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MAGNETIZED LRS BIANCHI TYPE-II UNIVERSE WITH BULK VISCOUS FLUID IN BARBER'S SECOND SELF CREATION THEORY

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Abstract: Exact solution of Barber's field equations of self-creation theory of gravitation in the presence of cosmic string source with bulk viscous fluid and magnetic field is obtained for LRS Bianchi type-II space-time. To obtain the determinate model of the universe, we have assumed that the coefficient of bulk viscosity ξ is inversely proportional to the expansion scalar θ in the model. We have also used the power law relation between Barber's scalar function ϕ and average scale factor R. It is found that the scalar function ϕ affects the other physical parameters of the model and contributes very high to matter density and particle density at late times of the universe. Some geometrical and physical aspects of the model are also discussed.

Keywords: Bianchi type-II universe, Self creation theory, Scalar function, Electromagnetic field, Cosmic string.

1. Introduction

Various cosmological problems are being studied by cosmologists to reveal the evolution of the universe. Einstein's general theory of relativity [7] has provided a sophisticated theory of gravitation. It has been very successful in describing gravitational phenomena. It has also served as a basis for models of the universe. Mach's principle is not substantially accommodated i.e. inertial properties of matter is not taken satisfactorily into consideration in Einstein's [8] general relativity and therefore there have been many interesting attempts to generalize general theory of relativity by incorporating the Mach's principle and other features. One of them is Barber's [3] second self-creation theory of gravitation, developed by coupling the scalar field $\phi(t)$ with trace of energy-momentum tensor T_{ij} in order to accommodate the Mach's principle substantially by the theory. Barber's theory is a variable G-theory and predicts local effects, which are within the observational limits. In it, the Newtonian gravitational parameter G is not a constant but a function of time parameter t. Also, the scalar field $\phi(t)$ does not gravitate directly but divides the matter tensor acting as a reciprocal gravitational constant, which is not the case in Einstein's general theory of relativity. This theory is capable of verification or falsification. It can be done by observing the behaviour of both bodies of degenerate matter and photons.

Right from Barber [3], the following cosmologists have studied the theory and developed the models of the universe and investigated geometrical and physical aspects of the universe in self-creation cosmology theory. Pimentel [16] and Soleng [23] have discussed the Robertson-Walker solutions in Barber's second self-creation theory of gravitation by using power law relation between the expansion factor of the universe and the scalar field. Reddy and Venkateswarlu [19] have presented Bianchi type VIo cosmological solutions in Barber's second theory of gravitation both in vacuum as well as in the presence of perfect fluid with pressure equal to energy density. Venkateswarlu and Reddy [25] have also investigated spatially homogeneous and anisotropic Bianchi type-I cosmological macro models when the source of gravitational field is a perfect fluid. Shanthi and Rao [21] have investigated Bianchi type-II and III space-times in Barber's second theory of gravitation, both in the vacuum as well in the presence of stiff-fluid. Ram and Singh [22] have investigated spatially homogenous and isotropic R-W model of the universe in Barber's second self-creation theory of gravitation in the presence of perfect fluid by using "Gamma law" equation of state. Pradhan and Pandey [17] have studied bulk viscous cosmological models in Barber's second self-creation theory. Cosmological models in self-creation theory of gravitation have been widely discussed in the literature (for example see Reddy [20], Rao and Vinutha [18], Katore et al. [12], Borkar and Ashtankar [4], Jaiswal and Tiwari [9], Pawar et al. [14] and Chauhan [6]). Johri and Desikan [11] studied cosmological models with constant deceleration parameter in Brans-Dicke theory.

Pawar and Solanke [15] have studied magnetized anisotropic dark energy models in Barber's second self-creation theory. Jain and Jain [10] have discussed Bianchi type-I magnetized radiating cosmological model in self-creation theory of gravitation. Borkar and Ashtankar [5] have investigated LRS Bianchi type-II string cosmological model in the presence of Magnetic field in Barber's second self-creation theory. Recently, Advani and Jain [1] have studied cosmological model of Bianchi type-I involving magnetic radiation in self-creation theory of gravitation with constant deceleration parameter. Bali and Jain [2] investigated magnetofluid cosmological model in Bianchi Type I space-time representing expanding and shearing universe.

