Journal of Rajasthan Academy of Physical Sciences
ISSN: 0972-6306; URL: http://raops.org.in
Vol.23, No.1&2, January-June, 2024, 14-29

ON THE ONSET OF STATIONARY CONVECTION IN ELECTROTHERMAL CONVECTION IN A DIELECTRIC FLUID SATURATED DENSELY PACKED POROUS LAYER

Chitresh Kumari[1], Jitender Kumar[2], Harjinder Singh[3] and **Jyoti Prakash*[1]**
[1]Department of Mathematics and Statistics
Himachal Pradesh University, Shimla-171005 (H.P.), India.
[2]Department of Mathematics, Baba Balak Nath College, Hamirpur-176039 (H.P.), India.
[3]Department of Mathematics, Kanwar Durga Chand Govt. Degree College Jaisinghpur, Kangra-176090 (H.P.), India.
*Email: jpsmaths67@gmail.com

Abstract: The electrothermoconvection in a dielectric fluid layer saturating a densely packed porous medium heated from below (or above) is studied analytically using linear stability theory. Sufficient conditions for the occurrence of stationary convection are derived for the two configurations in terms of the parameters of the systems alone. The results hold for free boundaries for all wave numbers and for rigid boundaries for a particular case. To the best of author's knowledge, the results derived herein are not reported in the literature as far as the domain of electrothermoconvection studies is concerned. Further, it is also shown that the results for Rayleigh-Bénard convection and Rayleigh-Bénard convection in densely packed porous medium follow as a consequence.

Keywords: Dielectric fluids, Electrothermoconvection, Linear stability theory, Stationary convection, Oscillatory convection, Porous medium, Darcy model

Mathematics Subject Classification: 76E06, 76E25, 76S99

1. Introduction

The thermal convection problem in a horizontal fluid layer has been thoroughly investigated and well documented since Bénard's experimental work [1] and Lord Rayleigh's theoretical efforts [29]. The onset of natural convection in different physical configurations, e.g., single diffusive systems, double diffusive systems or multicomponent diffusive systems under the influence of external magnetic field has also been widely investigated when the fluid under consideration is finitely/highly electrically conducting. The influence of magnetic field on thermal convection of an electrically conducting fluid is elaborated in an excellent way, in his famous book, by Chandrasekhar [7]. However, the effect of electric field becomes significant if the fluid is dielectric.

Several theoretical and experimental investigations on the convective instabilities in a dielectric fluid layer in the presence of an electric field has been carried out in the recent past due to its practical importance in various fields such as chemical engineering, material science processing, biomechanics of the design of artificial organs and purification of ground water pollution [31]. Electrothermoconvection in a dielectric fluid layer saturating porous medium under the influence of an external electric field is of particular interest in the light of its possibility of reducing the fluid viscosity which results in increasing the petroleum production and a control of heat and mass transfer in high voltage devices by electric field [20]. The first observations of convection produced in a fluid layer heated from above were reported by Gross and Porter [8] and Turnbull [35] wherein a dielectric fluid is kept under the influence of a uniform electric field. Roberts [30] studied electroconvection by assuming the dielectric constant as a function of temperature. Castellanos et al. [5] investigated oscillatory and steady convection in dielectric liquid layers subjected to unipolar injection and temperature gradient. Maekawa et al. [16] investigated the convective instability problem in AC and DC electric fields using linear stability theory. Exhaustive reviews in this domain of enquiry have been presented by Jones [12] and Saville [32]. For further studies one may refer to Guan et al. [9], Chandrappa et al. [6], Peng et al. [24], Li et al.[15] and Pavithra and Nair [22].

Beginning from the initial work of the Horton and Rogers [11], extensive investigations have been made on thermal/thermohaline convection in fluid saturated porous media due to its wide applications in engineering and technology, for e.g., radioactive waste management, thermal insulations, geophysical systems, solid–matrix compact heat exchangers etc. For the copious literature related to this domain of research, one may be referred to Vafai [37] and Nield and Bejan [21]. Electrothermoconvection in a dielectric fluid layer saturating porous medium under the influence of an external electric field is of particular significance in the light of its possibility of reducing the fluid viscosity which results in increasing the petroleum production and a control of heat and mass transfer in high voltage devices by electric field [20]. Many researchers have contributed in investigating the electrothermoconvection in dielectric fluid layer saturating a porous medium. For broad view of the subject, one may refer to Rudraiah and Gayathri [31] and Nield and Bejan [21].

