac{\partial^2 C}{\partial z^2} - \frac{\alpha_H}{H^{1/2}} z^{1/2} C \quad (6)$$

3. Method of Solution

The partial differential equation (6) along with initial and boundary conditions, are made dimensionless by using the following dimensionless quantities:

$\bar{x} = \frac{K_z x}{U_H H^2}$, $\bar{y} = \frac{y}{H}$, $\bar{z} = \frac{z}{H}$, $\overline{h_s} = \frac{h_s}{H}$, $\bar{C} = \frac{U_H H^2 C}{Q}$, $\bar{\alpha} = \frac{\alpha H^2}{K_z}$, $\bar{w} = \frac{w_s H}{K_z}$, $\overline{Q'} = \frac{Q'}{Q}$, $\bar{\delta}(\bar{y}) = H\delta(y)$ and $\bar{\delta}(\bar{z} - \overline{h_s}) = H\delta(z - h_s)$.

On dropping the bars $(-)$, the dimensionless form of equation (6) is rewritten as follows:

$$z^{1/2} \frac{\partial C}{\partial x} - w \frac{\partial C}{\partial z} = \beta \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} - \alpha z^{1/2} C \text{ , where } \beta = \frac{K_y}{K_z} \quad (7)$$

Similarly, boundary conditions can be re-written as follows:

$$C(x, y, z) = \frac{Q'\delta(y)\delta(z - h_s)}{z^{1/2}}, x = 0 \quad (8)$$

$$C(x, y, z) = 0, \qquad y \to \pm\infty \quad (9)$$

$$C(x, y, z) = 0, \qquad z = 0 \quad (10)$$

$$C(x, y, z) = 0, \qquad z = 1 \quad (11)$$

In order to solve the partial differential equation (7), the method of separation of variables is employed. Thus, we take

$$C(x, y, z) = L(x)M(y)N(z) \quad (12)$$

as trial solution, where $L(x)$,$M(y)$ and $N(z)$ are lonely functions of x, y and z respectively. Then, equation (7) is reduced to the following ordinary differential equations:

$$\frac{dL}{dx} = -\lambda^2 L \tag{13}$$

$$\frac{d^2M}{dy^2} = \frac{\eta^2}{\beta} M \tag{14}$$

$$\frac{d^2N}{dz^2} + w\frac{dN}{dz} - \left((\alpha - \lambda^2)z^{1/2} - \eta^2\right)N = 0 \tag{15}$$

where λ^2 and η^2 are two separation constants.

The solution of equation (13) is given by

$$L(x) = C_1 exp(-\lambda^2 x) \tag{16}$$

where C_1 is an arbitrary constant of integration.

The solution of equation (14) is given by

$$M(y) = C_2(e^{-(\frac{\eta}{\sqrt{\beta}})y} + e^{(\frac{\eta}{\sqrt{\beta}})y}) \tag{17}$$

where $C_2$is an arbitrary constant of integration.

To find a solution of equation (15), we simplify it by using $z = \frac{S^2}{4}$ i.e., $s = 2z^{\frac{1}{2}}$ so that $\frac{dN}{dz} = \frac{2}{s}\frac{dN}{ds}$ and $\frac{d^2N}{dz^2} = \frac{4}{s^2}\frac{d^2N}{ds^2} - \frac{4}{s^3}\frac{dN}{ds}$

Putting these values of $\frac{dN}{dz}$ and $\frac{d^2N}{dz^2}$ in equation (15), we have

$$\frac{4}{s^2}\frac{d^2N}{ds^2} - \frac{4}{s^3}\frac{dN}{ds} + 2\frac{w}{s}\frac{dN}{ds} + \left(A\frac{s}{2} + \eta^2\right)N = 0\ ,\ A = -(\alpha - \lambda^2) \tag{18}$$

For solving equation (18), we apply Frobenius series method, so let us take

$$N(s) = \sum_{n=0}^{\infty} c_n s^{k+n}\ ,\ c_0 \neq 0 \text{ i.e., } N(s) = cs^k + c_1 s^{k+1} + c_2 s^{k+2} + \cdots \tag{19}$$

Putting the values of $N, \frac{dN}{ds}, \frac{d^2N}{ds^2}$ in (18), we obtain the following indicial equation:

$k(k-2) = 0 \Rightarrow k = 0\ and\ 2$ which gives the indicial roots as $k = 0\ and\ 2$.