Motivated by the above discussions, in this paper, we shall focus upon the problem of establishing a formalism for studying LRS Bianchi type-II anisotropic model with string bulk viscous fluid along with magnetic field. The outline of the paper is as follows : The metric and field equations are presented in Section 2. In Section 3, we deal with an exact solution of the field equations. In Section 4, we describe some physical and geometric properties of the model. Finally, conclusions are summarized in the last Section 5.

2. The Metric and Field Equations

We consider LRS Bianchi type-II metric of the form

$$ds^2 = -dt^2 + A^2\left(dx^2 + dz^2\right) + B^2\left(dy - xdz\right)^2 , \tag{1}$$

where the metric potentials A and B are functions of t alone. The field equations in Barber's second self-creation theory [3] are

$$R_{ij} - \frac{1}{2} R g_{ij} = -\frac{8\pi T_{ij}}{\phi} \tag{2}$$

and

$$\Box\phi = \frac{8}{3}\pi\,\eta T' , \tag{3}$$

where ϕ is the Barber's scalar function of t , T' is the trace of the energy-momentum tensor, η is a coupling constant to be determined from experiments.

The energy-momentum tensor $T_i^{\,j}$ for a cloud of strings with bulk viscous fluid and electromagnetic field $E_i^{\,j}$ is given by

$$T_i^{\,j} = \rho v_i v^j - \lambda x_i x^j - \xi v^l_{;l}\left(g_i^{\,j} + v_i v^j\right) + E_i^{\,j} , \tag{4}$$

where v_i and x_i satisfy the conditions

$$v_i v^i = -1 = -x_i x^i , \quad v_i x^i = 0 . \tag{5}$$

Here ρ is the proper energy density for a cloud strings with particles attached to them, λ is the string tension density, v^i is the four-velocity of the particles, x^i is a unit space-like vector representing the direction of string and ξ is the coefficient of bulk viscosity.

If the particle density of the configuration is denoted by ρ_p, then

$$\rho = \rho_p + \lambda . \tag{6}$$

The electromagnetic field $E_i^{\,j}$ [12] is defined by

$$E_i^{\,j} = \bar{\mu}\left[|h|^2 \left(v_i v^j + \frac{1}{2} g_i^{\,j} \right) - h_i h^j \right] , \tag{7}$$

where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl} v^j , \tag{8}$$

where F^{kl} is the electromagnetic field tensor and $\in_{ijkl}$ is the Levi-Civita tensor.

The expansion scalar θ is given by

$$\theta = v^l_{;l} = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B}. \tag{9}$$

In a co-moving coordinate system, we assume

$$v^i = (0,0,0,1), \qquad x^i = \left(\frac{1}{A},0,0,0\right). \tag{10}$$

We assume that the magnetic field is in yz-plane. Therefore, the current is flowing along *x*-axis. Thus F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. We have $h_1 \neq 0,\ h_2 = h_3 = h_4 = 0.$

Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \tag{11}$$

and $F^{ij}_{;j} = 0$ (12)

lead to

$$F_{23} = H = \text{constant}. \tag{13}$$

The components of electromagnetic field with the help of equations (7), (8) and (13) are obtained as

$$E^1_1 = \frac{-H^2}{2\bar{\mu}A^2B^2} = -E^2_2 = -E^3_3 = E^4_4. \tag{14}$$

For the line element (1) the field equations (2) and (3) lead to the following system of equations:

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = \left[\lambda + \xi\theta + \frac{H^2}{2\bar{\mu}A^2B^2}\right]\frac{8\pi}{\phi}, \tag{15}$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3B^2}{4A^4} = \left[\xi\theta - \frac{H^2}{2\bar{\mu}A^2B^2}\right]\frac{8\pi}{\phi}, \tag{16}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{B^2}{4A^4} = \left[\rho + \frac{H^2}{2\bar{\mu}A^2B^2}\right]\frac{8\pi}{\phi}, \tag{17}$$

$$\ddot{\phi}+\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{\phi}=\frac{8\pi\eta}{3}(\lambda+3\xi\theta+\rho). \quad (18)$$

Here and in what follows an over dot denotes ordinary differentiation with respect to *t*.

3. Solution of the Field Equations

The field equations (15) - (18) are a system of four equations with six unknown parameters A, B, ρ, λ, ξ and ϕ. Two additional constraints relating these parameters are required to obtain explicit solution of the system. We firstly assume the power law relation between average scale factor R and the Barber's scalar function ϕ [11] as

$$\phi \propto R^n \Rightarrow \phi = cR^n, \quad (19)$$

where *n* is any integer and *c* is the constant of proportionality.