The occurrence of stationary convection or, in particular, the validity of the principle of the exchange of stabilities disallows the manifestation of oscillatory motions in a stability problem. The validity of this principle removes the unsteady terms from the perturbation equations resulting into notable mathematical simplicity of the problem. Pellew and Southwell [23] established the validity of this principle for the classical Rayleigh-B$\acute{e}$nard problem. Later, Banerjee et al. [2] derived a sufficient condition for the validity of the exchange principle for magnetohydrodynamic Rayleigh- B$\acute{e}$nard problem which was further extended to different complex hydrodynamic configurations by Gupta et al.[10], Prakash and Manan [25], Prakash [26] and Prakash et al. [27],[28].

Different types of instability patterns were observed experimentally by Gross and Porter [8] and Turnbull [36] for horizontal dielectric fluid layer when heated from above; the former observed a stationary convection whereas the latter demonstrated the occurrence of oscillatory motions. The theoretical verification of the occurrence of stationary convection, as observed by Gross and Porter [8], was tried by Roberts [30] where he quoted without giving any mathematical proof that the principle of the exchange of stabilities can easily be proved for the case of free boundaries, whereas for the case of rigid boundaries it is not obvious. He further could not succeed in explaining the unsteady quasi–cellular motions as observed by Malkus and Veronis [17] in very similar conditions used by Gross and Porter [8].

Turnbull [35] and Bradley [3] on the other hand, predicted overstability using a quadratic conductivity model and a linear conductivity model respectively. Castellanos and Velarde [4] investigated the effect of a temperature dependent dielectric constant on the stability of a liquid layer subjected to an electric field, weak unipolar injection and temperature gradient and predicted that overstable modes appear only when the heating is from above. Martin and Richardson [19] investigated the linear instability of a unipolar charge injection model and predicted numerically that stationary convection is dominant if thermal gradient is weakly stabilizing whereas oscillatory convection is dominant if the thermal gradient is strongly stabilizing. Later, Martin and Richardson [18] derived a conductivity model and predicted the occurrence of overstability by investigating numerically the linear instabilities for linear quadratic and Arhenius-type conductivity variations. Bradley [3] discussed B$\acute{e}$nard type situation, i.e., when fluid layer heated from below by assuming Prandtl number P_r to be finite and predicted that the layer cannot be unstable for all top-heavy density distributions (all adverse temperature gradients), although it is plausible that they will be overstable if the electric field is sufficiently strong.

In the present communication, an attempt has been made to derive analytically necessary conditions for occurrence of oscillatory motions and hence sufficient conditions for the manifestation of stationary convection in terms of the parameters of the system alone, for the cases of electrothermoconvection of a fluid layer, saturating densely packed porous medium, heated from below and above separately for the case of free boundaries and for a particular case of rigid boundaries. These results are in agreement with Turnbull [35], Bradley [3], Castellanos and Velarde [4]; and Martin and Richardson [18],[19]that oscillatory instability may exists in electrothermoconvection configurations in certain parameter regime of the systems. The results derived in the present paper are not reported in the literature to the best of authors knowledge.

2. Formulation of the Problem

We consider a dielectric fluid layer, saturating a densely packed porous medium, of infinite horizontal extension and finite vertical depth d statically confined between the horizontal boundaries $z = 0$ and $z = d$ which are respectively maintained at uniform temperatures T_0 and $T_1 (< T_0)$. A uniform vertical AC electric field is applied across the fluid layer; the lower surface is grounded and upper surface is kept at an alternating $60Hz$

potential with root mean square value V_1. The Darcy model has been employed to mathematically analyse this problem (see Fig. 1).

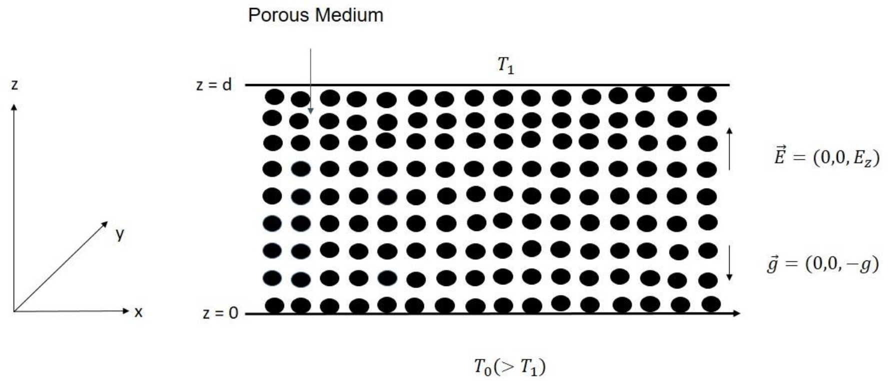

Fig. 1: Geometrical Configuration of the Problem

The basic equations governing the flow of dielectric fluid for the present model are given by [31]

