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BIANCHI TYPE-V DARK ENERGY COSMOLOGICAL MODEL IN MODIFIED SCALE COVARIANT THEORY OF GRAVITATION

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Abstract In this paper, we study a spatially homogeneous and anisotropic Bianchi type-V dark energy cosmological model in modified scale covariant theory of gravitation. Exact solution of generated Einstein's field equations are obtained by assuming (i) a special form of average scale factor of the model corresponding to the constant negative value of deceleration parameter and (ii) the interaction term between dark energy and other different part of matter in the universe. The physical and dynamical behaviours of the model are discussed. The model describes the accelerated phase of expanding universe.

Keywords: Bianchi type-V cosmological model, dark energy, scale covariant theory.

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1. Introduction

A number of recent observational data such as high red-shift Ia supernovae [12, 9], cosmic microwave background anisotropy [8, 14], large scale structure[4] etc. all have given convincing indications that our physical universe is undergoing a phase of accelerated expansion.

This late-time cosmic accelerated expansion of the universe is believed to be due to the presence of a mysterious cosmic fluid with negative pressure, called dark energy (DE), which is a kind of repulsive force acting as antigravity responsible for gearing up the universe. Though the exact nature of DE is still not known, much attentions now a days are focused to solve problems in respect of DE in general relativity and cosmology. For describing the accelerated expansion of the universe, we need DE in the universe valid on all scales of the universe. The thermodynamical studies of DE reveals that the constituents of DE may be a kind of massless particles (bosons and fermions) whose collective behavior resembles with a kind of radiation fluid having negative pressure.

The DE has been conventionally characterized by the equation of state (EoS) parameter $\omega = \frac{p}{\rho}$, where ρ is energy density, p is the pressure, which is not necessarily time independent. Cosmologists have proposed so far various candidates of DE to fit the current observations. The simplest candidate for DE is the cosmological constant for which $\omega = -1$. But the cosmological constant problem is long standing problem in physics as it is ploughed with the fine tunning problem. A number of other dynamically evolving scalar field proposals of DE have been studied such as quintessence $(\omega > -1)$, k-essence $\left(-1 < \omega < -\frac{1}{3}\right)$, tachyon $(-1 < \omega < 0)$. According to Caldwell[4], the matter with $\omega < -1$ gives rise to Big-Rip type of future singularity. Bamba et al.[1] have presented a review of different DE isotropic cosmologies with early deceleration and late-time acceleration of universe.

The limitations of general relativity in providing satisfactory explanation of various phases of evolution of the universe have led several cosmologists to adopt various hypotheses to formulate alternative or modified theories of gravitation. Such theories are expected to bring out a number of aspects of mathematics and physical interests associated with them. Canuto et al.[5,6] formulated a scale covariant theory of gravitation by associating the mathematical operation of scale transformation with the physics of using different dynamical systems to measure space-time distances. They obtained the generalized Einstein's field equations invariant under the scale-transformations and studied several astrophysical tests. This theory is a viable alternative to general relativity which allows a natural interpretation of the possible variation of the gravitational constant. Beesham [2] discussed power asymptotic singularities in this theory with special attention to the Friedmann model and Kasner model and generalized the general relativistic results.

Reddy et al.[11] presented an LRS Bianchi type-I cosmological model with negative constant deceleration parameter. Ram et al.[10] discussed spatially homogeneous Bianchi type-V universe filled with a perfect fluid having a big-bang singularity at the initial time $t = 0$. Recently, Singh et al.[13] studied the behavior and contribution of DE to the accelerated expansion of the universe in the modified scale covariant theory of gravitation by introducing on interaction term in the field of equations by considering FRW spherically symmetric space-time. In this paper, we investigate spatially homogeneous and anisotropic Bianchi type-V cosmological models dominated by DE in the modified scale covariant theory of gravitation. The plan of the paper is as follows: In Section-2, we consider the metric and present the general expressions of field equations. In section-3, we present an exact solution of the field equations using a special form of the average scale function obtained on assuming that the deceleration parameter in the universe is negative constant for an accelerated expanding model. We also find energy density and pressure of DE and other matter present in the universe by introducing an interaction term in the field equations. In section-4, we present relevant physical discussions of the DE cosmological model using physical and kinematical parameters. Section-5 contains some concluding remarks.