Secondly, referring to Thorne [24] observations of the velocity-red shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today to within 30 percent. More precisely, the red-shift studies place the limit $\frac{\sigma}{H} \le 0.30$, where σ is shear and H is Hubble constant. Following Bali and Jain [2], we assume that the expansion (θ) in the model is proportional to the shear (σ), which is physically plausible condition. This condition leads to

$$A = R^m, \quad (20)$$

where *m* is a constant.

The spatial volume for the model (1) is given by

$$R^3 = A^2 B. \quad (21)$$

From equations (20) and (21), we get

$$B = R^{3-2m}. \quad (22)$$

To obtain the determinate model of the universe, we assume that the coefficient of bulk viscosity (ξ) is inversely proportional to expansion scalar (θ). This condition leads to

$$\xi\theta = K, \quad (23)$$

where *k* is a proportionality constant.

By using equations (19) - (23) in equation (16), we get

$$2\ddot{R}+(3m-2)\frac{\dot{R}^2}{R}=\frac{3}{4mR^{8m-7}}+\left[\frac{K}{R^{n-1}}-\frac{H^2}{2\bar{\mu}R^{n-2m+5}}\right]\frac{8\pi}{mc}. \quad (24)$$

Now on putting $\dot{R} = f(R)$ in equation (24), we get

$$\frac{d(f^2)}{dR} + (3m-2)\frac{f^2}{R} = \frac{3}{4mR^{8m-7}} + \left[\frac{K}{R^{n-1}} - \frac{H^2}{2\overline{\mu}R^{n-2m+5}}\right]\frac{8\pi}{mc}. \tag{25}$$

On integration, equation (25) leads to

$$f^2 = \left(\frac{dR}{dt}\right)^2 = \left[\frac{K}{(3m-n)R^{n-2}} - \frac{H^2}{2\overline{\mu}(5m-n-6)R^{n-2m+4}}\right]\frac{8\pi}{mc} - \frac{3}{4m(5m-6)R^{8(m-1)}} + \frac{L}{R^{3m-2}}, \tag{26}$$

where L is an integrating constant.

With the help of equations (20), (22) and (26), the metric (1) reduces to

$$ds^2 = -\left[\left\{\frac{K}{(3m-n)R^{n-2}} - \frac{H^2}{2\overline{\mu}(5m-n-6)R^{n-2m+4}}\right\}\frac{8\pi}{mc} - \frac{3}{4m(5m-6)R^{8(m-1)}} + \frac{L}{R^{3m-2}}\right]^{-1} dR^2 + R^{2m}\left(dx^2 + dz^2\right) + R^{6-4m}(dy - xdz)^2. \tag{27}$$

After making a suitable transformation of co-ordinates the metric (27) takes the form

$$ds^2 = -\left[\left\{\frac{K}{(3m-n)T^{n-2}} - \frac{H^2}{2\overline{\mu}(5m-n-6)T^{n-2m+4}}\right\}\frac{8\pi}{mc} - \frac{3}{4m(5m-6)T^{8(m-1)}} + \frac{L}{T^{3m-2}}\right]^{-1} dT^2 + T^{2m}\left(dx^2 + dz^2\right) + T^{6-4m}(dy - xdz)^2. \tag{28}$$

4. The Geometric and Physical Significance of Model

The cosmic scale factors A, B and the spatial volume V of the model (28) are given by

$$A = T^m, \quad B = T^{3-2m} \text{ and } V = A^2 B = T^3. \tag{29}$$

The scale factors A, B and volume V are increasing function of cosmic time T. Initially, when T $\to$ 0, the scale factors A, B and volume V tend to zero value and finally, when T $\to\infty$, they attain infinite values. This shows that the universe starts evolving with zero volume and expanding with infinite volume at final stage.

The scalar function ϕ for the model (28) is given by

$$\phi = cT^n. \tag{30}$$

We find that the scalar function ϕ is an increasing function of cosmic time T. Therefore, during evolution of the universe the scalar field is growing and affects the behaviour of the physical parameters of the model.

