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LRS BIANCHI TYPE II STRING COSMOLOGICAL MODEL FOR BAROTROPIC BULK VISCOUS FLUID DISTRIBUTION WITH DARK ENERGY

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Abstract: The current work deals with locally rotationally symmetric (LRS) Bianchi type II string cosmological models for bulk viscous fluid and barotropic equation of state with dark energy(Λ). To achieve a deterministic solution, we have assumed that the coefficient of shear viscosity$(\sigma)$is proportional to expansion(θ) and $\xi\theta = K$(constant), as considered by Zimdahl [26], where ξ is coefficient of bulk viscosity. Also (Λ) is taken proportional to R^{-3}. Some geometrical and physical characteristics are also discussed for these models.

Keywords: LRS Bianchi Type-II, Bulk Viscous, Barotropic Fluid, Dark Energy, General relativity.

1. Introduction

Kibble [8] and Zel'dovich [25] have demonstrated that cosmic strings are important source of density perturbations, necessary for the creation of large scale structures in the universe. Letelier [9] and Stachel [21] were the first to approach strings in a general relativistic manner, which led to numerous studies on homogeneous string cosmological models exhibiting various Bianchi symmetries. String cosmological models have been studied by numerous researchers in various contexts; these include Bali [2], Bali and Dave [3], Priyokumar et al. [15], Bali and Pradhan [4]. Jain et al. [7] have discussed a magnetized string cosmological model for perfect fluid distribution for LRS Bianchi type I.

When creating cosmological models, Bianchi type II space-time is important, especially when portraying the early phases of the universe's evolution. Asseo and Sol [1], Roy and Banerjee [16], Yadav et al. [24] are among the scholars who have examined Bianchi type II space time in their researches. In barotropic fluid, the pressure is the function of density only. It is expressed as $p = \gamma\rho; 0 \leq \gamma \leq 1$, where γ is a dimensionless parameter that defines the properties of the cosmic fluid. The compatibility of commonly used equation of state parameterizations in dark energy models with barotropic fluid frameworks has been studied by Perković and Štefančić [13]. Inflationary bulk viscous Bianchi type VIII space time for barotropic fluid distribution has been investigated by Singh et al. [20].

A step towards a more thorough and accurate description of the cosmos is the inclusion of bulk viscosity in cosmological models. After Misner [11] emphasized the importance of viscosity in cosmological models, many more researches were conducted. The Bianchi type II cosmological model was examined by Banerjee et al. [5] for both bulk and shear viscosity. A Bianchi type II cosmological models with and without bulk viscosity were investigated by Sharma [19]. Tyagi and Sharma [23] described Bianchi type II string cosmological models involving bulk viscous fluid.

The observed accelerated expansion of the cosmos is thought to be caused by dark energy, a hypothetical type of energy. This concept is supported by astronomical observations of type Ia-supernovae, large scale structures and the measurements of cosmic microwave background (CMB) anisotropy. The nature of dark energy has been explained by Caldwell [6], Peebles [12], and Saha [17]. The spatially homogenous and completely anisotropic Bianchi type I space-time was examined by Tiwari et al. [22] while taking into account a variable cosmic constant (Λ) and deceleration parameter (q) in the presence of both bulk and shear viscosity. Pradhan et al. [14] studied a new class of dark energy models in Bianchi type II space time, focusing on a variable equation of state and a constant deceleration parameter. In the context of Bianchi type II space-time. Samdurkar et al. [18] found solutions to the Einstein field equations with dark energy, which are represented as an inhomogeneous equation of state with bulk viscous fluid.

In this paper, we have examined the LRS Bianchi type-II string cosmological models for barotropic bulk viscous fluid distribution with dark energy, which is motivated by the research work previously provided. In order to obtain the deterministic model of the universe, we assumed that σ (shear) is proportional to θ (expansion) which leads to $A = B^n$, where n is a constant, A and B are metric potentials and $\xi\theta = K$, where ξ is coefficient of bulk viscosity. The paper includes a thorough discussion of the situations of the dust fluid model$(\gamma = 0)$, the stiff fluid model$(\gamma = 1)$ and the radiation-dominated fluid$\left(\gamma = \frac{1}{3}\right)$. A detailed assessment of the physical parameters has been conducted.