2. Field Equations and General Expressions

To formulate the modified scale covariant theory, Canuto et al.[5,6] proposed a modification of Einstein's theory of gravitation in which the gravitational constant is a variable. They obtained the generalized field equations invariant under scale transformations and studied several astrophysical tests. In this theory, Einstein's field equations are valid in gravitational units whereas physical quantities are measured in atomic units. The metric tensors in two systems of units are related by the conformal transformation

$$\bar{g}_{ij} = \phi^2 g_{ij}, i,j = 1,2,3,4 \tag{1}$$

where the gauge function ϕ is the function of coordinates, bar denotes gravitational units and unbarred denotes atomic units. Using (1), Einstein's field equations are transformed into

$$R_{ij} - \frac{1}{2}Rg_{ij} + f_{ij}(\phi) = -8\pi G T_{ij} + \Lambda(\phi) g_{ij} \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i,j} \text{-} 4\phi_{,i}\phi_{,j} - g_{ij}\left(\phi\phi_{,l}^{,l} - \phi^{,l}\phi_{,l}\right) \tag{3}$$

The energy momentum tensor T_{ij} of the fluid contained in the universe is taken of the form

$$T_{ij} = \left(\rho_\alpha + \rho_m + p_d + p_m\right)u_i u_j - \left(p_d + p_m\right)g_{ij} \tag{4}$$

where ρ_d and p_d are respectively the energy density and fluid pressure for the dark energy, and ρ_m and p_m are respectively the energy density and pressure of other matters including dark matter. Here $u^i = (1,0,0,0)$ is the four velocity flow vector satisfying $u^i u_i = 1$.

We consider the spatially homogeneous and anisotropic Bianchi type-V metric of the form

$$ds^2 = dt^2 - A^2dx^2 - e^{2mx}\left(B^2dy^2 + C^2dz^2\right) \tag{5}$$

where A, B, C are cosmic scale functions of time t and m is a constant.

In comoving coordinates, the field equations (2) for the metric (5) together (3) and (4) can be written explicitly as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}-\frac{2\dot{A}\dot{\phi}}{A\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{\ddot{\phi}}{\phi}-\frac{\dot{\phi}^2}{\phi^2}=-8\pi G\left(p_d+p_m\right), \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{m^2}{A^2}-\frac{2\dot{B}\dot{\phi}}{B\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{\ddot{\phi}}{\phi}-\frac{\dot{\phi}^2}{\phi^2}=-8\pi G\left(p_d+p_m\right), \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}-\frac{2\dot{C}\dot{\phi}}{C\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{\ddot{\phi}}{\phi}-\frac{\dot{\phi}^2}{\phi^2}=-8\pi G\left(p_d+p_m\right), \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{3m^2}{A^2}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\frac{\ddot{\phi}}{\phi}+\frac{3\dot{\phi}^2}{\phi^2}=8\pi G\left(\rho_d+\rho_m\right), \tag{9}$$

$$\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0. \tag{10}$$

where an overdot denotes ordinary derivative with respect to cosmic time t.

The conservation equation takes the form

$$\left(\dot{\rho}_d+\dot{\rho}_m\right)+\left(\rho_d+\rho_m+p_d+p_m\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\left(\rho_d+\rho_m\right)\left(\frac{\dot{G}}{G}+\frac{\dot{\phi}}{\phi}\right)$$
$$+3\left(p_d+p_m\right)\frac{\dot{\phi}}{\phi}=0 \tag{11}$$

This equation can be separated into two equations

$$\dot{\rho}_d+\left(\rho_d+p_d\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\rho_d\left(\frac{\dot{G}}{G}+\frac{\dot{\phi}}{\phi}\right)+3p_d\frac{\dot{\phi}}{\phi}=Q, \tag{12}$$

and

$$\dot{\rho}_m+\left(\rho_m+p_m\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\rho_m\left(\frac{\dot{G}}{G}+\frac{\dot{\phi}}{\phi}\right)+3p_m\frac{\dot{\phi}}{\phi}=-Q, \tag{13}$$

where Q is the interaction term which describes the energy flow rate between dark matter and dark energy/vacuum energy. The case $Q<0$ represents the decay of energy from dark matter to dark energy. When $Q=0$, there is no interaction between dark sectors. The case $Q>0$ corresponds to decay of energy from dark energy to dark matter[13].