The energy density (ρ), the string tension density (λ) and the particle density (ρ_p) for the model (28) are given by

$$\rho = \frac{3(m-2)K}{(n-3m)} + \frac{(n-2m)H^2}{2\bar{\mu}(5m-n-6)T^{6-2m}} + \left(\frac{(m-3)}{(5m-6)T^{8m-6}} - \frac{3m(m-2)L}{T^{3m}}\right)\frac{\phi}{8\pi}, \tag{31}$$

$$\lambda = -\frac{3(mn-6m-n+6)}{2m(n-3m)} + \frac{(mn-14m^2+22m+3n)H^2}{2\bar{\mu}m(5m-n-6)T^{6-2m}} + \left(\frac{(m^2-38m+54)}{m(5m-6)T^{8m-6}} + \frac{18(m^2-3m+2)L}{T^{3m}}\right)\frac{\phi}{32\pi}, \tag{32}$$

$$\rho_p = \frac{3(2m^2+mn-10m-n+6)K}{2m(n-3m)} + \frac{(10m^2+mn-22m-3n)H^2}{2\bar{\mu}m(5m-n-6)T^{6-2m}} + \left(\frac{(3m^2+26m-54)}{m(5m-6)T^{8m-6}} - \frac{(5m^2-13m+6)L}{T^{3m}}\right)\frac{\phi}{32\pi}, \tag{33}$$

From the above equations we observe that at the initial epoch i.e. when $T \to 0$, the energy density, string tension and particle density are infinitely large and tend to small values when $T \to \infty$. It is also observed that when $T \to \infty$, the energy density and particle density tend to small positive values but never approach to zero due to the presence of bulk viscosity, whereas the string tension density tends to a small negative value which shows that the string phase in the universe switched-off or disappeared as predicted by astronomical observations. Since scalar function ϕ is an increasing function of cosmic time T and so in the early universe its contribution to energy density and particle density is very small, but at the later times it is playing the important role and affect the behaviour of these parameter.

The dynamical scalars θ and σ have the values given by

$$\theta = 3\left[\left\{\frac{K}{(3m-n)} - \frac{H^2}{2\bar{\mu}(5m-n-6)T^{6-2m}}\right\}\frac{8\pi}{m\phi} - \frac{3}{4m(5m-6)T^{8m-6}} + \frac{L}{T^{3m}}\right]^{\frac{1}{2}}, \tag{34}$$

$$\sigma = \sqrt{3}(m-1)\left[\left\{\frac{K}{(3m-n)} - \frac{H^2}{2\bar{\mu}(5m-n-6)T^{6-2m}}\right\}\frac{8\pi}{m\phi} - \frac{3}{4m(5m-6)T^{8m-6}} + \frac{L}{T^{3m}}\right]^{\frac{1}{2}}. \tag{35}$$

It is seen that the expansion scalar θ is a decreasing function of cosmic time T. It attains a very large value in the beginning and a small value in the end of the model. Thus, the model starts evolving with highest expansion and the expansion continuously decreases and approaches to a small finite value at late times and is affected by the scalar function ϕ. That is our universe continue to expand at a finite rate in the end of the model. With regard to the nature of shear σ, we can put the similar argument as that of expansion scalar θ. Furthermore, since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ provided $m \neq 1,$ the model does not approach isotropy for large values of T. The shear scalar σ is zero when m = 1, hence m = 1 is the isotropy condition.

5. Conclusion

In this paper we have presented a new exact solution of Barber's second self-creation theory of gravitation for LRS Bianchi type-II space-time in the presence of cosmic string source with viscous fluid and magnetic field. The model starts evolving with zero volume at T = 0 and expand with cosmic time T. It is found that the Barber's scalar function ϕ affects the behaviour of physical parameters. At late times scalar function ϕ contributes very high to energy density and matter density and due to this contribution these densities decreases at slightly slower rates. The expansion scalar θ in the model is inversely proportional to cosmic time T and the scalar function ϕ. This suggest that in the presence of scalar function expansion scalar decreases at high rate at late times. Also $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ provided $m \neq 1,$ the model does not approach isotropy at any time. Initially string tension density λ is very high but it tends to a small negative value when T→∞. Therefore, the strings disappear from the universe at late times (i.e. at present epoch).

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References

[1] Advani, P. and Jain, N. (2021). Cosmological model of Bianchi type-I involving magnetic radiation in self-creation theory of gravitation with constant declaration parameter, Prespacetime J. **12**, 489-500.