2. The Metric and Field Equations

The line element for LRS Bianchi type II space - time is considered as

$$ds^2 = -dt^2 + B^2(dx + zdy)^2 + A^2(dy^2 + dz^2) \quad (1)$$

Where A andB are function of"t" alone and$\sqrt{-g} = A^2B$.

The energy momentum tensor T_i^j for bulk viscous fluid containing one-dimensional cosmic string is given by

$$T_i^j = (\bar{p} + \rho)v_i v^j + \bar{p}g_i^j - \lambda x_i x^j \quad (2)$$

where λ is the string tension density, ρ is matter density, $\bar{p}$ is effective pressure, x^i is unit space like vector satisfying the direction of string and v^i is the four - velocity vector which satisfies the relation

$$g_{ij}v^i v^j = -x^i x_i = -1 \text{ and } v^i x_i = 0 \tag{3}$$

The effective pressure $\bar{p}$ is related to equilibrium pressure p by relation

$$\bar{p} = p - \xi\theta \tag{4}$$

where ξ is the co-efficient of bulk viscosity.

In a co-moving coordinate system, we have

$$v^i = (0,0,0,1) \text{and } x^i = \left(\frac{1}{B}, 0,0,0\right) \tag{5}$$

The Einstein field equation in the geometrized unit$(8\pi G = c = 1)$is given by

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -T_i^j \tag{6}$$

where R_i^j is Ricci tensor and$R = g^{ij}R_{ij}$ is Ricci scalar.

Einstein's field equation (6) for the metric (1) and energy momentum tensor (2) leads to the following system of equations

$$\frac{2A_{44}}{A} - \frac{3B^2}{4A^4} + \frac{{A_4}^2}{A^2} + \Lambda = -p + \xi\theta + \lambda \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} + \Lambda = -p + \xi\theta \tag{8}$$

$$\frac{2A_4 B_4}{AB} + \frac{{A_4}^2}{A^2} - \frac{B^2}{4A^4} + \Lambda = \rho \tag{9}$$

In above equations the suffix 4 denotes differentiation with respect to "t".

The average scale factor R for metric (1) is given by

$$R^3 = A^2 B \tag{10}$$

The scalar expansion θ and shearσ are given by

$$\theta = v^i_{;i} = \frac{2A_4}{A} + \frac{B_4}{B} \tag{11}$$

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{A_4}{A} - \frac{B_4}{B}\right] \tag{12}$$

The generalized mean Hubble's parameter H is defined as

$$H = \frac{1}{3}\left(H_x + H_y + H_z\right) \tag{13}$$

H_x, H_y, H_z are directional Hubble parameters in the direction of x, y and z respectively, given by

$$H_x = \frac{B_4}{B} \text{ and } H_y = H_z = \frac{A_4}{A} \tag{14}$$

Therefore, $H = \frac{1}{3}\left(\frac{2A_4}{A} + \frac{B_4}{B}\right)$ (15)

The deceleration parameter is given by

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \quad (16)$$

3. Solution of field equations

Here we have system of three equations (7)-(9), in eight unknowns $A, B, p, \rho, \lambda, \Lambda, \xi$ and θ. To obtain a deterministic solution, we assume

i. θ is proportional to shear σ which leads to

$$A = B^n \quad (17)$$

where n is constant.

ii. Λ is inversely proportional to R^3 which leads to

$$\Lambda = \frac{\alpha}{R^3} = \frac{\alpha}{A^2 B} \quad (18)$$

where α is the constant of proportion.

iii. barotropic fluid condition

$$p = \gamma\rho; 0 \leq \gamma \leq 1 \quad (19)$$

iv. $\xi\theta = K$ = constant (21)

