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VISCOUS BIANCHI TYPE I UNIVERSE WITH COSMOLOGICAL TERM Λ

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Abstract. We examine homogeneous and anisotropic Bianchi type I cosmological models with viscous fluid and time-dependent cosmological term Λ. A law of variation for the Hubble parameter, which is related to the cosmic time t and yields a variable deceleration parameter, is assumed to solve the field equations. Our models present an initial epoch with decelerating expansion followed by late time acceleration consistent with observation.

Keywords: Bianchi I, Viscosity, Variable cosmological term, Hubble parameter.

1. Introduction

The late time cosmic acceleration of our universe is strongly supported by observational data. Observations of supernovae Ia [35], the data from Baryon Acoustic Oscillations (BAO) [26] and Cosmic Microwave Background Radiation (CMBR) [12] measurements confirm that the universe is expanding with acceleration at present time. The existence of dark energy as a fluid with negative pressure that accelerates the universe at present time has been established. It contributes about 70% of the energy density. The nature of dark energy component remains one of the deepest mysteries of cosmology. It is the substance responsible for an anti-gravity force. Cosmological term Λ is the most favored candidate of dark energy representing energy density of vacuum. It provides a correct description of the universe evolution but suffers from fine-tuning and coincidence problems. The discrepancy between the observed value and the theoretically predicted value motivates to study cosmological models with time dependent cosmological term Λ. An initial large value of Λ would explain inflation and galaxy formation, while subsequent slow decay of Λ would produce a small present value Λ_0 to be reconciled with observations suggesting $\Omega_\Lambda \approx .07$ [31]. For this purpose some phenomenological models have been introduced [23]. Cosmological scenarios with time varying Λ were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated in the last two decades [1,5-8,15,16,25,36].

In the investigation of relativistic cosmological models, distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of matter. However, observation of large entropy per baryon and the remarkable degree of isotropy of the cosmic background radiation suggest the analysis of dissipative effects in cosmology. Dissipative effects, including both bulk and shear viscosity, play a very important role in the evolution of the universe. The viscosity theory of relativistic fluids was first suggested by Eckart [9] and later developed by Israel and Stewart [11]. In the framework of full causal theory, the characteristic of the evolution equation is very complicated. Therefore, the conventional theory [13] is still applied to phenomena which are quasi-stationary. Dissipative process can be modelled involving both bulk and shear viscosity. The inclusion of dissipative terms in the energy-momentum tensor of cosmic fluids seems to be the best motivated generalization of the matter term of the gravitational field equations. Misner [20,21] suggested the strong dissipation due to the neutrino viscosity may considerably reduce the anisotropy of the black-body radiation. The viscosity mechanism in cosmology can explain the anomalously high entropy per baryon in the present universe [38,39]. Bulk viscosity associated with grand-unified-theory phase transition [14] may lead to an inflationary scenario [10,24,37]. Cosmological models with viscous fluid source have been widely considered by many researchers [2,3,17-19,27,30,33,34].

In order to define a class of initial conditions that could lead to the observed high degree of isotropy of the present universe, anisotropic cosmological models were investigated. It has been argued that a class of anisotropic universe models does exist, which develops towards isotropic Friedmann universe due to evolutionary processes. It is of interest to study cosmological models with a richer structure, both geometrically and physically than standard FRW models.

In this paper, we investigate Bianchi type-I models, the simplest anisotropic generalization of FRW models with flat space slices. Matter content is taken to be viscous fluid. We obtain exact solutions of Einstein field equations assuming a functional form of the Hubble parameter. Cosmological consequences of the resulting models have been discussed.

2. Metric and Field Equations

We consider homogeneous and anisotropic Bianchi I space-time represented by the line element

$$ds^2 = -dt^2 + A^2(t)\, dx^2 + B^2(t)\, dy^2 + C^2(t)\, dz^2, \tag{1}$$

where A, B, C are directional scale factors.

We assume matter content of the universe to be viscous fluid represented by energy momentum tensor

$$T_i^j = (\rho + \bar{p}) v_i v^j + \bar{p} g_i^j - 2\eta \sigma_i^j, \tag{2}$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta v^i_{;i}. \tag{3}$$

Here ρ is the matter energy density, p, the isotropic pressure, ζ and η are coefficients of bulk and shear viscosity respectively, v^i , the flow vector of the fluid satisfying $v_i v^i = -1$. Cosmic fluid satisfies linear equation of state

$$p = \omega\rho, 0 \leq \omega \leq. \tag{4}$$

Expansion scalar θ, shear tensor σ_{ij} and shear scalar σ are defined by

$$\theta = v^i_{;i}, \tag{5}$$

$$\sigma_{ij} = \frac{1}{2}\left(v_{i;k}h^k_j + v_{j;k}h^k_i\right) - \frac{1}{3}\theta h_{ij}, \tag{6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{7}$$

where $h_{ij} = g_{ij} + v_i v_j$ is the projection tensor and semicolon(;) stands for covariant derivative. We choose the coordinates to be co-moving, so that

$$v^1 = v^2 = v^3 = 0, v^4 = 1. \tag{8}$$

The Einstein's field equations (in gravitational units$8\pi G = c = 1$) with time dependent cosmological term $\Lambda(t)$ are given by

$$R^j_i - \frac{1}{2}Rg^j_i = -T^j_i + \Lambda g^j_i. \tag{9}$$

For the metric (1) and matter distribution (2), Einstein field equations (9), read as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - 2\eta\frac{\dot{A}}{A} = -p + \left(\zeta - \frac{2}{3}\eta\right)\theta + \Lambda, \tag{10}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - 2\eta\frac{\dot{B}}{B} = -p + \left(\zeta - \frac{2}{3}\eta\right)\theta + \Lambda, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - 2\eta\frac{\dot{C}}{C} = -p + \left(\zeta - \frac{2}{3}\eta\right)\theta + \Lambda, \tag{12}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = \rho + \Lambda. \tag{13}$$

