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BIANCHI TYPE-VI_0 MATHEMATICAL COSMOLOGICAL MODEL WITH VARIABLE DECELERATION PARAMETER

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Abstract: In Bianchi type-VI_0 cosmological model, we have investigated with deceleration parameter $q=-1+H$, which yields scale factor $a=e^{\sqrt{2t+k}}$. The model has non-singular origin and expands exponentially with cosmic time t. We observe that the equation of state parameter (ω) is time dependent. The jerk parameter (j) provides a perfect diagnosis of how dark energy model is closed to ΛCDM dynamics which is in agreement with the recent observations. We also discussed statefinder parameter {r, s} which predicts that the universe in 1 the model originates from Einstein era {r, s}={∞, -∞} to ΛCDM model {r, s}={1, 0}. The physical and geometrical properties of cosmological model are also discussed.

Key Words: Bianchi type-VI_0, EOS parameter, Variable deceleration parameter, jerk parameter, statefinder parameter

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1. Introduction

Bianchi Type VI_0 space-time is of particular interest because this is simple generalisation of Bianchi Type I space-time Bali et al. [2008], Tikekar and Patel [1996] investigated some string cosmological models in Bianchi Type VI_0 space-time.

Many observations of supernova experiments concluded that the expansion of the universe is accelerating, Riess et al. [21], Perlmutter et al. [18, 19, 20], Yadav [32], Schmidt et al. [24], Granavich et al. [12]. These observations indicates the presence of unknown fluid-dark energy that opposes the self attraction of the matter. This dark energy is the unknown form of energy could not have been detected directly and it does not cluster like ordinary matter. To study cosmological models, one of the important observational quantity is deceleration parameter q. This deceleration parameter q and Hubble's constant H_0 play a significant role in describing the nature of evolution of the

universe. Berman [7] proposed a special law of variation for Hubble constant by assuming deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2}$ to find the cosmological solution.

The equation of state parameter (ω=p/ρ, where p is the pressure and ρ is the energy density) is not necessarily constant, which is considered as an important quantity in describing the dynamics of the universe. The current cosmological data from large scale-structure (Komastu et al. [15]) does not support the possibility of ω<<-1. If ω=-1 (quitessence dark energy) (Steinhardt et al. [27]) less than- 1 (phantom dark energy) (Caldwell [9]). Tiwari and Sonia [30] investigated the Bianchi Type-I string cosmological model with bulk viscosity and time dependent Λ term. Sharma and Sharma [25] investigated the new study of cosmological model with cosmological constant. Kujat et al. [17], Bartelmann et al. [6] considered the equation of state parameter (ω) as a constant with phase wise value -1, 0, -1/3 and 1 for vacuum fluid, dust fluid, radiation and stiff dominated universe respectively. But, ω is a function of red shift or time (Jimenez [13], Dass et al. [11]). We don't expect the universe to have these characteristics in its early phases because the universe at smaller scales is neither homogeneous nor isotropic. Additionally, astronomical measurements in the late 1980s showed that, contrary to earlier expectations, the predictions of FRW models do not always match our requirements. Therefore, it is thought that the early stages of the universe's evolution can be studied in spatially homogenous and anisotropic Bianchi space times (I-IX). Because it is a straightforward generalization of Bianchi Type I space time, Bianchi Type VI_0 space time is of particular relevance among these. Bianchi Type VI_0 cosmological models, which can be istropized in specific circumstances, provide a better explanation of some cosmic difficulties, such as helium abundance.

In this paper, we study Bianchi Type-VI_0 spacetime with EOS parameters (γ and δ). To obtain the explicit solution, we assume deceleration parameter q=-1+H (H=Hubble parameter).

2. The Metric and Field Equations

Consider Bianchi Type-VI_o space time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{-2x} dz^2 \tag{1}$$

Where A, B and C are functions of cosmic time t. This implies that the model is totally anisotropic and spatially homogeneous.