After using conditions (17) – (20) in equations (8) and (9), we get

$$B_{44} + \left(\frac{n^2+2n\gamma+n^2\gamma}{n+1}\right)\frac{{B_4}^2}{B} = \frac{(\gamma-1)}{4(n+1)B^{4n-3}} + \frac{(1+\gamma)\alpha}{(n+1)B^{2n}} + \frac{KB}{(n+1)} \quad (21)$$

Now we consider the following cases -

I. Dust fluid model ($\boldsymbol{\gamma = 0}$)

On putting $\gamma = 0$ in (21), we get

$$B_{44} + \frac{n^2 {B_4}^2}{(n+1)B} = \frac{-1}{4(n+1)B^{4n-3}} - \frac{\alpha}{(n+1)B^{2n}} + \frac{KB}{(n+1)} \quad (22)$$

On putting $B_4 = f(B)$ and $B_{44} = ff'$ in equation (22), we get

$$\frac{df^2}{dB} + \frac{2n^2 f^2}{(n+1)B} = \frac{-1}{2(n+1)B^{4n-3}} - \frac{2\alpha}{(n+1)B^{2n}} + \frac{2KB}{(n+1)} \quad (23)$$

On integrating (23), we obtain

$$f^2 = \frac{1}{4(n^2-2)B^{4n-4}} + \frac{2\alpha}{(n-1)B^{2n-1}} + \frac{KB^2}{(n^2+n+1)} + \frac{L}{B^{\frac{2n^2}{n+1}}} \quad (24)$$

whereL is integrating constant. Equation (24) leads to

$$\int \frac{dB}{\sqrt{\frac{1}{4(n^2-2)B^{4n-4}} + \frac{2\alpha}{(n-1)B^{2n-1}} + \frac{KB^2}{(n^2+n+1)} + \frac{L}{\frac{2n^2}{B^{n+1}}}}} = \int dt + L' = t + L' \tag{25}$$

Where L' is the integrating constant. Value of B can be obtained from equation (25). Hence by the appropriate transformation of co-ordinates i.e. $B = T, x = X, y = Y$ and $z = Z$ metric (1) becomes

$$ds^2 = -\frac{dT^2}{\frac{1}{4(n^2-2)T^{4n-4}} + \frac{2\alpha}{(n-1)T^{2n-1}} + \frac{KT^2}{(n^2+n+1)} + \frac{L}{\frac{2n^2}{T^{n+1}}}} + T^2(dX + ZdY)^2$$

$$+ T^{2n}\left(dY^2 + dZ^2\right) \tag{26}$$

II. Stiff fluid model($\gamma = 1$)

On putting$\gamma = 1$ in equation (21), we obtain

$$B_{44} + \frac{(2n^2+2n){B_4}^2}{(n+1)B} = -\frac{2\alpha}{(n+1)B^{2n}} + \frac{KB}{(n+1)} \tag{27}$$

On putting$B_4 = f(B)$ and $B_{44} = ff'$in equation (27), we get

$$\frac{df^2}{dB} + \frac{4nf^2}{B} = -\frac{4\alpha}{(n+1)B^{2n}} + \frac{2KB}{(n+1)} \tag{28}$$

On integrating (28), we have

$$f^2 = -\frac{4\alpha}{(n+1)(2n+1)B^{2n-1}} + \frac{KB^2}{(n+1)(2n+1)} + \frac{M}{B^{4n}} \tag{29}$$

where M is the integrating constant. From equation (29) we have

$$\int \frac{dB}{\sqrt{-\frac{4\alpha}{(n+1)(2n+1)B^{2n-1}} + \frac{KB^2}{(n+1)(2n+1)} + \frac{M}{B^{4n}}}} = \int dt + M' = t + M' \tag{30}$$

where M' is the constant of integration.