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\rho_0\left(\frac{1}{\phi}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\phi^2}\left(\vec{q}.\nabla\right)\vec{q}\right) = -\nabla p + \rho\vec{g} - \frac{\mu}{k}\vec{q} + \vec{F_e}, \tag{2}$$

$$A\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla\right)T = \kappa\nabla^2 T, \tag{3}$$

where $\vec{q} = (u,v,w)$, ρ_0, t, ϕ, A, p, ρ, $\vec{g} = (0,0,-g)$, μ, T, k, κ and $\vec{F_e}$ respectively denote the velocity, reference density, time, porosity of the medium, ratio of heat capacities, pressure, fluid density, acceleration due to gravity, fluid viscosity, temperature, permeability of the porous medium, thermal diffusivity and the force of electric origin which can be expressed as [13]

$$\vec{F_e} = \rho_e\vec{E} - \frac{1}{2}\left(\vec{E}.\vec{E}\right)\nabla\varepsilon + \frac{1}{2}\nabla\left(\rho\frac{\partial\varepsilon}{\partial\rho}\vec{E}.\vec{E}\right). \tag{4}$$

Here ρ_e is the free charge density, ε is the dielectric constant and $\vec{E} = (0,0,E_z)$ is the root mean square value of the electric field. In Equation (4), the first, second and third terms respectively represent Coulomb force, the Dielectrophoretic force and the Electrostrictive force.The first term is neglected as compared to second term since the Coulomb force due to a free charge, is of negligible order as compared with the Dielectrophoretic force term for most dielectric fluids in a 60-Hz AC electric field [34].

The third term is grouped with the pressure $'p'$ in Equation (2). Equation (2), thus simplifies to

$$\rho_0\left(\frac{1}{\phi}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\phi^2}\left(\vec{q}.\nabla\right)\vec{q}\right)=-\nabla P+\rho\vec{g}-\frac{\mu}{k}\vec{q}-\frac{1}{2}\left(\vec{E}.\vec{E}\right)\nabla\varepsilon, \tag{5}$$

where $P=p-\frac{1}{2}\left(\rho\frac{\partial\varepsilon}{\partial\rho}\vec{E}.\vec{E}\right)$ is the modified pressure.

The equation of state is given by

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right], \tag{6}$$

where α is the coefficient of volume expansion and T_0 is the reference temperature.

The Maxwell's equations relevant to the present context are

$$\nabla\times\vec{E}=0, \tag{7}$$

$$\nabla.\left(\varepsilon\vec{E}\right)=0. \tag{8}$$

In the light of Equation (7), $\vec{E}$ can be written as

$$\vec{E}=-\nabla V, \tag{9}$$

where V is root mean square value of the electric potential.

The dielectric constant is considered as a linear function of temperature and is given by

$$\varepsilon=\varepsilon_0\left[1-\gamma\left(T-T_0\right)\right], \tag{10}$$

where ε_0 is the value of dielectric constant at the reference temperature T_0 and $\gamma\left(>0\right)$ is the thermal expansion coefficient of dielectric constant and is considered to be small [16].

Now following the linear stability theory [7], using basic state solutions, linearized perturbation equations, normal mode technique ascribing to all the quantities describing the perturbation a dependence on x, y and t of the form $\exp\left[i\left(k_x x+k_y y\right)+nt\right]$, we derive the non-dimensional equations given by

$$\left(\frac{n}{P_r}+D_a^{-1}\right)\left(D^2-a^2\right)w=-R_t a^2\theta-R_{ea}a^2\left(\theta+D\Phi\right), \tag{11}$$

$$\left(D^2-a^2-An\right)\theta=-w, \tag{12}$$

$$\left(D^2 - a^2\right)\Phi = -D\theta, \tag{13}$$

where $D = \dfrac{d}{dz}$ is the differentiation with respect to vertical coordinate z, $a^2 = k_x^2 + k_y^2$ is square of the resultant wave number, $n\left(= n_r + in_i\right)$ is the complex growth rate, w is $z-$component of the perturbation velocity, θ is perturbation temperature, Φ is perturbation electrical potential, $P_r = \dfrac{\nu\phi}{\kappa}$ is the Prandtl number, $D_a = \dfrac{k}{d^2}$ is the Darcy number, $R_t = \dfrac{\alpha g \Delta T d^3}{\nu\kappa}$ is the thermal Rayleigh number and $R_{ea} = \dfrac{\gamma^2 \varepsilon_0 E_0^2 \left(\Delta T\right)^2 d^2}{\mu\kappa}$ is the electric Rayleigh number.

The boundaries are considered to be free and rigid. Hence the boundary conditions are given by

$$w = D^2 w = \theta = D\Phi = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{14}$$

(both the boundaries are free)

$$w = Dw = \theta = \Phi = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{15}$$

(both the boundaries are rigid)

3. Mathematical Analysis

Now we derive sufficient conditions for the occurrence of stationary convection in a dielectric fluid layer saturating a densely packed porous medium, separately, for the cases when (I) the fluid layer is heated from below and (II) the fluid layer is heated from above.