From equations (10)-(13), we obtain

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 4\eta\sigma^2. \tag{14}$$

We define average scale factor S as

$$S = (ABC)^{\frac{1}{3}}. \tag{15}$$

Volume scale factor V is given by

$$V = S^3 = ABC. \tag{16}$$

Generalized Hubble parameter H and generalized deceleration parameter q are defined as

$$H = \frac{\dot{S}}{S} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{17}$$

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{18}$$

Here $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's factors along *x, y* and *z* directions respectively. Expansion scalar θ and components of shear tensor σ_j^i for the metric (1) are given by

$$\theta = 3H, \tag{19}$$

$$\sigma_1^1 = H_1 - H, \sigma_2^2 = H_2 - H, \sigma_3^3 = H_3 - H, \sigma_4^4 = 0. \tag{20}$$

Shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2]. \tag{21}$$

From equations (10)-(13), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{C}}{C} + 2\eta\right) = 0, \tag{22}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A} + 2\eta\right) = 0. \tag{23}$$

Integrating (22) and (23), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{S^3} e^{-2\int \eta dt}, \tag{24}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{S^3} e^{-2\int \eta dt}, \tag{25}$$

From (24) and (25) together with (17), we get

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{k_2 + 2k_1}{3S^3} e^{-2\int \eta dt}, \tag{26}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{k_2 - k_1}{3S^3} e^{-2\int \eta dt}, \tag{27}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{2k_2 + 2k_1}{3S^3} e^{-2\int \eta dt}. \tag{28}$$

Equations (10)-(14) can be written in terms of H, σ and q as

$$\bar{p} = H^2(2q - 1) - \sigma^2 + \Lambda, \tag{29}$$

$$\rho + \Lambda = 3H^2 - \sigma^2, \tag{30}$$

$$\dot{\rho} + 3(\rho + \bar{p})H + \dot{\Lambda} = 4\eta\sigma^2. \tag{31}$$

From equations (29) and (30), we have

$$\dot{H} = -3H^2 + \frac{1}{2}(\rho - p)H + \frac{3}{2}\zeta H + \Lambda, \tag{32}$$

which is the Raychaudhuri equation for present distribution. This equation shows that the shear viscosity does not affect the mean expansion rate but bulk viscosity and positive Λ increase the rate of expansion.

3. Solution and Discussion

Over the years, it has been difficult and fascinating problem for cosmologists to explain the expansion history of the universe. To describe the dynamics of the universe, Hubble parameter H and deceleration parameter q are important observational quantities. The present value $H_0$of Hubble parameter sets the present time scale of expansion while q_0, the present day value of deceleration parameter tells us that the expansion of the present universe is accelerating rather than going to decelerate as expected before the supernovae of Ia observations. The law of variation for Hubble parameter was initially proposed by Berman [4] for FRW models that yields a constant value of deceleration parameter. Singh [32] studied cosmological models by assuming a functional relation between Hubble parameter H and average scale factor S. We assume a functional form for the Hubble parameter H which yields a model of the universe that describes an early deceleration followed by late time acceleration.

We consider

$$H = m + n\,coth\,t, \tag{33}$$

where m and n are constants. For this assumption, we obtain scale factor S, spatial volume V, expansion scalar θ and deceleration parameter q as

$$V = S^3 = (sinh\,t)^{3n} e^{3mt}, \tag{34}$$

$$\theta = 3(m + n\,coth\,t), \tag{35}$$

$$q = -1 + \frac{n}{(m\,sinh\,t + n\,cosh\,t)^2}\,. \tag{36}$$

We observe that spatial volume is zero and expansion scalar is infinite at $t = 0$, which shows that the universe starts evolving with zero volume at initial epoch with an infinite rate of expansion. At $t = 0$, $q = -1 + \frac{1}{n} > 0$ provided $0 < n < 1$and for $t = \infty$, $q = -1$. Therefore the model universe represents initial deceleration and late time accelerating expansion. The derived model can be utilized to describe the dynamics of the universe consistent with recent cosmological observations. In order to obtain analytical models, we assume specific form of coefficients of shear viscosity η and bulk viscosity ζ in the following models :

3.1 Model I

We assume that $\eta = \eta_0$ and $\zeta = \zeta_0$ are constants.