Value of B can be obtained from equation (30). Hence by the appropriate transformation of co-ordinates i.e. $B = T, x = X,\ y = Y$ and $z = Z$, metric (1) becomes

$$ds^2 = -\frac{dT^2}{\left[-\frac{4\alpha}{(n+1)(2n+1)T^{2n-1}} + \frac{KT^2}{(n+1)(2n+1)} + \frac{M}{T^{4n}}\right]} + T^2(dX+ZdY)^2$$

$$+T^{2n}(dY^2+dZ^2) \tag{31}$$

III. Radiation dominated model $\left(\boldsymbol{\gamma = \frac{1}{3}}\right)$

On putting $\gamma = \frac{1}{3}$ in equation (21), we get

$$B_{44} + \frac{2n(2n+1)B_4^2}{3(n+1)B} = -\frac{1}{6(n+1)B^{4n-3}} - \frac{4\alpha}{3(n+1)B^{2n}} + \frac{KB}{(n+1)} \tag{32}$$

On putting $B_4 = f(B)$ and $B_{44} = ff'$ in equation (32), we have

$$\frac{df^2}{dB} + \frac{4n(2n+1)f^2}{3(n+1)B} = -\frac{1}{3(n+1)B^{4n-3}} - \frac{8\alpha}{3(n+1)B^{2n}} + \frac{2KB}{(n+1)} \quad (33)$$

Equation (33) leads to

$$f^2 = \frac{1}{4(n^2-n-3)B^{4n-4}} - \frac{8\alpha}{(2n^2+n+3)B^{2n-1}} + \frac{3KB^2}{(4n^2+5n+3)} + \frac{N}{B^{\frac{8n^2+4n}{(3n+3)}}} \quad (34)$$

where N is the integrating constant.

On integrating, equation (34) leads to

$$\int \frac{dB}{\sqrt{\frac{1}{4(n^2-n-3)B^{4n-4}} - \frac{8\alpha}{(2n^2+n+3)B^{2n-1}} + \frac{3KB^2}{(4n^2+5n+3)} + \frac{N}{B^{\frac{8n^2+4n}{(3n+3)}}}}} = \int dt + N' = t + N' \quad (35)$$

where N' is the constant of integration.

Value of B can be obtained from equation (35). Hence by the appropriate transformation of co-ordinates i.e. $B = T$, $x = X$, $y = Y$ and $z = Z$, metric (1) becomes

$$ds^2 = -\frac{dT^2}{\left[\frac{1}{4(n^2-n-3)T^{4n-4}} - \frac{8\alpha}{(2n^2+n+3)T^{2n-1}} + \frac{3KT^2}{(4n^2+5n+3)} + \frac{N}{T^{\frac{8n^2+4n}{(3n+3)}}}\right]}$$
$$+T^2(dX + ZdY)^2 + T^{2n}(dY^2 + dZ^2) \quad (36)$$

4. Physical and geometrical characteristics

For the model (26), energy density (ρ), pressure (p), string tension density(λ), expansion (θ), shear(σ), Hubble directional parameter (H_x, H_y and H_z), Hubble parameter (H) and deceleration parameter (q) are given by

$$\rho = \frac{(n+1)}{2(n^2-2)T^{4n-2}} + \frac{(2n^2+5n-1)\alpha}{(n-1)T^{2n+1}} + \frac{n(n+2)K}{(n^2+n+1)} + \frac{(n^2+2n)L}{T^{\frac{2n^2+2n+2}{n+1}}} \quad (37)$$

$$p = 0 \quad (38)$$

$$\lambda = -\frac{n(n-2)}{4(n^2-2)T^{4n-2}} + \frac{2n\alpha}{T^{2n+1}} + \frac{3n^2K}{(n^2+n+1)} - \frac{n(n-1)(n+2)L}{(n+1)T^{\frac{2n^2+2n+2}{n+1}}} \quad (39)$$

$$\theta = (2n+1)\left(\sqrt{\frac{1}{4(n^2-2)T^{4n-2}} + \frac{2\alpha}{(n-1)T^{2n+1}} + \frac{K}{(n^2+n+1)} + \frac{L}{T^{\frac{2n^2+2n+2}{n+1}}}}\right) \quad (40)$$