Case I. When the fluid layer is heated from below

Subcase (i). Free boundaries

We prove the following theorem:

Theorem1. If $\left(w, \theta, \Phi, n\right)$, $\mathrm{P}_r > 0$, $R_t > 0$, $R_{ea} > 0$, $\mathrm{A} > 0$, $\mathrm{D}_a^{-1} > 0$, $n = n_r + in_i$, $n_r \geq 0$ is a solution of Equations (11)-(13) together with the boundary conditions (14) and $\dfrac{P_r R_{ea} A}{\pi^4} \leq 1$, then, we must have $n_i = 0$.

In particular, $n_r = 0$ implies $n_i = 0$ if $\dfrac{P_r R_{ea} A}{\pi^4} \leq 1$.

***Proof*:** Multiplying Equation (11) by w^* (the superscript * henceforth denotes the complex conjugation) and integrating the resulting equation from $z = 0$ to $z = 1$, we get

$$\left(\frac{n}{P_r}+D_a^{-1}\right)\int_0^1 w^*\left(D^2-a^2\right)wdz$$
$$=-\left(R_t+R_{ea}\right)a^2\int_0^1 w^*\theta dz-R_{ea}a^2\int_0^1 w^* D\Phi dz \tag{16}$$

Using Equations (12),(13) and the boundary conditions (14), we have

$$-a^2\left(R_t+R_{ea}\right)\int_0^1 w^*\theta dz = a^2\left(R_t+R_{ea}\right)\int_0^1 \theta\left(D^2-a^2-An^*\right)\theta^* dz, \tag{17}$$

and $$-R_{ea}a^2\int_0^1 w^* D\Phi dz = R_{ea}a^2\int_0^1 D\Phi\left(D^2-a^2-An^*\right)\theta^* dz$$

$$=R_{ea}a^2\int_0^1 D\Phi D^2\theta^* dz-R_{ea}a^2\left(a^2+An^*\right)\int_0^1 \theta^* D\Phi dz$$

$$=R_{ea}a^2\int_0^1 D\Phi D^2\theta^* dz+R_{ea}a^2\left(a^2+An^*\right)\int_0^1 \Phi D\theta^* dz$$

$$=R_{ea}a^2\int_0^1 D\Phi D^2\theta^* dz-R_{ea}a^2\left(a^2+An^*\right)\int_0^1 \Phi\left(D^2-a^2\right)\Phi^* dz. \tag{18}$$

Utilizing Equations (17) and (18) in Equation (16), we obtain

$$\left(\frac{n}{P_r}+D_a^{-1}\right)\int_0^1 w^*\left(D^2-a^2\right)wdz$$
$$=a^2\left(R_t+R_{ea}\right)\int_0^1 \theta\left(D^2-a^2-An^*\right)\theta^* dz \tag{19}$$
$$+R_{ea}a^2\int_0^1 D\Phi D^2\theta^* dz-R_{ea}a^2\left(a^2+An^*\right)\int_0^1 \Phi\left(D^2-a^2\right)\Phi^* dz$$

Integrating the various terms of Equation (19), by parts, from $z=0$ to $z=1$ with the help of the boundary conditions (14), we get

$$\left(\frac{n}{P_r}+D_a^{-1}\right)\int_o^1\left(|Dw|^2+a^2|w|^2\right)dz$$
$$=a^2\left(R_t+R_{ea}\right)\int_0^1\left(|D\theta|^2+a^2|\theta|^2+An^*|\theta|^2\right)dz \tag{20}$$
$$-R_{ea}a^2\int_0^1 D\Phi D^2\theta^* dz-R_{ea}a^2\left(a^2+An^*\right)\int_0^1\left(|D\Phi|^2+a^2|\Phi|^2\right)dz.$$

Now multiplying Equation (13) by $D\theta^*$ both sides and integrating, we have

$$\int_0^1 D^2\theta^* D\Phi dz+a^2\int_0^1 \Phi D\theta^* dz=\int_0^1|D\theta|^2 dz. \tag{21}$$

Further, multiplying the complex conjugate of Equation (13) by Φ and integrating to obtain

$$\int_0^1\left(|D\Phi|^2+a^2|\Phi|^2\right)dz=\int_0^1 \Phi D\theta^* dz. \tag{22}$$

Equations (21) and (22) clearly implies that $\int_0^1 D^2\theta^* D\Phi dz$ is real.