For this assumption, equations (26)-(28) become

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{k_2 + 2k_1}{3S^3} e^{-2\eta_0 t}, \tag{37}$$

$$\frac{\dot{B}}{B}=\frac{\dot{S}}{S}+\frac{k_2-k_1}{3S^3}e^{-2\eta_0 t}, \tag{38}$$

$$\frac{\dot{C}}{C}=\frac{\dot{S}}{S}-\frac{2k_2+2k_1}{3S^3}e^{-2\eta_0 t}. \tag{39}$$

Therefore

$$\sigma_1^1=\frac{k_2+2k_1}{3S^3e^{2\eta_0 t}}, \tag{40}$$

$$\sigma_2^2=\frac{k_2-k_1}{3S^3e^{2\eta_0 t}}, \tag{41}$$

$$\sigma_3^3=-\frac{2k_2+k_1}{3S^3e^{2\eta_0 t}} \tag{42}$$

and

$$\sigma=\frac{k}{S^3e^{6\eta_0 t}}\,. \tag{43}$$

For the model, matter density ρ, vacuum energy density Λ and shear scalar σ have the following expressions:

$$(1+\omega)\rho=2n\,csc\,h^2\,t-\frac{2k^2}{(sinh\,t)^{6n}e^{(6m+4\eta_0)t}}+3(m+n\,coth\,t)\zeta_0, \tag{44}$$

$$\Lambda=3(m+n\,coth\,t)^2-\frac{2n\,csc\,h^2 t}{1+\omega}+\frac{(1-\omega)k^2}{(1+\omega)(sinh\,t)^{6n}e^{(6m+4\eta_0)t}}-\frac{3(m+n\,coth\,t)\zeta_0}{1+\omega}, \tag{45}$$

$$\sigma=\frac{k}{(sinh\,t)^{6n}e^{(3m+2\eta_0)t}}. \tag{46}$$

We observe that matter density ρ, cosmological term Λ and shear σ all diverge at $t=0$. The model starts with a big-bang from its singular state at $t=0$ and continues to expand till $t=\infty$. As $t\to\infty$, $\rho\to\frac{3(m+n)\zeta_0}{1+\omega}$, $\Lambda\to 3(m+n)^2-\frac{3(m+n)\zeta_0}{1+\omega}$ and $\sigma\to 0$. We find that the effect of bulk viscosity is to increase the value of matter density and to decrease the value of vacuum energy density. The presence of bulk viscosity does not allow the matter density to become zero in infinitely for future. Also, at late times cosmological term Λ tends to a genuine cosmological constant. We also observe that our solution does not exactly tend to a deSitter universe on account of bulk viscosity. For the model

$$\frac{\sigma}{\theta}=\frac{k}{3(m+n\,coth\,t)(sinh\,t)^{3n}e^{(3m+2\eta_0)t}}. \tag{47}$$

We observe that anisotropy $\frac{\sigma}{\theta}$ decreases with time and tends to zero for $t=\infty$. Hence, the model approaches isotropy for large t. We notice that the anisotropy parameter decreases faster with time due to the presence of shear viscosity. Therefore, the shear viscosity plays an important role in the process of isotropization of large scale structure of the universe.

3.2 Model II

We propose the form of shear and bulk viscosities as

$$\eta = \frac{1}{\eta_0 + t} \tag{48}$$

and

$$\zeta = \frac{1}{\zeta_0 + t}, \tag{49}$$

where η_0 and ζ_0 are constants.

Equations (26)-(28) together with (48) yield

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{k_2 + 2k_1}{3S^3(\eta_0 + t)^2}, \tag{50}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{k_2 - k_1}{3S^3(\eta_0 + t)^2}, \tag{51}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{2k_2 + 2k_1}{3S^3(\eta_0 + t)^2}. \tag{52}$$

Therefore

$$\sigma_1^1 = \frac{k_2 + 2k_1}{3S^3(\eta_0 + t)^2}, \tag{53}$$

$$\sigma_2^2 = \frac{k_2 - k_1}{3S^3(\eta_0 + t)^2}, \tag{54}$$

$$\sigma_3^3 = -\frac{2k_2 + k_1}{3S^3(\eta_0 + t)^2} \tag{55}$$

and

$$\sigma = \frac{k}{S^3(\eta_0 + t)^2}. \tag{56}$$

Matter density ρ, vacuum density Λ and shear scalar σ are given by

$$(1+\omega)\rho = 2n\, csc\, h^2\, t - \frac{2k^2}{(sinh\, t)^{6n} e^{6mt} (\eta_0 + t)^4} + \frac{3(m + n\, coth\, t)}{(\zeta_0 + t)^4}, \tag{57}$$

$$\Lambda = 3(m + n\, coth\, t)^2 - \frac{2n\, csc\, h^2 t}{1+\omega} + \frac{(1-\omega)k^2}{(1+\omega)(sinh\, t)^{6n} e^{6mt} (\eta_0 + t)^4} - \frac{3(m + n\, coth\, t)}{(1+\omega)(\zeta_0 + t)}, \tag{58}$$

$$\sigma = \frac{k}{(sinh\, t)^{3n} e^{3mt} (\eta_0 + t)^2}. \tag{59}$$

We observe that the model has singularity at $t = 0$. It starts with a big-bang at $t = 0$. At $t = 0$, ρ, Λ and σ are all infinite whereas $\eta = \frac{1}{\eta_0}$ and $\zeta = \frac{1}{\zeta_0}$. In the limit of large times, ρ, η, ζ and σ become zero but $\Lambda \to (m+n)^2$. We find that cosmological term Λ is a decaying function of time and it approaches a small value at late times. Thus, solution tends to de Sitter universe with $H = \sqrt{\frac{\Lambda}{3}} = m + n$ for the large values of *t*. Viscous effect in the model vanishes at late times. For the model, anisotropy

$$\frac{\sigma}{\theta} = \frac{k}{3(m+n\,coth\,t)(sinh\,t)^{3n}e^{3mt}(\eta_0+t)^2}. \tag{60}$$

For the large values of *t*, $\frac{\sigma}{\theta} = 0$ implying that the model approaches isotropy at late times. The shear viscosity is found to be responsible for faster removal of initial anisotropies in the universe.