For the metric (1), we now define certain physical quantities of dynamical interest in cosmology. The spatial volume V and the average scale factor a are defined by

$$V = a^3 = ABC . \tag{14}$$

The generalized mean Hubble parameter H is defined by

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{15}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble parameters in the direction of x, y and z -axes respectively. The expansion scalar θ and shear scalar σ are given by

$$\theta = 3H, \tag{16}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 \text{-} 3H^2\right). \tag{17}$$

The anisotropy parameter A_m of the expansion is characterized by the mean and directional Hubble's parameter defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 . \tag{18}$$

The anisotropy parameter of expansion results in isotropic expansion for $A_m = 0$.

An important observational quantity in cosmology is the deceleration parameter q defined as

$$q = \frac{-a\ddot{a}}{\dot{a}^2} . \tag{19}$$

The sign of q indicates whether the universe inflates or not. The positive sign of q corresponds to standard decelerating model whereas the negative sign indicates inflation.

3. Solutions of Field Equations and Model

Berman [3] proposed a special law of variation of Hubble's parameter which yields a constant value of the deceleration parameter in the universe. It may be noted that most of the well known models in general relativity and alternative theories of gravitation including inflationary models are with constant deceleration parameter. For an accelerating model of the universe, we take this constant as negative. Then (19) gives the solution for the average scale factor as

$$a = \left(c_1 t + c_2\right)^{\frac{1}{1+q}} \tag{20}$$

where c_1 and c_2 are integration constants. This equation implies that the condition of accelerated expansion is $1+q>0$. We shall use (20) to obtain the exact solutions of the field equations (6)-(10).
Integration of (10) gives

$$A^2 = BC \tag{21}$$

On absorbing the integration constant in C, from (14) and (20), we obtain

$$A = \left(c_1 t + c_2\right)^{\frac{1}{1+q}} \tag{22}$$

Now, subtracting (7) from (6), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \left(\frac{\dot{C}}{C} + \frac{2\dot{\phi}}{\phi}\right)\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = 0 \tag{23}$$

This equation can be written in the form

$$\frac{d}{dt}\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{2\dot{\phi}}{\phi}\right)\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right) = 0 \tag{24}$$

Integration of (24) twice provides

$$B = \mathrm{M}\ \mathrm{a}\ \exp\left[X\int \frac{dt}{a^3\phi^2}\right] \tag{25}$$

where M and X are integration constants. Similarly subtracting (8) from (6) and integrating the result twice, we obtain

$$C = M^{-1}\mathrm{a}\ \exp\left(-X\int \frac{dt}{a^3\phi^2}\right). \tag{26}$$

It is clear that the explicit solution of A, B and C can be obtained if the gauge function ϕ is known. Regarding the gauge function ϕ, we assume that (Johri and Desikan[7])

$$\phi = \beta a^{\alpha} \tag{27}$$

where α and β are constants. Inserting values of *a* and ϕ in (25) and (26), we obtain the solutions for scale factors A, B and C are given by

$$A=\left(c_1t+c_2\right)^{\frac{1}{1+q}}, \tag{28}$$

$$B=M\left(c_1t+c_2\right)^{\frac{1}{1+q}}\exp\left[\frac{(1+q)X}{(q-2\alpha-2)\beta^2}\left(c_1t+c_2\right)^{\frac{-(2\alpha-q+2)}{1+q}}\right], \tag{29}$$

$$C=M^{-1}\left(c_1t+c_2\right)^{\frac{1}{1+q}}\exp\left[\frac{-(1+q)X}{(q-2\alpha-2)\beta^2}\left(c_1t+c_2\right)^{\frac{-(2\alpha-q+2)}{1+q}}\right]. \tag{30}$$