$$\sigma = \frac{n-1}{\sqrt{3}}\left(\sqrt{\frac{1}{4(n^2-2)T^{4n-2}} + \frac{2\alpha}{(n-1)T^{2n+1}} + \frac{K}{(n^2+n+1)} + \frac{L}{T^{\frac{2n^2+2n+2}{n+1}}}}\right) \quad (41)$$

$$H_x = \sqrt{\frac{1}{4(n^2-2)T^{4n-2}} + \frac{2\alpha}{(n-1)T^{2n+1}} + \frac{K}{(n^2+n+1)} + \frac{L}{T^{\frac{2n^2+2n+2}{n+1}}}} \tag{42}$$

$$H_y = H_z = n\left(\sqrt{\frac{1}{4(n^2-2)T^{4n-2}} + \frac{2\alpha}{(n-1)T^{2n+1}} + \frac{K}{(n^2+n+1)} + \frac{L}{T^{\frac{2n^2+2n+2}{n+1}}}}\right) \tag{43}$$

$$H = \frac{2n+1}{3}\left(\sqrt{\frac{1}{4(n^2-2)T^{4n-2}} + \frac{2\alpha}{(n-1)T^{2n+1}} + \frac{K}{(n^2+n+1)} + \frac{L}{T^{\frac{2n^2+2n+2}{n+1}}}}\right) \tag{44}$$

$$q = -1 + \frac{3}{2n+1}\left\{\frac{\frac{1-2n}{4(n^2-2)T^{4n-2}} - \frac{(2n+1)\alpha}{(n-1)T^{2n+1}} + \frac{(n^2+n+1)L}{(n+1)T^{\frac{2n^2+2n+2}{n+1}}}}{\frac{1}{4(n^2-2)T^{4n-2}} + \frac{2\alpha}{(n-1)T^{2n+1}} + \frac{K}{(n^2+n+1)} + \frac{L}{T^{\frac{2n^2+2n+2}{n+1}}}}\right\} \tag{45}$$

For this model the energy condition $\rho \geq 0$ gives

$$\frac{(n+1)}{2(n^2-2)T^{4n-2}} + \frac{(2n^2+5n-1)\alpha}{(n-1)T^{2n+1}} + \frac{(n^2+2n)K}{(n^2+n+1)} + \frac{(n^2+2n)L}{T^{\frac{2n^2+2n+2}{n+1}}} \geq 0 \tag{46}$$

For the model (31), energy density (ρ), pressure (p), string tension density(λ), expansion (θ), shear(σ), Hubble directional parameter $(H_x, H_y \text{ and } H_z)$, Hubble parameter (H) and deceleration parameter (q) are given by

$$\rho = \frac{(1-5n-2n^2)\alpha}{(n+1)(2n+1)T^{2n+1}} - \frac{1}{4T^{4n-2}} + \frac{(n^2+2n)M}{T^{4n+2}} + \frac{(n^2+2n)K}{(n+1)(2n+1)} \tag{47}$$

$$p = \frac{(1-5n-2n^2)\alpha}{(n+1)(2n+1)T^{2n+1}} - \frac{1}{4T^{4n-2}} + \frac{(n^2+2n)M}{T^{4n+2}} + \frac{(n^2+2n)K}{(n+1)(2n+1)} \tag{48}$$

$$\lambda = \frac{4n\alpha(1-n)}{(n+1)(2n+1)T^{2n+1}} + \frac{3n^2K}{(n+1)} - \frac{n(n+2)M}{T^{4n+2}} \tag{49}$$

$$\theta = (2n+1)\left(\sqrt{-\frac{4\alpha}{(n+1)(2n+1)T^{2n+1}} + \frac{K}{(n+1)(2n+1)} + \frac{M}{T^{4n+2}}}\right) \tag{50}$$