3.3 Model III

We assume the form of coefficient of shear viscosity given by Saha [28] as

$$\eta = 3\eta_0\frac{\dot{S}}{S},\ \eta_0 = \text{constant} \tag{61}$$

and bulk viscosity

$$\zeta = \zeta_0 + \zeta_1\frac{\dot{S}}{S} + \zeta_2\frac{\ddot{S}}{S}, \tag{62}$$

proposed by Mostafapoor and Grϕn [22], ζ_0, ζ_1 and ζ_2 being constants. Equations (26)-(28) together with (61) reduces to

$$\frac{\dot{A}}{A} = \frac{\dot{S}}{S} + \frac{k_2+2k_1}{3S^{3+6\eta_0}}, \tag{63}$$

$$\frac{\dot{B}}{B} = \frac{\dot{S}}{S} + \frac{k_2-k_1}{3S^{3+6\eta_0}}, \tag{64}$$

$$\frac{\dot{C}}{C} = \frac{\dot{S}}{S} - \frac{2k_2+2k_1}{3S^{3+3\eta_0}}. \tag{65}$$

We obtain

$$\sigma_1^1 = \frac{k_2+2k_1}{3S^{3+6\eta_0}}, \tag{66}$$

$$\sigma_2^2 = \frac{k_2-k_1}{3S^{3+6\eta_0}}, \tag{67}$$

$$\sigma_3^3 = -\frac{2k_2+k_1}{3S^{3+6\eta_0}} \tag{68}$$

and

$$\sigma = \frac{k}{S^{3+6\eta_0}}. \tag{69}$$

Coefficient of shear viscosity η, shear scalar σ and anisotropy parameter $\frac{\sigma}{\theta}$ for the model come out to be

$$\eta = 3\eta_0(m + n\,coth\,t), \tag{70}$$

$$\sigma = \frac{k}{(sinh\,t)^{(3+6\eta_0)n}e^{(3+6\eta_0)mt}}, \tag{71}$$

$$\frac{\sigma}{\theta}=\frac{k}{3(m+n\,coth\,t)(sinh\,t)^{(3+6\eta_0)n}e^{(3+6\eta_0)mt}}\,. \tag{72}$$

We observe that model starts with a big-bang from its singular state at $t=0$ with σ and η all infinite. At late times, $\sigma\to 0$ and $\eta\to 3\eta_0(m+n)$. Coefficient of shear viscosity tends to genuine constant for large values of *t*. For $t\to\infty$, $\frac{\sigma}{\theta}\to 0$. Therefore the model approaches isotropy at late times. We notice that presence of shear viscosity accelerates the process of isotropization.

We now discuss the different cases of bulk viscosity.

Case 1 : $\zeta_0\neq 0,\zeta_1=0$ and $\zeta_2=0$.

For this choice, we obtain matter density ρ, cosmological term Λ and coefficient of bulk viscosity ζ as

$$(1+\omega)\rho=2n\,csc\,h^2\,t-\frac{2k^2}{(sinh\,t)^{(6+12\eta_0)n}e^{(6+12\eta_0)mt}}+3(m+n\,coth\,t)\zeta_0\,, \tag{73}$$

$$\Lambda=3(m+n\,coth\,t)^2-\frac{2n\,csc\,h^2t}{1+\omega}+\frac{(1-\omega)k^2}{(1+\omega)(sinh\,t)^{(6+12\eta_0)n}e^{(6+12\eta_0)mt}}-\frac{3(m+n\,coth\,t)}{(1+\omega)}\zeta_0\,, \tag{74}$$

$$\zeta=\zeta_0. \tag{75}$$

The model has singularity at $t=\infty$. Matter density ρ and vacuum energy density Λ are infinite at the initial singularity. At late times, $\rho\to\frac{3(m+n)}{(1+\omega)}\zeta_0$ and $\Lambda\to 3(m+n)^2-\frac{3(m+n)}{(1+\omega)}\zeta_0$. Cosmological term Λ is decreasing function of time and it approaches a small positive value as time progresses. Due to bulk viscosity matter density does not vanish in the infinitely far future.

Case 2 : $\zeta_1\neq 0$ and $\zeta_0=\zeta_2=0$.