Applying the scale transformation (i,e on putting $c_1=1$, $c_2=0$ and $M=1$), the solutions of the metric functions can be written as

$$A=t^{\left(\frac{1}{1+q}\right)} \tag{31}$$

$$B=t^{\frac{1}{1+q}}\exp\left[\frac{(1+q)X}{(q-2\alpha-2)\beta^2}t^{\frac{-(2\alpha-q+2)}{1+q}}\right], \tag{32}$$

$$C=t^{\frac{1}{1+q}}\exp\left[\frac{-(1+q)X}{(q-2\alpha-2)\beta^2}t^{\frac{-(2\alpha-q+2)}{1+q}}\right]. \tag{33}$$

Hence, the metric of our solutions can be written as

$$ds^2=dt^2-t^{\frac{2}{1+q}}dx^2-e^{2mx}[t^{\frac{2}{1+q}}\exp\left\{\frac{2(1+q)X}{(q-2\alpha-2)\beta^2}t^{\frac{-(2\alpha-q+2)}{1+q}}\right\}dy^2$$
$$+t^{\frac{2}{1+q}}\exp\left\{\frac{-2(1+q)X}{(q-2\alpha-2)\beta^2}t^{\frac{-(2\alpha-q+2)}{1+q}}\right\}dz^2]. \tag{34}$$

The spatial volume V of the model (34) is

$$V=t^{\frac{3}{1+q}}, \tag{35}$$

The directional Hubble's parameters and the mean Hubble parameter are obtained as

$$H_1=\frac{1}{(1+q)t},H_2=\frac{1}{(1+q)t}+\frac{X}{\beta^2t^{\left(\frac{2\alpha+3}{1+q}\right)}},H_3=\frac{1}{(1+q)t}-\frac{X}{\beta^2t^{\left(\frac{2\alpha+3}{1+q}\right)}} \tag{36}$$

and

$$H = \frac{1}{(1+q)t} \tag{37}$$

The expansion scalar θ and shear scalar σ^2 are obtained as

$$\theta = \frac{3}{(1+q)t} \tag{38}$$

$$\sigma^2 = \frac{2X^2}{\beta^4 t^{\left(\frac{4\alpha+6}{1+q}\right)}}. \tag{39}$$

The anisotropic parameter of expansion has the value

$$A_m = \frac{2X^2(1+q)^2}{3\beta^4 t^{\left(\frac{4\alpha+4-2q}{1+q}\right)}} \tag{40}$$

The gauge function ϕ has the solution

$$\phi = \beta t^{\left(\frac{\alpha}{1+q}\right)}. \tag{41}$$

We now obtain the pressure and energy density of dark energy and dark matter in our model for two different forms of the interaction term Q. Generally the interaction is assumed as directly proportional to the density of dark sector i,e $Q \propto \rho$, where ρ is the density of dark energy or dark matter or the combination of both dark energy and dark matter.

Case-I When $Q = lH\rho_d$ and $p_d = \omega\rho_d$ where l is proportionality constant and ω is the EoS parameter. Putting the value of H and ϕ in (12) and integrating the result, we obtain

$$\rho_d = \frac{k_1}{G\beta^{(3\alpha+1)}} t^{\left(\frac{-(3\alpha\omega+3\omega+\alpha+2)}{1+q}\right)} \tag{42}$$

where k_1 is an arbitrary constant. Using the equation of state, the pressure p_d is given by

$$p_d = \frac{\omega k_1}{G\beta^{(3\alpha+1)}} t^{\left(\frac{-(3\alpha\omega+3\omega+\alpha+2)}{1+q}\right)} \tag{43}$$

Substituting the value of p_d in (6) and ρ_d in (9), we find that

$$p_m = \frac{1}{8\pi G}\left[\frac{m^2}{t^{\left(\frac{2}{1+q}\right)}} - \frac{X^2}{\beta^4 t^{\left(\frac{4\alpha+6}{1+q}\right)}} + \frac{\alpha q + 2q - 1}{(1+q)^2 t^2}\right) - \frac{\omega k_1}{G\beta t^{\left(\frac{3\alpha\omega+3\omega+\alpha+2}{1+q}\right)}} \tag{44}$$

$$\rho_m = \frac{1}{8\pi G}\left[\frac{2\alpha^2 + 3\alpha + \alpha q + 2q + 2}{(1+q)^2 t^2} - \frac{3m^2}{t^{\frac{2}{1+q}}} - \frac{X^2}{\beta^4 t^{\frac{4\alpha+6}{1+q}}}\right) - \frac{k_1}{G\beta^{3\omega+1} t^{\left(\frac{3\alpha\omega+3\omega+\alpha+2}{1+q}\right)}}. \tag{45}$$

At $t = 0$, ρ_m and p_m are infinite, which tend to zero as $t \to \infty$.