Now equating imaginary parts of both sides of Equation (21) and cancelling $n_i\left(\neq 0\right)$ throughout, we get

$$\frac{1}{P_r}\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz+\left(R_t+R_{ea}\right)a^2A\int_0^1|\theta|^2 dz$$
$$-R_{ea}a^2A\int_0^1\left(|D\Phi|^2+a^2|\Phi|^2\right)dz=0. \tag{23}$$

It is evident from Equation (23) that one cannot conclude from it, as in Pellew and Southwell's [23] case that $n_r=0$ implies $n_i=0$ for all a^2. This is due to the fact that the last term in the left hand side of this equation misbehave as regards its sign. Thus the need arises to derive a sufficient condition for the occurrence of stationary convection for the present problem in hand. We proceed in the following manner:

Multiplying Equation (13)by Φ^* and integrating, by parts, we have

$$\int_0^1\left(|D\Phi|^2+a^2|\Phi|^2\right)dz=-\int_0^1 \theta D\Phi^* dz$$

$$\leq \int_0^1 |\theta||D\Phi|\,dz \leq \left(\int_0^1 |\theta|^2\,dz\right)^{\frac{1}{2}}\left(\int_0^1 |D\Phi|^2\,dz\right)^{\frac{1}{2}}, \tag{24}$$

(Using the Schwartz inequality)

which implies that

$$\int_0^1 |D\Phi|^2\,dz \leq \left(\int_0^1 |\theta|^2\,dz\right)^{\frac{1}{2}}\left(\int_0^1 |D\Phi|^2\,dz\right)^{\frac{1}{2}}.$$

Thus

$$\left(\int_0^1 |D\Phi|^2\,dz\right)^{\frac{1}{2}} \leq \int_0^1\left(\int_0^1 |\theta|^2\,dz\right)^{\frac{1}{2}}. \tag{25}$$

Combining inequality (24) with inequality (25), we obtain

$$\int_0^1\left(|D\Phi|^2 + a^2|\Phi|^2\right)dz \leq \int_0^1 |\theta|^2\,dz. \tag{26}$$

Now multiplying Equation (12) by θ^* throughout and integrating the resulting equation, by parts, and making use of boundary conditions (14), we have from real part of the final equation

$$\int_0^1\left(|D\theta|^2 + a^2|\theta|^2 + An_r|\theta|^2\right)dz = \text{Real part of } \int_0^1 \theta^* w\,dz$$

$$\leq \int_0^1 |\theta||w|\,dz$$

$$\leq \left(\int_0^1 |\theta|^2\,dz\right)^{\frac{1}{2}}\left(\int_0^1 |w|^2\,dz\right)^{\frac{1}{2}}. \tag{27}$$

(Utilizing the Schwartz inequality)

Since $n_r \geq 0,$ we get

$$\int_0^1 |D\theta|^2 dz \leq \left(\int_0^1 |\theta|^2\,dz\right)^{\frac{1}{2}}\left(\int_0^1 |w|^2\,dz\right)^{\frac{1}{2}}. \tag{28}$$

Since $w(0)=0=w(1)$ and $\theta(0)=0=\theta(1)$, we have by Rayleigh-Ritz inequality [33]

$$\int_0^1 |Dw|^2\, dz \geq \pi^2 \int_0^1 |w|^2\, dz \tag{29}$$

and

$$\int_0^1 |D\theta|^2\, dz \geq \pi^2 \int_0^1 |\theta|^2\, dz. \tag{30}$$

Inequalities (28) and (30), thus yields

$$\left(\int_0^1 |\theta|^2\, dz\right)^{\frac{1}{2}} \leq \frac{1}{\pi^2}\left(\int_0^1 |w|^2\, dz\right)^{\frac{1}{2}}. \tag{31}$$

Further, using inequality (31) in inequality (27), we obtain

$$\int_0^1 \left(|D\theta|^2 + a^2|\theta|^2 + An_r|\theta|^2\right)dz \leq \frac{1}{\pi^2}\int_0^1 |w|^2\, dz.$$

Since $n_r \geq 0,$ therefore the above inequality, upon using inequality (29), yields

$$a^2\int_0^1 |\theta|^2 dz \leq \frac{1}{\pi^4}\int_0^1 |Dw|^2\, dz. \tag{32}$$

Combining inequalities (26) and (32), we obtain

$$a^2\int_0^1 \left(|D\Phi|^2 + a^2|\Phi|^2\right)dz \leq \frac{1}{\pi^4}\int_0^1 |Dw|^2\, dz. \tag{33}$$

Using inequalities (33) in Equation (23), we get

$$\left(\frac{1}{P_r} - \frac{R_{ea}A}{\pi^4}\right)\int_0^1 |Dw|^2 dz + \frac{1}{P_r}a^2\int_0^1 |w|^2 dz + \left(R_t + R_{ea}\right)a^2\int_0^1 |\theta|^2 dz \leq 0$$

and thus for $n_r \geq 0,\ n_i \neq 0,$ we necessarily have

$$\frac{P_r R_{ea} A}{\pi^4} > 1, \tag{34}$$

which is a necessary condition for overstability to manifest.