In this case, we obtain

$$(1+\omega)\rho=2n\,csc\,h^2\,t-\frac{2k^2}{(sinh\,t)^{(6+12\eta_0)n}e^{(6+12\eta_0)mt}}+3(m+n\,coth\,t)^2\zeta_1, \tag{76}$$

$$\Lambda=\frac{3(\omega+1-\zeta_1)(m+n\,coth\,t)^2}{1+\omega}-\frac{2n\,csc\,h^2t}{1+\omega}+\frac{(1-\omega)k^2}{(1+\omega)(sinh\,t)^{(6+12\eta_0)n}e^{(6+12\eta_0)mt}}, \tag{77}$$

$$\zeta=\zeta_1(m+n\,coth\,t). \tag{78}$$

The model has singularity at $t=\infty$. It starts evolving with a big-bang at $t=\infty$ with ρ,Λ and ζ all infinite. In the limit of large times (i.e. $t\to\infty$), $\rho\to\frac{3(m+n)^2}{(1+\omega)}\zeta_0$, $\Lambda\to\frac{3(m+n)^2(1+\omega-\zeta_1)}{(1+\omega)}$ and $\zeta\to\zeta_1(m+n)$. In the absence of bulk viscosity, the energy density tends to zero for $t\to\infty$. Cosmological term Λ and coefficient of bulk viscosity tend to a genuine constants for large values of *t*.

Case 3 : $\zeta_0 \neq 0, \zeta_1 \neq 0$ and $\zeta_2 = 0$.

For this assumption, we obtain

$$(1+\omega)\rho = 2n\,csc\,h^2\,t - \frac{2k^2}{(sinh\,t)^{(6+12\eta_0)n} e^{(6+12\eta_0)mt}} + 3(m + n\,coth\,t)[\zeta_0 + \zeta_1(m + n\,coth\,t)], \quad (79)$$

$$\Lambda = \frac{3(\omega+1-\zeta_1)(m+n\,coth\,t)^2}{1+\omega} - \frac{2n\,csc\,h^2 t}{1+\omega} + \frac{(1-\omega)k^2}{(1+\omega)(sinh\,t)^{(6+12\eta_0)n} e^{(6+12\eta_0)mt}} - \frac{3(m+n\,coth\,t)}{1+\omega}\zeta_0, \quad (80)$$

$$\zeta = \zeta_0 + \zeta_1(m + n\,coth\,t). \quad (81)$$

We observe that at $t = \infty$, ρ, Λ and ζ are all infinite. In the limit of large times (i.e. $t \to \infty$), $\rho \to \frac{3(m+n)}{(1+\omega)}[\zeta_0 + \zeta_1(m+n)], \Lambda \to \frac{3(m+n)^2(1+\omega-\zeta_1)}{(1+\omega)} - \frac{3(m+n)\zeta_0}{(1+\omega)}$ and $\zeta \to \zeta_0 + \zeta_1(m+n)$. We find that bulk viscosity increases the matter densityρ and decreases vacuum energy density Λ. We also observe that presence of bulk viscosity prevents the model to tend to a de Sitter universe and matter density to become negligible asymptotically. Coefficient of bulk viscosity tends to a genuine constant for large values of *t*.

Case 4 : $\zeta_0 \neq 0, \zeta_1 \neq 0$ and $\zeta_2 \neq 0$.

For this choice, we obtain ρ, Λ and ζ as

$$(1+\omega)\rho = 2n\,csc\,h^2\,t[2 - 3\zeta_2(m + n\,coth\,t)] - \frac{2k^2}{(sinh\,t)^{(6+12\eta_0)n} e^{(6+12\eta_0)mt}}$$
$$+3(m + n\,coth\,t)^2 . [\zeta_0 + \zeta_1(m + n\,coth\,t) + \zeta_2(m + n\,coth\,t)^2] \quad (82)$$

$$\Lambda = \frac{3(\omega + 1 - \zeta_1)(m + n\,coth\,t)^2}{1+\omega} + \frac{(1-\omega)k^2}{(1+\omega)(sinh\,t)^{(6+12\eta_0)n} e^{(6+12\eta_0)mt}}$$
$$- \frac{n\,csc\,h^2 t[2-3\zeta_2(m+n\,coth\,t)]}{1+\omega} - \frac{3(m+n\,coth\,t)[\zeta_0+\zeta_2(m+n\,coth\,t)^2]}{1+\omega} \quad (83)$$

$$\zeta = \zeta_0 + (m + n\,coth\,t)[\zeta_1 + \zeta_2(m + n\,coth\,t)] - \zeta_2 n\,csc\,h^2\,t. \quad (84)$$

We observe that the model starts with big-bang from its singular state $t = 0$ where ρ, Λ and ζ all diverge. As $t \to \infty$, $\rho \to \frac{3(m+n)}{(1+\omega)}[\zeta_0 + \zeta_1(m+n) + \zeta_2(m+n)^2], \Lambda \to \frac{3(m+n)^2(1+\omega-\zeta_1)}{(1+\omega)} - \frac{3(m+n)}{(1+\omega)}[\zeta_0 + \zeta_2(m+n)^2]$ and $\zeta \to \zeta_0 + (m+n)[\zeta_1 + \zeta_2(m+n)$. We find that the cosmological term Λ is very large at initial time and leads to small value at late times. On account of bulk viscosity, matter density ρ increases and vacuum energy density Λ decreases. Coefficient of bulk viscosity tends to a genuine constant for large values of *t*. Because of bulk viscosity, matter density does not become negligible and the model does not tend to a de Sitter universe for large values of *t*.