To determine the diagonal form of energy-momentum tensor, the generalization form of EOS parameter of perfect fluid is

$$T_{ij} = diag[T_{oo}, T_{11}, T_{22}, T_{33}] \tag{2}$$

Allowing for anisotropy in the presence of fluid, and thus in its EOS parameter, gives new possibilities for the evolution of the energy source. To see this, we parametrize the energy momentum tensor given in equation (2) as follows:

$$T_{ij} = diag[\rho, -p_x, -p_y, -p_z]$$
$$= diag[1, -\omega_x, -\omega_y, -\omega_z]\rho$$
$$T_{ij} = diag[1, -\omega, -(\omega+\rho), -(\omega+\gamma)]\rho \tag{3}$$

Where p_x, p_y, p_z are directional pressure, ρ is energy density, ω_x, ω_y and ω_z are the directional EOS parameters along x, y, z axis respectively, ω is the deviation free EOS parameter, δ and γ are skewness parameters. ω, δ, γ are not necessarily constant, they might be function of cosmic time t.

The Einstein's field equations are

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{4}$$

Where 8πg=1, R_{ij}, R, g_{ij} and T_{ij} are Ricci tensor, Ricci scalar, metric tensor and energy momentum tensor respectively.

The field equation (4), with equation (3) for the line element (1) give rise to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\omega\rho \tag{5}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = -(\omega+\delta)\rho \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\omega+\gamma)\rho \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \rho \tag{8}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{9}$$

We define average scale factor 'a' for Bianchi type VI_0 space time as

$$a = (ABC)^{1/3} \tag{10}$$

Also shear scalar (σ), expansion scalar (θ), Hubble's parameter (H), deceleration parameter (q) and Anisotropic parameter $(\bar{A})$ as

$$\sigma^2 = \frac{1}{2}(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2) \tag{11}$$

$$\theta = 3H \tag{12}$$

$$H = \frac{\dot{a}}{a} \tag{13}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{14}$$

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2\sigma^2}{3H^2} \tag{15}$$

Where $\Delta H_i = H_i - H$ (i=x, y, z) represents the directional Hubble's parameters.

3. Solution of the Field Equations

Integrating equation (9), we get

$$C = kB \tag{16}$$

Where k is constant.

From equations (6), (7) and (16) we obtain that the skewness parameters along y and z axis are equal i.e. γ=δ

Equations (5)-(9) are reduced to

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = -\omega\rho \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\omega + \gamma)\rho \tag{18}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \rho \tag{19}$$

Equations (17), (18), (19) are three equations with five unknowns A, B, ω, ϒ, δ. To get the determinate solution's, we require two more conditions. First, we assume that the expansion scalar (θ) is proportional to shear scalar (σ) and using equation (16) we get

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\alpha\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{20}$$

Where α is constant, which yields to

$$\frac{\dot{A}}{A}=k_1\frac{\dot{B}}{B} \tag{21}$$

Where $k_1=\frac{2\alpha\sqrt{3}+1}{1-\alpha\sqrt{3}}$

From equation (21), we get

$$A=k_2B^{k_1} \tag{22}$$

Where k_2 is constant. Without any loss of generality we assume $k_1=k_2=1$ (Tiwari and Sonia [30])

$$\Rightarrow A=B \tag{23}$$

The observations of the velocity red-shift relation for extragalactic source says that Hubble expansion of the universe is isotropic at the time within ≈30 (Kristian and Sachs [16], Kantowski and Sachs [14], Tiwari et al. [29]). To put more precisely, red shift studies the limit $\frac{\sigma}{H}\leq 0.3$ on the ratio of the shear tensor (σ) and Hubble parameter (H) in the nearest of our galaxy today.

Second, we assume that the deceleration parameter q is a linear function of Hubble parameter (H) i.e. $q=\alpha'+\beta H$ (24)

Where α', β are constants and $H=\frac{\dot{a}}{a}$

Recent observations have suggested that the present universe is accelerating and value of deceleration parameter (q) lies between 0 and -1. For the sake of simplicity taking α=-1, β=1, the equation (24) becomes

$$a=e^{\sqrt{2t+k_3}} \tag{25}$$

Where k_3 is constant. This shows that at t=0 the scale factor 'a' tends to constant, hence the model is non-singular origin.