$$\sigma = \frac{n-1}{\sqrt{3}}\left(\sqrt{-\frac{4\alpha}{(n+1)(2n+1)T^{2n+1}} + \frac{K}{(n+1)(2n+1)} + \frac{M}{T^{4n+2}}}\right) \tag{51}$$

$$H_x = \sqrt{-\frac{4\alpha}{(n+1)(2n+1)T^{2n+1}} + \frac{K}{(n+1)(2n+1)} + \frac{M}{T^{4n+2}}} \tag{52}$$

$$H_y = H_z = n\left(\sqrt{-\frac{4\alpha}{(n+1)(2n+1)T^{2n+1}} + \frac{K}{(n+1)(2n+1)} + \frac{M}{T^{4n+2}}}\right) \tag{53}$$

$$H = \frac{2n+1}{3}\left(\sqrt{-\frac{4\alpha}{(n+1)(2n+1)T^{2n+1}} + \frac{K}{(n+1)(2n+1)} + \frac{M}{T^{4n+2}}}\right) \tag{54}$$

$$q = -1 + \frac{3}{2n+1}\left\{\frac{\frac{2\alpha}{(n+1)T^{2n+1}} - \frac{(2n+1)M}{T^{4n+2}}}{-\frac{4\alpha}{(n+1)(2n+1)T^{2n+1}} + \frac{K}{(n+1)(2n+1)} + \frac{M}{T^{4n+2}}}\right\} \tag{55}$$

For this model the energy condition $\rho \geq 0$ gives

$$\frac{(1-5n-2n^2)\alpha}{(n+1)(2n+1)T^{2n+1}} - \frac{1}{4T^{4n-2}} + \frac{(n^2+2n)M}{T^{4n+2}} + \frac{(n^2+2n)K}{(n+1)(2n+1)} \geq 0 \tag{56}$$

For the model (36), energy density (ρ), pressure (p), string tension density(λ), expansion(θ), shear(σ), Hubble directional parameters$(H_x, H_y \text{ and } H_z)$, Hubble parameter (H) and deceleration parameter (q) are given by

$$\rho = \frac{3(n+1)}{4(n^2-n-3)T^{4n-2}} - \frac{3(2n^2+5n-1)\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{(n^2+2n)N}{T^{\frac{8n^2+10n+6}{3n+3}}} + \frac{3(n^2+2n)K}{(4n^2+5n+3)} \tag{57}$$

$$p = \frac{(n+1)}{4(n^2-n-3)T^{4n-2}} - \frac{(2n^2+5n-1)\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{(n^2+2n)N}{3T^{\frac{8n^2+10n+6}{3n+3}}} + \frac{(n^2+2n)K}{(4n^2+5n+3)} \tag{58}$$

$$\lambda = \frac{n(2-n)}{4(n^2-n-3)T^{4n-2}} - \frac{8n\alpha(n-1)}{(2n^2+n+3)T^{2n+1}} + \frac{9n^2K}{(4n^2+5n+3)} + \frac{(n^3-n^2-6n)N}{3(n+1)T^{\frac{8n^2+10n+6}{3n+3}}} \tag{59}$$

$$\theta = (2n+1)\left(\sqrt{\frac{1}{4(n^2-n-3)T^{4n-2}} - \frac{8\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{3K}{(4n^2+5n+3)} + \frac{N}{T^{\frac{8n^2+10n+6}{(3n+3)}}}}\right) \tag{60}$$

$$\sigma = \frac{n-1}{\sqrt{3}}\left(\sqrt{\frac{1}{4(n^2-n-3)T^{4n-2}} - \frac{8\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{3K}{(4n^2+5n+3)} + \frac{N}{T^{\frac{8n^2+10n+6}{(3n+3)}}}}\right) \tag{61}$$

$$H_x = \sqrt{\frac{1}{4(n^2-n-3)T^{4n-2}} - \frac{8\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{3K}{(4n^2+5n+3)} + \frac{N}{T^{\frac{8n^2+10n+6}{(3n+3)}}}} \tag{62}$$