3.4 Model IV

We consider

$$\eta = 3\eta_0 \frac{\dot{S}}{S} \tag{85}$$

and

$$\zeta = \zeta_0 \rho, \tag{86}$$

where ζ_0 is constant [17]. For this assumption, we obtain ρ, Λ and ζ as

$$\rho = \frac{1}{1+\omega+3\zeta_0(m+n\coth t)}\left[2n\,csc\,h^2\,t - \frac{2k^2}{(\sinh t)^{(6+12\eta_0)n} e^{(6+12\eta_0)mt}}\right], \tag{87}$$

$$\Lambda = 3(m+n\coth t)^2 - \frac{2n\,csc\,h^2 t}{1+\omega+3\zeta_0(m+n\coth t)} + \frac{k^2(1-\omega-3\zeta_0(m+n\coth t))}{[1+\omega+3\zeta_0(m+n\coth t)]\left[(\sinh t)^{(6+12\eta_0)n} e^{(6+12\eta_0)mt}\right]}, \tag{88}$$

$$\rho = \frac{\zeta_0}{1+\omega+3\zeta_0(m+n\coth t)}\left[2n\,csc\,h^2\,t - \frac{2k^2}{(\sinh t)^{(6+12\eta_0)n} e^{(6+12\eta_0)mt}}\right], \tag{89}$$

This model also starts with a big-bang from its singular state $t \to 0$ with ρ, Λ and ζ all infinite. In the limit of large times, ρ and ζ become zero whereas $\Lambda \to 3(m+n)^2$. We find that cosmological term Λ is a decaying function of time and it approaches a small value at late times. We also observe that our model tends asymptotically to a de Sitter universe with $H = \sqrt{\frac{\Lambda}{3}} = m + n$ for large values of *t*.

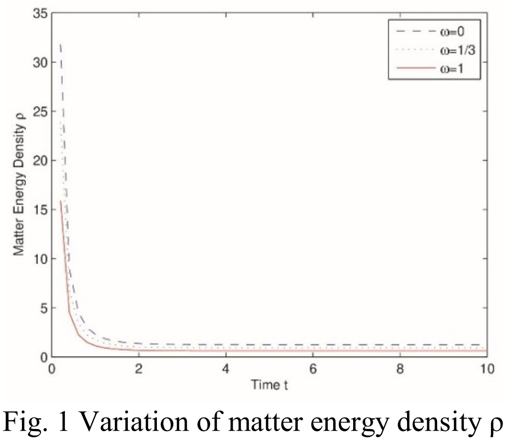

Fig. 1 Variation of matter energy density ρ with cosmic time t in Model I.

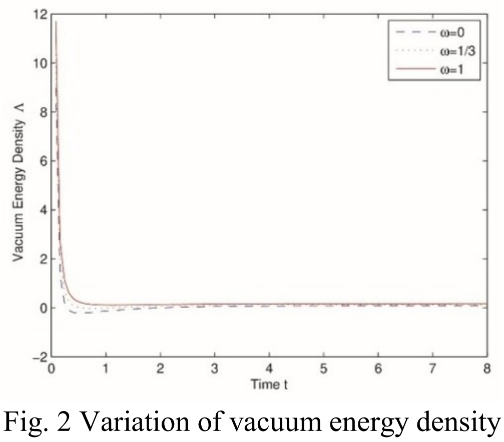

Fig. 2 Variation of vacuum energy density Λ with cosmic time t in Model I.

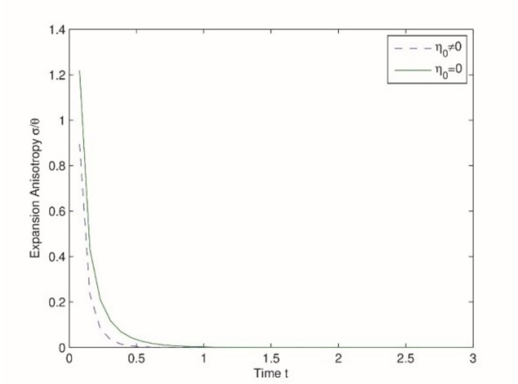

Fig. 3 Variation of expansion anisotropy σ/θ with cosmic time t in Model I.

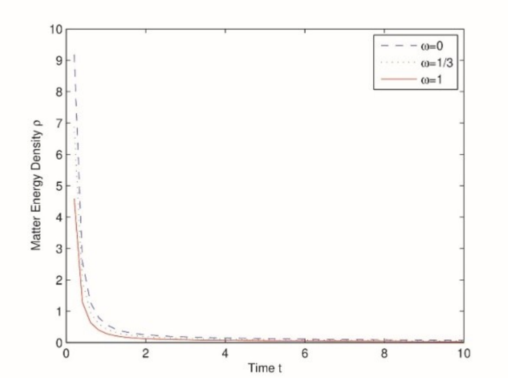

Fig. 4 Variation of matter energy density ρ with cosmic time t in Model II.

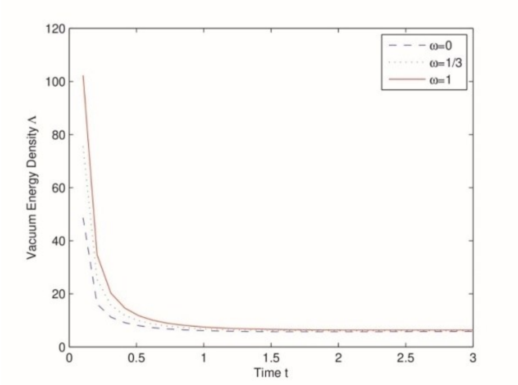

Fig. 5 Variation of vacuum energy density Λ with cosmic time t in Model II.