The recurrence relation is found as follows:

$$c_n = \frac{\frac{-A}{8}c_{n-5} - \frac{\eta^2}{4}c_{n-4} - \frac{w}{2}c_{n-2}(k+n-2)}{(k+n)(k+n-2)} \tag{20}$$

Values of constants can be obtained by using equation (20) as follows:

For $n = 1$, we get $c_1 = 0$

For $n = 2$, we get $c_2 = -\frac{wc_0}{2(k+2)}$

For $n = 3$, we get $c_3 = 0$

For $n = 4$, we get $c_4 = -\frac{\frac{\eta^2}{4}c_0}{(k+4)(k+2)} - \frac{\frac{w}{2}c_2}{(k+4)}$

For $n = 5$, we get $c_5 = \frac{\frac{-\lambda^2}{8}c_0}{(k+5)(k+3)}$

For $n = 6$, we get $c_6 = \frac{-\frac{\eta^2}{4}c_2}{(k+6)(k+4)} - \frac{\frac{w}{2}c_4}{(k+6)}$, etc.

Thus, for *k=0*, we have $c_2 = \frac{-wC_0}{2.2}$,$c_4 = \frac{-\frac{\eta^2}{4}c_0}{2.4} - \frac{\frac{w}{2}c_2}{4}$, $c_5 = \frac{\frac{-\lambda^2}{8}c_0}{5.3}$, $c_6 = \frac{-\frac{\eta^2}{4}c_2}{6.4} - \frac{\frac{w}{2}c_4}{6}$, etc.

and for *k=2*, we have $c_2 = \frac{-wc_0}{2.4}$, $c_4 = \frac{-\frac{\eta^2}{4}c_0}{4.6} - \frac{\frac{w}{2}c_2}{6}$, $c_5 = \frac{\frac{-\lambda^2}{8}c_0}{5.7}$, $c_6 = \frac{-\frac{\eta^2}{4}c_2}{6.8} - \frac{\frac{w}{2}c_4}{8}$, etc.

Putting these values of constants in (19), we find the solution of (15) in the following form:

$$N(z) = k_2\Big[4z - 2wz^2 - 0.665z^3(-w^2+\eta^2) - 0.45696z^{\frac{7}{2}}\lambda^2 - 0.16665z^4w^3 + 0.333\eta^2wz^4 + 0.228z^{9/2}\lambda^2w + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\Big] \quad (21)$$

The above is obtained since k_1=0 after using the boundary condition (10).

Again, by using the boundary condition (10), we get the eigen-value equation in the following form:

$$4 - 2w - 0.665(-w^2 + {\eta_m}^2) - 0.45696{\lambda_m}^2 + 0.4569\alpha - 0.1666w^3 + 0.3333w{\eta^2}_m + 0.228w{\lambda^2}_m - 0.228w\alpha + \cdots = 0 \quad (22)$$

Since the above equation is uniformly convergent in [0, 1] for ${\lambda^2}_m \le {\eta^2}_m$, therefore, we take the value of separation constants so as ${\lambda^2}_m/{\eta^2}_m \le 1$.

Now, applying the boundary condition (9) in equation (17), the solution *M(y)* is obtained as

$$M(y) = C_2\left[H(y)e^{-(\frac{\eta}{\sqrt{\beta}})y} + H(-y)e^{(\frac{\eta}{\sqrt{\beta}})y}\right] \quad (23)$$

where *H(y)* is the unit step function which is given by

$$H(y) = \begin{cases} 0, & y < 0 \\ 1, & y > 0 \end{cases} \quad (24)$$

Substituting the values of *L(x)*, *M(y)* and *N(z)* respectively from equations (16), (23) and (21) in equation (12), the expression for concentration *C (x, y, z)* is given by

$$C(x,y,z) = \sum_{m=1}^{\infty}[k_m \exp(-{\lambda^2}_m x)\,\{H(y)e^{-(\frac{\eta m}{\sqrt{\beta}})y} + H(-y)e^{(\frac{\eta m}{\sqrt{\beta}})y}\}f_m(z)] \quad (25)$$

where $k_m = c_1c_2k_2$ and $f_m(z)$ is given by equation (21).