Hence if $\frac{P_r R_{ea} A}{\pi^4} \leq 1,$ we must have $n_i = 0,$

which is a sufficient condition for the occurrence of stationary convection.

This proves the theorem.

Theorem 1, from physical point of view, may be stated as: for electrothermoconvection problem in densely packed porous medium heated from below, an arbitrary neutral or unstable mode of the system is definitely non-oscillatory in character and in particular the principle of the exchange of stabilities is valid if $\frac{P_r R_{ea} A}{\pi^4} \leq 1.$

Subcase (ii). Rigid Boundaries

For the case of rigid boundaries, the boundary conditions are given by Equation (15).

Following the same procedure as is used in Theorem 1, we derive the same integrated Equation (20), in this case also. But for the case of rigid boundaries, the second integral in the right hand side of this equation cannot be dropped from the imaginary part without justification. This is elaborated as follows:

Multiplying Equation (13) by $D\theta^*$ and integrating the resulting equation over the vertical range of z, we get

$$\left[D\theta^* D\Phi\right]_0^1 - \int_0^1 D\Phi D^2\theta^* dz - a^2 \int_0^1 \Phi D\theta^* dz = -\int_0^1 |D\theta|^2 dz. \quad (35)$$

Since $\int_0^1 |D\theta|^2 dz$ is real and the integral $\int_0^1 \Phi D\theta^* dz$ is also real by Equation (22), therefore from Equation (35), two cases arise:

(a) When $\left[D\theta^* D\Phi\right]_0^1$ and $\int_0^1 D\Phi D^2\theta^* dz$ are both real, then on equating the imaginary parts of both sides of Equation (20) and cancelling $n_i\,(\neq 0)$ both sides, we obtain an equation exactly same as Equation (23) and the proof follows as is done for free boundaries to obtain the same result.

(b) When imaginary part of $\left[D\theta^* D\Phi\right]_0^1 =$ Imaginary part of $\int_0^1 D\Phi D^2\theta^* dz,$ the authors fail to derive any result for the occurrence of stationary convection and thus it is an open problem.

Special cases: It follows from Theorem 1 that an arbitrary neutral or unstable mode is nonoscillatory in character and in particular the principle of the exchange of stabilities is valid for

(i) Rayleigh-Bénard convection in densely packed porous medium $\left(R_t > 0,\ \mathrm{R}_{ea} = 0, A > 0\right)$ [14].

(ii) Rayleigh-Bénard convection $\left(R_t > 0,\ R_{ea} = 0,\ A = 1\right)$ [23].

Case II. When fluid layer is heated from above

Subcase (i). **Free boundaries**

We prove the following theorem:

Theorem 2. If $\left(w, \theta, \Phi, n\right),\ \mathrm{P}_r > 0, R_t < 0,\ R_{ea} > 0,\ A > 0,\ \ n = n_r + i n_i,\ n_r \geq 0$ is a solution of Equations (11)-(13) together with the boundary conditions (14) and $\frac{1}{\pi^4}\left(\left|R_t\right| + R_{ea}\right) A P_r \leq 1,$

then we must have $n_i = 0.$

In particular, $n_r = 0$ implies $n_i = 0$ if $\frac{1}{\pi^4}\left(\left|R_t\right| + R_{ea}\right) A P_r \leq 1.$

Proof**:** In the present case, $R_t < 0.$ Thus with $R_t = -\left|R_t\right|,$ Equation (11) becomes

$$\left(D^2 - a^2\right)\left(\frac{n}{P_r} + D_a^{-1}\right) w = \left|R_t\right| a^2 \theta - R_{ea} a^2 \left(\theta + D\Phi\right). \quad (36)$$

Now, following the same procedure as is used to prove Theorem 1, Equation (23) modifies to

$$\frac{1}{P_r}\int_0^1 \left(\left|Dw\right|^2 + a^2\left|w\right|^2\right) dz - \left|R_t\right| A a^2 \int_0^1 \left|\theta\right|^2 dz + R_{ea} a^2 A \int_0^1 \left|\theta\right|^2 dz - R_{ea} a^2 A \int_0^1 \left(\left|D\Phi\right|^2 + a^2\left|\Phi\right|^2\right) dz = 0. \quad (37)$$

Now utilizing inequalities (32) and (33) in (37), we obtain

$$\left(\frac{1}{P_r} - \frac{\left|R_t\right| A}{\pi^4} - \frac{R_{ea} A}{\pi^4}\right)\int_0^1 \left|Dw\right|^2 dz + \frac{a^2}{P_r}\int_0^1 \left|w\right|^2 dz + R_{ea} a^2 \int_0^1 \left|\theta\right|^2 dz \leq 0,$$

and therefore for $n_r \geq 0,\ \mathrm{n}_i \neq 0,$ we necessarily have

$$\frac{1}{\pi^4} A P_r \left(\left|R_t\right| + R_{ea}\right) > 1, \quad (38)$$

which is a necessary condition for overstability to occur.