$$H_y = H_z = n\left(\sqrt{\frac{1}{4(n^2-n-3)T^{4n-2}} - \frac{8\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{3K}{(4n^2+5n+3)} + \frac{N}{T^{\frac{8n^2+10n+6}{(3n+3)}}}}\right) \tag{63}$$

$$H = \frac{2n+1}{3}\left(\sqrt{\frac{1}{4(n^2-n-3)T^{4n-2}} - \frac{8\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{3K}{(4n^2+5n+3)} + \frac{N}{T^{\frac{8n^2+10n+6}{(3n+3)}}}}\right) \tag{64}$$

$$q = -1 + \frac{3}{2n+1}\left\{\frac{\frac{(1-2n)}{4(n^2-n-3)T^{4n-2}} + \frac{4\alpha(2n+1)}{(2n^2+n+3)T^{2n+1}} - \frac{(4n^2+5n+3)N}{(3n+3)T^{\frac{8n^2+10n+6}{(3n+3)}}}}{\frac{1}{4(n^2-n-3)T^{4n-2}} - \frac{8\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{3K}{(4n^2+5n+3)} + \frac{N}{T^{\frac{8n^2+10n+6}{(3n+3)}}}}\right\} \tag{65}$$

For this model the energy condition $\rho \geq 0$ gives

$$\frac{3(n+1)}{4(n^2-n-3)T^{4n-2}} - \frac{3(2n^2+5n-1)\alpha}{(2n^2+n+3)T^{2n+1}} + \frac{(n^2+2n)N}{T^{\frac{8n^2+10n+6}{3n+3}}} + \frac{3(n^2+2n)K}{(4n^2+5n+3)} \geq 0 \quad (66)$$

The cosmological constant Λ is given by

$$\Lambda = \frac{\alpha}{T^{2n+1}} \quad (67)$$

Rotation has a magnitude of zero, implying that

$$\omega = 0 \quad (68)$$

5. Conclusion

The expanding, shearing and non-rotating universe is represented by models (26), (31) and (36). Models begin to grow at the big bang. As time goes on, for $n > 0$, the expansion θ slows down, gets closer to zero as$T \to \infty$ and stops for $n = -\frac{1}{2}$.

It is discovered that the cosmological constant decreases with cosmic time. Additionally, the parameters of these models, including the energy density (ρ), pressure (p), string tension density (λ), expansion (θ), shear(σ), Hubble directional parameter $(H_x, H_y \text{ and } H_z)$and Hubble parameter (H), decrease with time T for $n > 0$ and become closer to zero as $T \to \infty$. The energy condition $\rho \geq 0$ holds true for all values of T.

Since $T \to \infty$; $\frac{\sigma}{\theta} \neq 0$ indicates anisotropic behavior of the models for $n \neq 1$. However these isotropize for $n = 1$. A point type singularity is observed as $T \to 0$, $g_{11} \to 0$, $g_{22} \to 0$, $g_{33} \to 0$ for $n > 0$ (MacCallum[10]). We observed that deceleration parameter $q \to -1$ as $T \to \infty$ for each of the three models, representing accelerating phase of the universe.

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References

[1] Asseo, E. and Sol, H. (1987). Extragalactic magnetic fields, Physics reports, **148**(6), 307-435.

[2] Bali, R. (2022). Creation field cosmological models for dust distribution and time-dependent cosmological term (Λ) in Bianchi type II space-time, International Journal of Geometric Methods in Modern Physics, **19**(14), 2250233.

[3] Bali, R. and Dave, S. (2003). Bianchi type-IX string cosmological models with bulk viscous fluid in general relativity, Astrophysics and Space Science, **288**(4), 503-509.

[4] Bali, R. and Pradhan, A. (2007). Bianchi type-III string cosmological models with time dependent bulk viscosity, Chinese Physics Letters, **24**(2), 585-588.