Now, applying the boundary condition (8) in equation (25), the following is obtained:

$$Q\delta(y)\delta(z-h_s) = \sum_{m=1}^{\infty} k_m\{H(y)e^{-(\frac{\eta_m}{\sqrt{\beta}})y} + H(-y)e^{(\frac{\eta_m}{\sqrt{\beta}})y}\}f_m(z)z^{1/2} \tag{26}$$

Multiplying (26) by $f_n(z)$ and then integrating w. r. t. z from 0 to 1 and w. r. t. y from $-\infty$ to ∞, and using the properties of Dirac-delta function: $\int_0^1 \delta(z-h_s)f_m(z)dz = f_m(h_s)$, $\int_{-\infty}^{\infty}\delta(y)dy = 1$ as well as using the orthogonality condition of eigen-functions i.e. $\int_0^1 z^{\frac{1}{2}} f_m(z)f_n(z)dz = 0, if\, m \neq n$, the constant k_m is obtained as follows:

$$k_m = \left(\frac{\eta_m}{\sqrt{\beta}}\right)\frac{Q' f_m(h_s)}{2\int_0^1 f_m{}^2(z)z^{1/2}dz} \tag{27}$$

Substituting the value of k_m in equation (25), we finally get

$$C(x,y,z) = \sum_{m=1}^{\infty}[Q' exp(-\lambda^2{}_m x)\left\{H(y)e^{-\frac{\eta_m}{\sqrt{\beta}}y} + H(-y)e^{\frac{\eta_m}{\sqrt{\beta}}y}\right\}\left(\frac{\eta_m}{\sqrt{\beta}}\right)\frac{f_m(h_s)f_m(z)}{2\int_0^1 z^{1/2} f_m{}^2(z)dz} \tag{28}$$

4. Results and Discussion

To see the effects of variable removal rate and variable wind velocity, the concentration $C(x,y,z)$ of non-buoyant air pollutants for suitable boundary conditions has been computed and displayed graphically in figures (1)-(4). The dimensionless concentration is calculated by using equation (28). The dimensionless parametric values used in the analysis are taken as follows:

Parameters	α	β	h_s	λ	Q'	H	w
Values	2.0	10.0	0.2	10.0	1.0	1.0	1.0

In figure (1), we have plotted the concentration of non- buoyant air pollutants against the downwind distance $(0 \leq x \leq 1)$ for different values of vertical distance $(z = 0.3, 0.4, 0.5)$, keeping the crosswind distance fixed at ± 1. From the graph shown in figure (1), it is seen that the concentration profile increases up to a certain level of downwind distance, and then it decreases regularly with increasing downwind distance. It is also seen that the concentration level of pollutants is higher for a higher value of vertical distance.

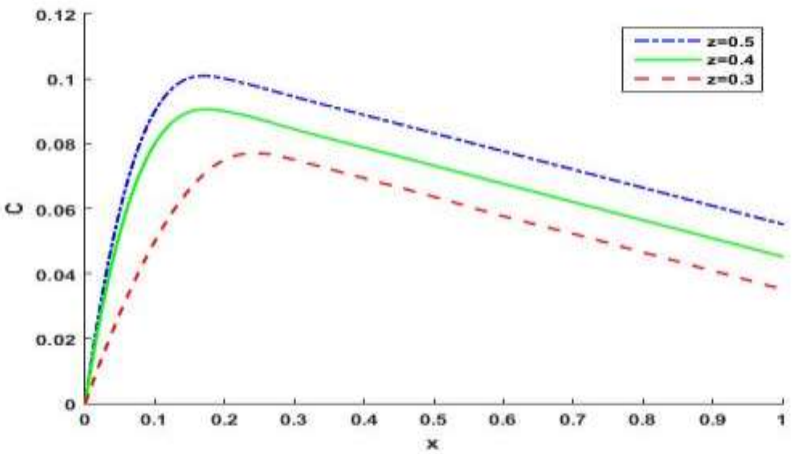

Fig 1: Variation of C $(x, \pm 1, z)$ with x for different values of z.

In figure (2), the concentration of non-buoyant air pollutants is plotted against the downwind distance *(0 ≤ x ≤ 1)* for different values of crosswind distance *(y= 0.6, 0.4, 0.2)*, keeping the value of vertical distance fixed at $z = 0.2$. From the graph shown in figure (2), it is seen that the concentration profile increases up to a certain level of downwind distance, and then it decreases with increasing downwind distance. From this figure, it is also seen that the concentration level is higher for lower value of crosswind distance.

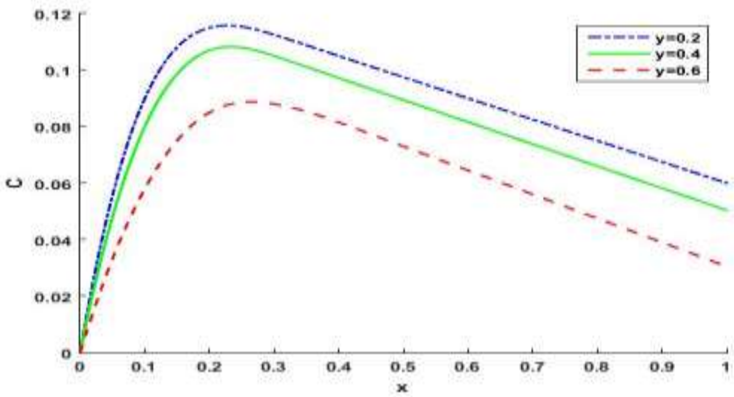

Fig 2: Variation of C(x, y, 0.2) with x for different values of y.