Hence if $\frac{1}{\pi^4} AP_r \left(|R_t| + R_{ea}\right) \leq 1,$ we must have $n_i = 0,$

which is a sufficient condition for establishment of stationary convection.

This establishes the theorem.

Theorem 2, from physical point of view, can be stated as: for electrothermoconvection problem in a horizontal dielectric fluid layer saturating as densely packed porous medium heated from above, an arbitrary neutral or unstable mode of the system is definitely non-oscillatory in character and in particular the principle of the exchange of stabilities is valid if $\frac{1}{\pi^4} AP_r \left(|R_t| + R_{ea}\right) \leq 1.$

Subcase (ii). Rigid boundaries

Similar arguments hold for the present case as were used in subcase (ii) of case I, that is, when $\left[D\theta^* D\Phi\right]_0^1$ and $\int_0^1 D\Phi D^2\theta^* dz$ are both real then we have the same result as obtained in Theorem 2 and when imaginary part of $\left[D\theta^* D\Phi\right]_0^1 =$ imaginary part of $\int_0^1 D\Phi D^2\theta^* dz$ then this is an open problem.

Special case: It follows from Theorem 2 that in the case of Rayleigh-B$\acute{e}$nard convection in densely packed porous medium $\left(R_t < 0,\ R_{ea} = 0\right)$ and Rayleigh-B$\acute{e}$nard convection $\left(R_t < 0, R_{ea} = 0, A = 1\right)$, it follows that $n_r < 0$ which implies that in both the cases the system is stable when heated from above.

4. Conclusion

Linear stability theory has been utilized to analyse electrothermoconvection of a dielectric fluid layer in densely packed porous medium heated from below and above separately using Darcy model. Necessary conditions for overstability and hence sufficient conditions for the occurrence of stationary convection are derived in both the configurations in terms of the parameters of the system alone. Further, the results for Rayleigh-B$\acute{e}$nard convection in densely packed porous medium and Rayleigh-B$\acute{e}$nard convection follow as a consequence. The results derived herein are in agreement with Turnbull [35], Bradley [3], Castellanos and Velarde [4]; and Martin and Richardson [18],[19] that oscillatory instability may exists in electrothermoconvection configurations in certain parameter regime of the systems.

Acknowledgement: The author is thankful to Referee for valuable comments and suggestions.

References

[1] Bénard, H. (1900). Les tourbillons celllaires dans une nappe liquid, Revue Générale Des Sciences Pures et Appliquées **11**, 1261-1271, 1309-1328.

[2] Banerjee, M.B., Gupta, J.R., Shandil, R.G., Sood, S.K., Banerjee, B. and Banerjee, K. (1985). On the principle of exchange of stabilities in the magnetohydrodynamic simple Bénard problem, Journal of Mathematical Analysis and Applications **108**, 216-222.

[3] Bradley, R., (1978). Overstable electroconvective instabilities, The Quarterly Journal of Mechanics and Applied Mathematics **31**, 381-390.

[4] Castellanos, A. and Velarde, M. G. (1981). Electrohydrodynamic stability in the presence of a thermal gradient, Physics of Fluids **24**, 1784–1786.

[5] Castellanos, A., Atten, P. and Velarde, M.G. (1984). Oscillatory and steady convection in dielectric liquid layers subjected to unipolar injection and temperature gradient, Physics of Fluids **27**, 1607-1615.

[6] Chandrappa, R., Chandrashekar, B., Venkatesh, V. and Sokalingam, M. (2022). Effect of electric field modulation on the onset of electroconvection in an anisotropic porous layer saturated with a dielectric fluid, Journal of Computational Applied Mechanics **53**, 510-523.

[7] Chandrasekhar, S. (1981). Hydrodynamic and Hydromagnetic Stability, Oxford university Press, Dover publication, Inc., New York.

[8] Gross, M.J. and Porter, J.E. (1966). Electrically induced convection in dielectric liquids, Nature **212**, 1343-1345.

[9] Guan, Y. He, X. Wang, Song, Q. Z., Zhang, M. and Wu, J. (2021). Monotonic instability and overstability in two-dimensional electrothermohydrodynamic flow, Physical Review Fluids **6**, 013702 (1-22).

[10] Gupta, J.R., Sood, S.K. and Bhardwaj, U.D. (1984). On Rayleigh-Bénard convection with rotation and magnetic field, Journal of Applied Mathematics and Physics **35**, 252-256.

[11] Horton, C.W. and Rogers, G.T. (1945). Convection current in porous medium. Journal of Applied Physics **16**, 367-370.