[5] Banerjee, A., Duttachoudhury, S.B. and Sanyal, A.K. (1986). Bianchi type-II cosmological model with viscous fluid, General Relativity and Gravitation, **18**(5), 461-477.

[6] Caldwell, R. R. (2002). A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state, Physics Letters B, **545**(1-2), 23-29.

[7] Jain, K., Chhajed, D. and Tyagi A. (2019). Magnetized LRS Bianchi Type-I massive string cosmological model for perfect fluid distribution with cosmological term, International Journal of Scientific Research in Physics and Applied Sciences, **7**(3), 167-172.

[8] Kibble, T. W. (1976). Topology of cosmic domains and strings, Journal of Physics A: Mathematical and General, **9**(8), 1387-1398.

[9] Letelier, P. S. (1979). Clouds of strings in general relativity, Physical Review D, **20**(6), 1294-1302.

[10] MacCallum, MAH (1971). A class of homogeneous cosmological models III asymptotic behaviour, Comm. Math. Phys., **20**, 57-84.

[11] Misner, C.W. (1967). Transport Processes in the Primordial Fireball, Nature, **214**(5083), 40-41.

[12] Peebles, P. J. and Ratra, B. (2003). The cosmological constant and dark energy, Reviews of Modern Physics, **75**(2), 559-606.

[13] Perković, D., and Štefančić, H. (2020). Barotropic fluid compatible parameterizations of dark energy. The European Physical Journal C, **80**(7), 629.

[14] Pradhan, A., Amirhashchi, H. and Jaiswal, R. (2011). A new class of LRS Bianchi type-II dark energy models with variable EoS parameter, Astrophysics and Space Science, **334**(2), 249-260.

[15] Priyokumar, S. K., Baro, J., and Jiten, B. (2021). Higher dimensional LRS Bianchi type-I string cosmological model with bulk viscosity in general relativity, Indian Journal of Science and Technology, **14**(16), 1239-1249.

[16] Roy, S. R. and Banerjee, S. K. (1995). Bianchi type II string cosmological models in general relativity, Classical and Quantum Gravity, **12**(8), 1943-1948.

[17] Saha, B. (2014). Isotropic and anisotropic dark energy models, Physics of Particles and Nuclei, **45**(2), 349-396.

[18] Samdurkar, S., Bawnerkar, S. and Rathod, Y. (2023). Role of inhomogeneous equation of state with bulk viscous fluid in Bianchi type II universe, Prespacetime Journal, **14**(6), 626-633.

[19] Sharma, S. (2012). Bianchi type II bulk viscous cosmological model in general relativity, Int. J. Theor. Appl. Sci, **4**(2), 36-41.

[20] Singh, J., Tyagi, A. and Singh, J. (2024). Bulk viscous inflationary universe for barotropic fluid distribution in Bianchi type VIII space time, Journal of Rajasthan Academy of Physical Sciences, **23**(3-4), 118-127.

[21] Stachel, J. (1980). Thickening the string. I. The string perfect dust, Physical Review D, **21**(8), 2171-2181.

[22] Tiwari, R. K., Beesham, A. and Shukla, B. K. (2017). Cosmological models with viscous fluid and variable deceleration parameter, The European Physical Journal Plus, **132**(1), 20.

[23] Tyagi, A. and Sharma, K. (2011). Locally rotationally symmetric Bianchi type-II magnetized string cosmological model with bulk viscous fluid in general relativity, Chinese Physics Letters, **28**(8), 089802.

[24] Yadav, A. K., Pradhan, A. and Singh, A. (2011). LRS Bianchi Type-II Massive String Cosmological Models in General Relativity, Rom. J. Phys., **56**, 1019-1034.

[25] Zeldovich, Y. B. (1980). Cosmological fluctuations produced near a singularity, Monthly Notices of the Royal Astronomical Society, **192**(4), 663-667.

[26] Zimdahl, W. (1996). Bulk viscous cosmology, Phys. Rev. D, **53**, 5483-5493.