Case-II When $Q = lH\rho_m$ and $p_m = \omega\rho_m$

Using this value of Q in (13) and integrating the result, we obtain

$$\rho_m = \frac{k_2}{G\phi^{(3\alpha+1)} t^{\frac{3\alpha\omega+3\omega+\alpha+l+3}{1+q}}} \tag{46}$$

where k_2 is the constant of integration. Using the equation of state, we obtain

$$p_m = \frac{\omega k_2}{G\phi^{(3\alpha+1)} t^{\frac{3\alpha\omega+3\omega+\alpha+l+3}{1+q}}} \tag{47}$$

Putting the value of p_m in (6), we find that

$$p_d = \frac{1}{8\pi G}\left[\frac{m^2}{t^{\frac{2}{1+q}}} - \frac{X^2}{\beta^4 t^{\frac{4\alpha+6}{1+q}}} + \frac{\alpha q + 2q - 1}{(1+q)^2 t^2}\right) - \frac{\omega k_2}{G\beta^{3\omega+1} t^{\frac{3\alpha\omega+3\omega+\alpha+l+3}{1+q}}} \tag{48}$$

From (9) and (47), we obtain

$$\rho_d = \frac{1}{8\pi G}\left[\frac{2\alpha^2 + 3\alpha + \alpha q + 2q + 1}{(1+q)^2 t^2} - \frac{3m^2}{t^{\frac{2}{1+q}}} - \frac{X^2}{\beta^4 t^{\frac{4\alpha+6}{1+q}}}\right) - \frac{k_2}{G\beta^{3\omega+1} t^{\frac{3\alpha\omega+3\omega+\alpha+l+3}{1+q}}}, \tag{49}$$

We again see that ρ_d and p_d are infinite at $t = 0$ and tend to zero as $t \to \infty$.

$$Q = \frac{lk_2}{G(1+q)\phi^{(3\omega+1)} t^{\frac{(3\alpha\omega+3\omega+\alpha+l+q+4)}{1+q}}}. \tag{50}$$

4. Discussion

We now discuss the physical and kinematical features of the DE model (34). From (34) we see that spatial volume is zero at $t = 0$ and it goes on increasing at t increases and ultimately tends to infinity as $t \to \infty$. At the epoch $t = 0$, the kinematical parameter H, θ and σ^2 are all infinite. Therefore the model has a big-bang type singularity at the time $t = 0$. The expansion scalar θ is a decreasing function of time which tends to zero as $t \to \infty$. The shear scalar σ also decreases as t increases and will tends to zero at late time if $\alpha > -\frac{3}{2}$. The anisotropy parameter A_m associated with the fluid is infinite at $t = 0$ which decreases with the increase of time which ultimately tends to zero at late time provided $4\alpha + 4\text{-}2q > 0$ i,e $\alpha > -\frac{3}{2}$, since $1 + q > 0$. Thus the model is anisotropic for finite time and becomes isotropic for large time if $\alpha > -\frac{3}{2}$.

The physical quantities ρ_d, p_d, ρ_m and p_m are infinite at $t = 0$ which are decreasing function of time and ultimately vanishes for large time.

5. Conclusion

In this paper, we have presented a spatially homogeneous and anisotropic Bianchi type-V dark energy cosmological model in scale covariant theory of gravitation. We have obtained the exact solutions of field equations by (i) applying a special law of variation of Hubble parameter and (ii) introducing a interaction term Q in the field equations. The model exhibits a big-bang type singularity at the time $t = 0$. We have discussed the physical behaviors of the model near the initial singularity and at late time. The density and pressure of the fluid contained in this model of universe are seen to decrease with time which are natural for a realistic universe. The model represents an accelerated expanding universe which is in good agreement with the present scenario and observations of modern cosmology.

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