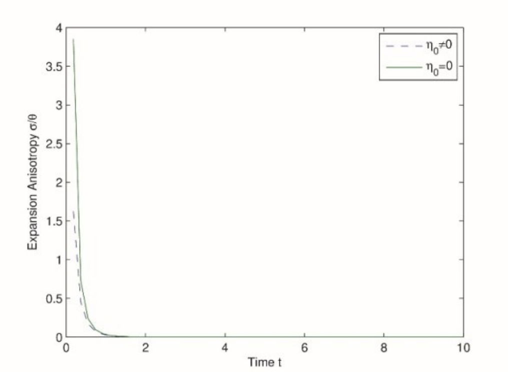

Fig. 6 Variation of expansion anisotropy σ/θ with cosmic time t in Model II.

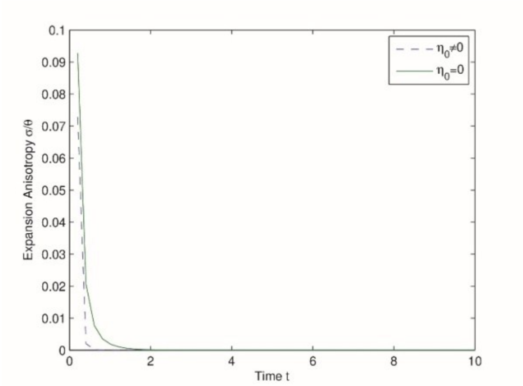

Fig. 7 Variation of expansion anisotropy σ/θ with cosmic time t in Model III.

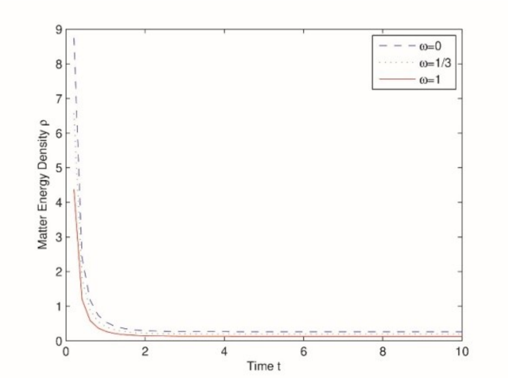

Fig. 8 Variation of matter energy density ρ with cosmic time t in model III (case 1).

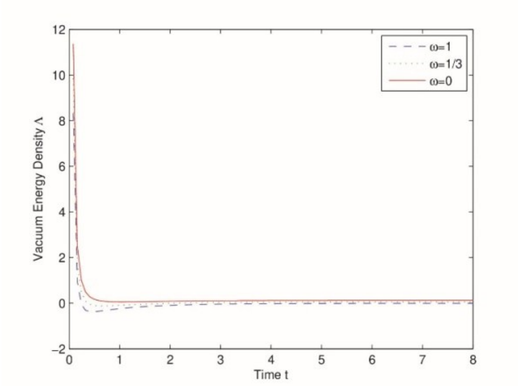

Fig. 9 Variation of vacuum energy density Λ with cosmic time t in model III (case 1).

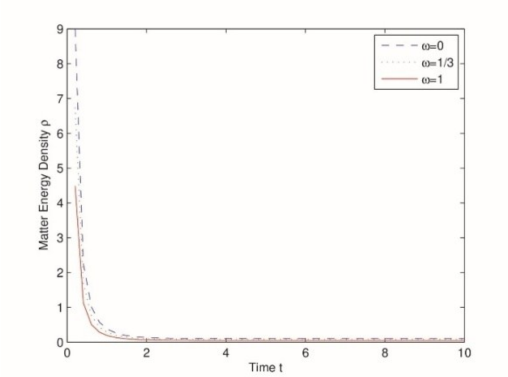

Fig. 10 Variation of matter energy density ρ with cosmic time t in model III (case 2).

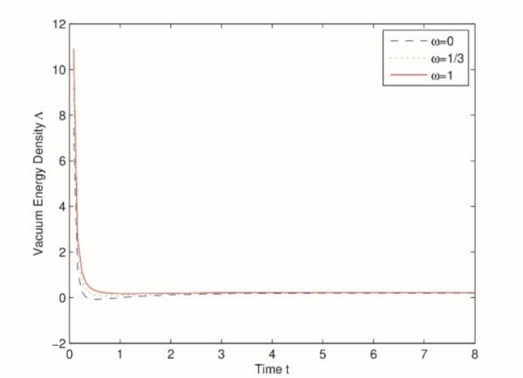

Fig. 11 Variation of vacuum energy density Λ with cosmic time t in model III (case 2).