[12] Jones, T.B. (1978). Electrohydrodynamically enhanced heat transfer in liquids—a review, in Advances in Heat Transfer, Academic Press, New York, 107-144.

[13] Landau, L. D. and Lifshitz, E. M. (1984). Course of Theoretical Physics—Electrodynamics of Continuous Media, Oxford: Butterworth Heinemann.

[14] Lapwood, E. R. (1948). Convection of a fluid in a porous medium. Mathematical Proceedings of the Cambridge Philosophical Society **44**, 508-521.

[15] Li, T.F., Luo, K. and Yi, H.L. (2019). Suppression of Rayleigh-Bénard secondary instability in dielectric fluids by unipolar charge injection. Physics of Fluids **31**, 064106(1-15).

[16] Maekawa, T., Abe, K. and Tanasawa, I. (1992). Onset of natural convection under an electric field, International Journal of Heat and Mass Transfer **35**, 613.

[17] Malkus, W. V. R. and Veronis, G. (1961). Surface electroconvection, Physics of Fluids **4**, 13–23.

[18] Martin, P.J. and Richardson, A.T. (1984). Conductivity models of electrothermal convection in a plane layer of dielectric liquid, Journal of Heat Transfer **106**, 131-136.

[19] Martin, P.J. and Richardson, A.T. (1982). Overstable electrothermal instabilities in a plane layer of dielectric liquid, Journal of Electrostatics **12**, 435-439.

[20] Moreno, R.Z., Bonet, E.J. and Trevisan, O.V. (1996). Electric alternating current effects on flow of oil and water in porous media. Proceedings of International Conference on Porous Media and Their Applications in Science, Hawaii: Engineering and Industry, 147-172.

[21] Nield, D.A. and Bejan, A. (2013). Convection in Porous Media, Fourth Edition, Springer, New York.

[22] Pavithra, H. M. and Nair, D.K. (2023). Effect of basic temperature gradients on electrothermal convection in a rotating layer of viscoelastic (Walter's b) fluid saturated porous medium heated from below, Power System Technology **47,** 41-62.

[23] Pellew, A. and Southwell, R.V. (1940). On the maintained convective motion in a fluid heated from below, Proceedings of the Royal Society of London. Series A **176**, 312-343.

[24] Peng, Y., Selvakumar, R. D. and Wu, J. (2024). Onset of Rayleigh-Bénard convection in dielectric liquids with electric conduction, Phys. Rev. Fluids **9**, 013902(1-24).

[25] Prakash, J. and Manan, S. (2016). A sufficient condition for the exchange principle in multicomponent convection problem in completely confined fluids, J. Raj. Acad. Phys. Sc. **15**, 245-253.

[26] Prakash, J. (2012). On stationary convection and oscillatory motions in ferromagnetic convection in a ferrofluid layer, Journal of Magnetism and Magnetic Materials **324**, 1523-1527.

[27] Prakash, J., Kumar R. and Kumar, P. (2016). A characterization theorem in rotatory hydrodynamic triply diffusive convection with viscosity variations, J. Raj. Acad. Phys. Sci. **15**, 139-148.

[28] Prakash, J., Bala, R., and Vaid, K. (2014). On the principle of the exchange of stabilities in rotatory triply diffusive convection, Proceedings of the National Academy of Sciences, India, Section A: Physical Sciences **84**, 433-439.

[29] Rayleigh, L. (1916). On convective currents in a horizontal layer of fluid when the higher temperature is on the underside, Philosophical Magazine **32**, 529-546.

[30] Roberts, P.H. (1969). Electrohydrodynamic convection, Quarterly Journal of Mechanics and Applied Mathematics **22**, 211-220.

[31] Rudraiah, N. and Gayathri, M. S. (2009). Effect of thermal modulation on the onset of electrothermoconvection in a dielectric fluid saturated porous medium, Journal of Heat Transfer **131**, 101009 (1-7).

[32] Saville, D.A. (1997). Electrohydrodynamics: The Taylor-Melcher leaky dielectric model, Annual Review of Fluid Mechanics **29**, 27-64.

[33] Schultz, M.H. (1973). Spine Analysis, Prentice-Hall Englewood cliffs NJ.

[34] Takashima, M. (1976). The effect of rotation on electrohydrodynamic instability, Canadian Journal of Physics **54**, 342–347.

[35] Turnbull, R. J. (1968a). Electroconvective instability with a stabilizing temperature gradient. I, Theory, Physics of Fluids **11**, 2588-2596.

[36] Turnbull, R. J. (1968b). Electroconvective instability with a stabilizing temperature gradient. II, Experimental results, Physics of Fluids **11**, 2597-2603.

[37] Vafai, K. (2005). Handbook of Porous media. Second edition. Taylor and Francis, CRC Press, Boca Raton, Florida, USA.