In figure (3), we have plotted the concentration of non-buoyant air pollutants against cross wind distance *(*$0 \le y \le 1$*)* for different values of downwind distance *(x=0.1, 0.3, 0.5)*, keeping the value of vertical distance fixed at *z = 0.2*. It is seen that the concentration profile increases up to a certain level, and attaining its peak value, it starts decreasing with an increase in the crosswind distance. From this figure, it is also seen that with an increase in downward distance, the concentration of air pollutants decreases.

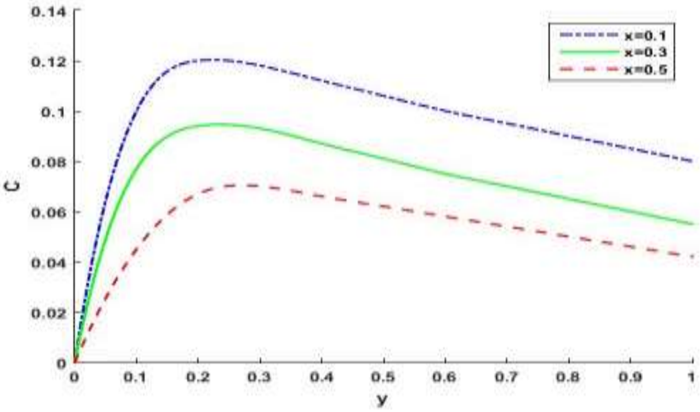

Fig 3: Variation of C(x, y, 0.2) with y for different values of x.

In figure (4), the concentration of non-buoyant air pollutants is plotted against the vertical distance for different downwind distances *(x= 0.1, 0.3, 0.5).* It is also seen that the concentration profile increases up to a certain level of vertical distance, and attaining its peak at $z = 0.5$, it starts decreasing with an increase in the vertical distance.

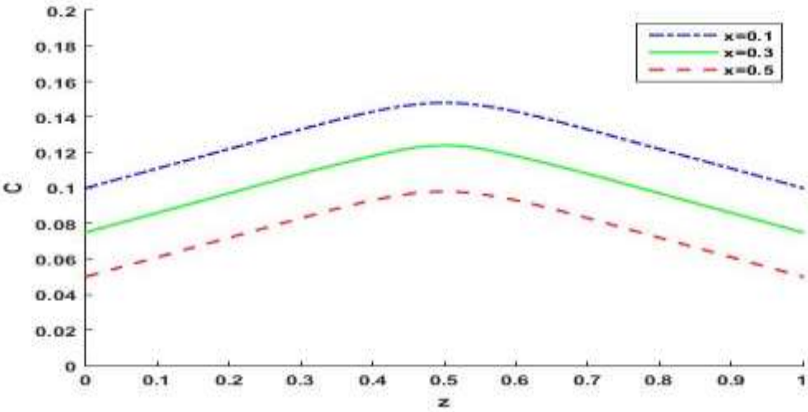

Fig 4: Variation of C(x, *0*, z) with z for different values of x.

5. Conclusions

This paper presents the construction of a mathematical model and its analytical solution for the steady state dispersion of non-buoyant air pollutants emitted from a continuous point source. We have demonstrated the dispersion of non-buoyant air pollutants by a continuous point source, where the wind velocity and removal rate are both assumed to follow a power law profile. The concentration profile of non-buoyant air pollutants is analyzed for different parametric values in relation to the downwind, crosswind, and vertical distances. Figures (1) and (2) respectively illustrate the uniform behavior of the concentration profile against the downward distance for $0 \leq x \leq 1$ for various vertical distances and crosswind distances while keeping constant the crosswind distance and the vertical distance, respectively. It is found that the concentration level becomes smaller as the downwind distance grows. Figure (3) shows a similar type of pattern for $0 \leq y \leq 1$ with a constant value of $z = 0.2$ for different values of $x = 0.1$, 0.2, and 0.3. From Figure 4, it is found that the concentration profile attains its peak value at $z = 0.5$ and decreases as the vertical distance increases. The concentration disperses symmetrically on both sides at $z = 0.5$.

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