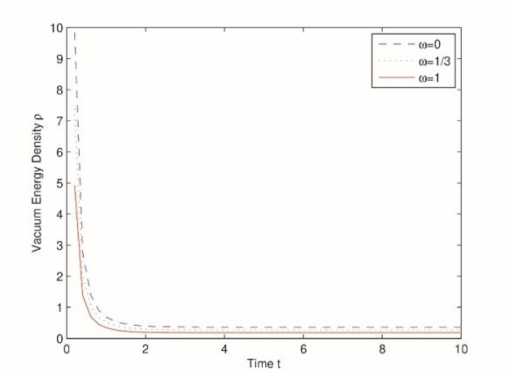

Fig. 12 Variation of matter energy density ρ with cosmic time t in model III (case 3).

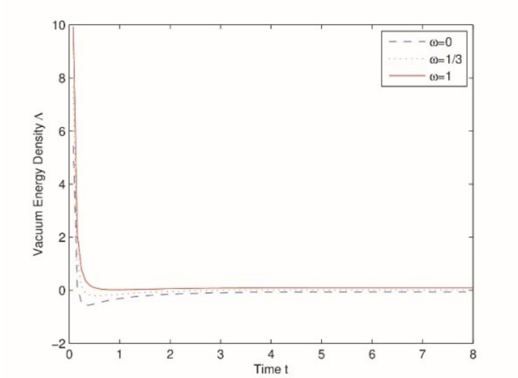

Fig. 13 Variation of vacuum energy density Λ with cosmic time t in model III (case 3).

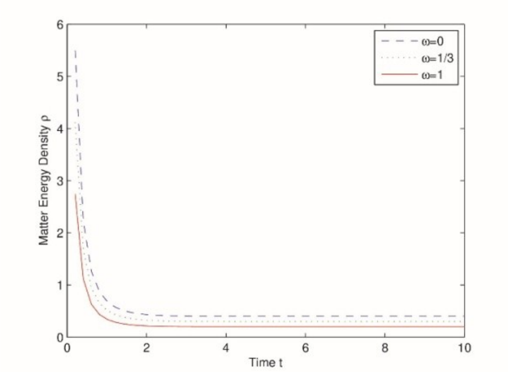

Fig. 14 Variation of matter energy density ρ with cosmic time t in model III (case 4).

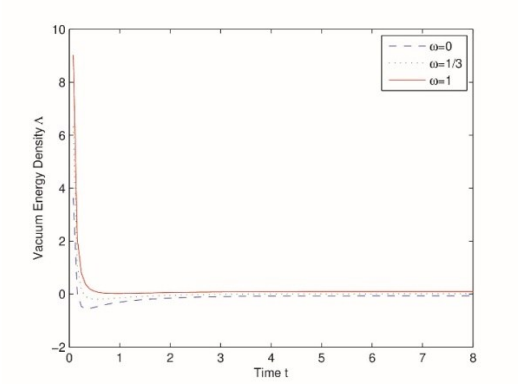

Fig. 15 Variation of vacuum energy density Λ with cosmic time t in model III (case 4).

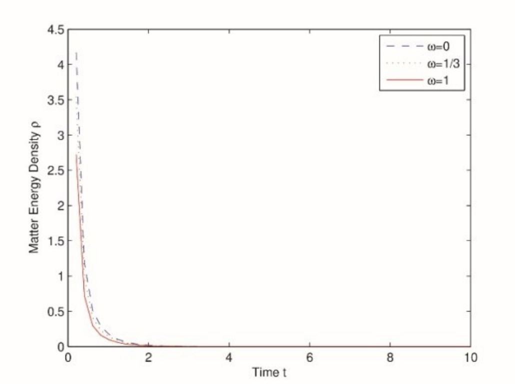

Fig. 16 Variation of matter energy density ρ with cosmic time t in model IV.

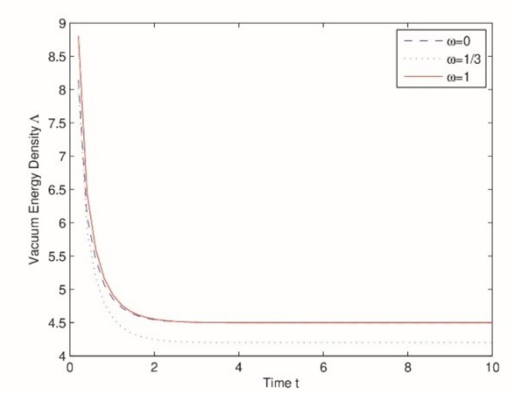

Fig. 17 Variation of vacuum energy density Λ with cosmic time t in model IV.

4. Conclusion

In this paper, cosmological models have been obtained by assuming a functional form for Hubble parameter which yields a model of the universe that represents initially decelerating and late time accelerating expansion. The vacuum energy density observed today falls below the value of the vacuum energy density predicted by quantum field theory by many order of magnitude. To explain the decay of the vacuum density, a number of dynamical models have been suggested in which cosmological term Λ varies with cosmic time t. These models give rise to an effective cosmological term which as long as the universe expands, decays from a huge value at initial times to the small value observed at present. We have discussed different forms of shear and bulk viscosity. We find that expansion anisotropy $\frac{\sigma}{\theta}$ in the model vanishes at late times. The shear viscosity is found to be responsible for faster removal of initial anisotropy in the universe. We observe that the proposed functional form for Hubble parameter H produces cosmological models that give the desired behavior of the universe. Variation of cosmological parameters ρ, Λ and σ/θ with cosmic time t in different models have been shown by graphical representations (Figs. 1-17).

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