[2] Bali, R and Jain, V.C. (1998). Generalized expanding and shearing anisotropic Bianchi type I magnetofluid cosmological model in general relativity, Astrophys. Space Sci. **262**, 145-155.

[3] Barber, G.A. (1982). On two self-creation cosmology, Gen. Relativ. Grav. **14**, 117-136.

[4] Borkar, M.S. and Ashtankar, N.K. (2011). Plane symmetric viscous fluid cosmological model with varying Λ-term in self-creation theory of gravitation, Tensor N.S. **73**, 228.

[5] Borkar, M.S. and Ashtankar, N.K. (2016). LRS Bianchi type-II cosmological model with string bulk viscous fluid and magnetic field in Barber's second self-creation theory, Appl. Appl. Math. **11**, 215-228.

[6] Chauhan, D.S. (2021). LRS Bianchi type-I models in self-creation cosmology, Research Guru **15**, 31-39.

[7] Einstein, A. (1916). Die Grundlage der allgemeinen Relativitätstheorie, Annalen der Physik **354**, 769-822.

[8] Einstein, A. (1915). Zur Allgemeinen Relativitättstheorie, Sitzungsber Preuss Akad Wiss Berlin **844**, 778-786.

[9] Jaiswal, J. and Tiwari, R.K. (2013). Shear free Bianchi type V string cosmological model in self creation cosmology, Int. J. Theo. Appl. Sci. **5**, 6-15.

[10] Jain, V.C. and Jain, N. (2015). Bianchi type-I magnetized radiating cosmological model in self creation theory of gravitation, Astrophys. Space Sci. **357**, 131.

[11] Johri, V.B. and Desikan, K. (1994). Cosmological models with constant deceleration parameter in Brans-Dicke theory, Gen. Relativ. Grav. **26**, 1217-1232.

[12] Katore, S.D., Saikh, A.Y., Sancheti, M.M. and Sarkate, N.K. (2011). Bianchi type-III cosmic strings and domain walls cosmological models in self-creation cosmology, Prespacetime J. **2**, 447-461.

[13] Lichnerowicz, A. (1967). Relativistic Hydrodynamics and Magneto hydrodynamics. p.13, Benjamin, New York.

[14] Pawar, D.D., Solanke, Y.S. and Bayaskar, S.N. (2014). Exact Kantowski-Sach cosmological models with constant EoS parameter in Barber's second self-creation theory, Prespecetime J. **5**, 60-68.

[15] Pawar, D.D. and Solanke, Y.S. (2014). Magnetized anisotropic dark energy models in Barber's second self-creation theory, Advan. High Energy Phys. 2014, Art. ID859638.

[16] Pimental, L.O. (1985). Exact self-creation cosmological solutions, Astrophys. Space Sci. **116**, 395-399.

[17] Pradhan, A. and Pandey, H.R. (2004). Bulk viscous cosmological models in Barber's second self-creation theory, Indian J. Pure Appl. Math. **35**, 513-523.

[18] Rao, V.U.M. and Vinutha, T. (2010). Plane symmetric string cosmological models in self-creation theory of gravitation, Astrophys. Space Sci. **325**, 59-62.

[19] Reddy, D.R.K. and Venkateswarlu, R. (1989). Bianchi type-VI_0 models in self-creation cosmology, Astrophys. Space Sci. **155**, 135-139.

[20] Reddy, D.R.K. (2006). Plane symmetric string cosmological models in self-creation theory of gravitation, Astrophys. space Sci. **305**, 139-141.

[21] Shanti, K. and Rao, V.U.M. (1991). Bianchi type-II and III models in self-creation cosmology, Astrophys. Space Sci. **179**, 147-153.

[22] Ram, S. and Singh, C.P. (1997). Early universe in self-creation cosmology, Astrophys. Space Sci. **257**, 123-129.

[23] Soleng, H.H. (1987). Self-creation cosmological solutions, Astrophys. Space Sci. **139**, 13-19.

[24] Thorne, K.S. (1967). Primordial element formation, Primordial magnetic field and the isotrophy of the universe, Astrophys Space Sci. **148**, 51.

[25] Venkateswarlu, R. and Reddy, D.R.K. (1990). Bianchi type-I models in self-creation theory of gravitation, Astrophys. Space Sci. **168**, 193-199.