Using equation (16), (23) and (25), we have

$$A = k_6 e^{\sqrt{2t+k_3}}$$

$$B = k_4 e^{\sqrt{2t+k_3}}$$

$$C = k_5 e^{\sqrt{2t+k_3}}$$

Where $k_4=k_5=k_6=-1$

$$\therefore A = B = C = -e^{\sqrt{2t+k_3}} \tag{26}$$

$$\therefore ds^2 = -dt^2 + (dx^2 + e^{2x}dy^2 + e^{-2x}dz^2)e^{(2t+k_3)} \tag{27}$$

4. Physical and Geometrical Properties of the Model

The Spatial volume (V), Shear scalar (σ), Expansion scalar (θ), Hubble parameter (H), anisotropic parameter $(\bar{A})$ and deceleration parameter (q) are given by

$$V = e^{3\sqrt{2t+k_3}} \tag{28}$$

$$\sigma = 0 \tag{29}$$

$$\theta = \frac{3}{\sqrt{2t+k_3}} \tag{30}$$

$$H = \frac{1}{\sqrt{2t+k_3}} \tag{31}$$

$$\bar{A} = 0 \tag{32}$$

$$q = -1 + \frac{1}{\sqrt{2t+k_3}} \tag{33}$$

Here we observe that at t=0, the scale factor is constant, hence the model has no singularity at origin. At $t = -\frac{k_3}{2} = t_1$ the expansion scalar θ and Hubble's parameter H are infinite, which shows that the universe starts evolving with zero volume at $t=t_1$ with an infinite rate of expansion. The anisotropic parameter $(\bar{A})$ and shear scalar (σ) does not exists.

From equation (19) and (26)

$$\rho = \frac{3}{2t+k_3} - k_4 e^{-2\sqrt{2t+k_3}} \tag{34}$$

$$\omega = -\frac{\left[\frac{-2+3\sqrt{2t+k_3}}{(2t+k_3)^{3/2}}\right] + k_4 e^{-2\sqrt{2t+k_3}}}{\frac{3}{2t+k_3} - k_4 e^{-2\sqrt{2t+k_3}}} \tag{35}$$

$$\gamma = \frac{-2k_4(2t+k_3)e^{-2\sqrt{2t+k_3}}}{3-k_4(2t+k_3)e^{-2\sqrt{2t+k_3}}} \tag{36}$$

From equation (35), we observe that equation of state parameter (ω) is time dependent.

5. The Jerk Parameter (j) and Statefinder Parameter {r,s}

The jerk parameter (j), third derivative of scale factor w.r.t. cosmic time t (Chiba and Nakamura [10], Sahni [23], Visser [31], Blandford et al. [9]) provides a perfect diagnosis of how dark energy model is closed to ΛCDM dynamics. The jerk parameter (j) is defined as :

$$j(t) = \frac{\dddot{a}}{aH^3} = \left(\frac{a^2H^2}{2H^2}\right)'' \tag{37}$$

Over dot and primes denotes derivative w.r.t. cosmic time t and scale factor respectively.

Equation (37) can be written as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H} \tag{38}$$

From equation (31), (33) and (38), we have

$$j(t) = 1 - \frac{3}{\sqrt{2t+k_3}} + \frac{3}{2t+k_3} \tag{39}$$

Sahni et al. [22, 23] and Alam et al. [1] introduced the Statefinder parameter {r, s} defined as

$$r = \frac{\dddot{a}}{aH^2} = 1 + 3\frac{\dot{H}}{H} + \frac{\ddot{H}}{H^3} \tag{40}$$

And

$$s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{41}$$

Now equation's (31), (33), (40) and (41) result into

$r = 1, s = 0$

Therefore statefinder parameter $\{r, s\} = \{1, 0\}$ which is the value for ΛCDM model.

We observe that when $t \to t_1$, {r, s}={∞, -∞} and when t→∞, {r, s}={1, 0} which depicts that the universe in the model starts from Einstein static era goes to ΛCDM model. This is in agreement with the recent observations (Singh and Baro [26], Bali [5], Bali et al. [3,4] Bali & Kumari [2], Tikekar & Patel [28]) which makes our model observationally acceptable.

6. Conclusion

In summary, we have studied a new class of anisotropic Bianchi type VI_o cosmological model in the presence of variable equation of state parameter (ω) by using the condition for time dependent deceleration parameter (q). As the time t increases, the rate of expansion θ decreases. Thus rate of expansion slows down with the passage of time. σ=0 implies that shear scalar does not exists. Here $\lim_{t \to \infty} \frac{\sigma}{\theta} = 0$, therefore the model describes a non-shearing, continuously expanding universe. It is also observe that anisotropic parameter $(\bar{A})$ does not exists. We also depicted that the universe in the model starts from Einstein static era and goes to ΛCDM model. Finally we notice that the jerk parameter (j) and statefinder parameter (r) also in the agreement with recent observations.

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