

CHEMICAL REACTION EFFECTS ON RADIATIVE MHD MIXED CONVECTION FLOW FROM A VERTICAL FLAT PLATE WITH OHMIC HEATING AND VISCOUS DISSIPATION IN SLIP FLOW REGIME

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Abstract: In this paper a steady mixed convection MHD flow of an incompressible and electrically conducting viscous fluid past an infinite vertical porous plate having constant suction under ohmic heating and viscous dissipation effects in the presence of chemical reaction with slip boundary conditions is examined. Also the flow is assumed to be having thermal radiation effect. Approximate solutions of governing equations for velocity, temperature and concentration are obtained and the effects of various parameters which appeared during non dimensionalization process are studied through the graphs made using Matlab software packages. Further the variation in skin friction, Nusselt number and Schmidt number along different flow parameters are also discussed with the help of graphs.

Keywords: MHD, Chemical reaction, Radiation, Mixed convection, Slip flow.

2010 Mathematical Subject Classification: 76A05, 76Dxx, 76E06, 76R50, 76W05, 80A32

1. Introduction

The motivation behind the study of MHD flow and heat transfer is its practical applications in the field of engineering and technology. Numerous researchers have contributed towards the different problems of MHD flow under thermal radiation effects due to its importance in industrial, geophysical and astrophysical sciences. Polymer production, manufacturing of ceramics, cooling of nuclear reactors, underground energy transport, high speed plasma wind are some of the examples of the practical importance of radiative MHD flows. Keeping in view of such varied applications, Cramer et al. [6] and Shercliff [22] has introduced the heat and mass transfer of MHD flows along vertical plates in the presence of magnetic field. Gribben [9] studied the magnetohydrodynamics boundary layer flows in the presence of pressure gradients. Pop [19] presented his studies

on unsteady hydromagnetic free convection flow from an infinite vertical porous plate using suction to be periodic. Further the Soundalgekar [23] investigated the fluctuating MHD flows with variable suction past an infinite wall. In earlier times Cogley et al. [5] studied the radiation effects through non gray fluids in equilibrium system. Hossain and Takhar [15] studied radiation effect on mixed convection flow past a vertical plate with surface temperature to be uniformed. Further V. Rajesh and Vijay [27] examined the mass transfer effects along radiation effects on MHD free convection flow along the exponentially accelerated vertical plate with variable temperature whereas Hakeem et al. [16] studied the radiation effects of an oscillatory flow of highly viscous free convective flow bounded by infinite plate. Also Takhar et al. [26] and Bestman et al. [2] formulated the solutions for radiation effects on MHD free convection flow past a semi infinite vertical plate and in rotating fluids. Further the results of radiation effects on an unsteady MHD convective heat and mass transfer flow past a semi infinite vertical permeable moving plate embedded in porous medium is studied by Ramachandra et al. [21]. Also Garg et al. [10] presented his studies on radiative free convection and mass transfer flow for an accelerated vertical plate in the presence of transverse magnetic field. Further Garg [11] studied the radiation effects of free convection flow on moving vertical porous plate with variable temperature.

Besides thermal radiation effects, the heat and mass transfer due to chemical reaction effects also play important role in many industrial processes involving chemical formulations and the designing of chemical processing equipments. Chambre et al. [4] presented his studies on the diffusion of chemically reactive species in laminar boundary layer flow developed in a hydrodynamic flow field. Sudheer et al. [24] studied the chemical reaction effects along with radiation effects on free convective flow through porous medium with variable suction and uniform magnetic field. Al-Odat et al. [1] observed the influence of chemical reaction on transient MHD free convection over a moving vertical plate.

The ohmic heating and viscous dissipation effects are other interesting aspects in the heat transfer of MHD free convective flow problems. One can find major applications of ohmic heating in the food processing industries and that of viscous dissipation in micro channels and micro tubes where it becomes significant due to high velocity gradients. Gebhart [13] and Pantokratoras [20] studied the viscous dissipation effects in natural convection along heated vertical plate. Also Hossain [14] worked on ohmic heating effects on MHD free convection flow past infinite plate having variable temperature. Further Chen [7] contributed to combined heat and mass transfer of MHD free convective flow adjacent to vertical surface with ohmic heating and viscous dissipation. Chaudhary et al. [8] investigated the Ohmic heating and viscous dissipation along with radiation effects on heat transfer in MHD mixed convection flow and mass transfer past an infinite vertical plate in the absence of porous medium. Recently Babu and Reddy [3] obtained the approximate solutions for mass transfer effects on MHD mixed convective flow from a vertical surface through porous medium with ohmic heating and viscous dissipation.

In the above studies, slip flow effect is not considered as in many practical situations the particle adjacent to the surface does not acquire the surface velocity which causes the slip along the surface. Such effects are not negligible. So taking into account the slip flow situations, Morques et al. [18] obtained the solutions for the MHD oscillatory flow of viscous fluid in a planner channel with slip conditions and Hayat et al. [17] studied the slip effects on second grade fluid past stretching sheet in porous medium. Further Singh et al. [25] studied the effect of slip velocity on MHD oscillatory flow through porous medium in a channel. Also Garg et al. [12] investigated the radiation effects on viscoelastic MHD oscillatory forced convection flow in a vertical channel with slip conditions.

In this paper we have aimed on viscous dissipation and ohmic heating effects on radiative MHD mixed convection flow past an infinite vertical plate under slip flow conditions. Further chemical reaction effects are also taken into account in the concentration equation.

2. Mathematical formulation

A two dimensional steady mixed convection flow of an incompressible and electrically conducting viscous fluid along an infinite vertical porous plate with constant suction velocity is considered in such a way that the x' -axis is taken along the plate and y' -axis is normal to it. Also transverse magnetic field of constant magnitude B_0 is applied along y' -axis. Since the plate is infinite in length so that all the physical variables are independent of x' -co ordinate. Here u' and v' are taken as components of velocity along the plate and normal to plate respectively. Also the magnetic Reynolds number is considered too small to neglect the induced magnetic field. Further the flow is assumed under chemical reaction and radiation effects along with viscous dissipation and ohmic heating taking slip conditions at boundary. The governing equations for conservation of mass, momentum, energy and concentration with above stated assumptions are given by

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v^0(\text{constant}) \quad (1)$$

$$v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty) + g\beta'(C' - C_\infty) - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu}{K'} u' \quad (2)$$

$$v' \frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 - \frac{1}{\rho C_p} \frac{\partial Q'}{\partial y'} + \frac{\sigma B_0^2 u'^2}{\rho C_p} \quad (3)$$

$$v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K_1(C' - C_\infty) \quad (4)$$

along with slip boundary conditions as

$$u' = L_1 \frac{\partial u'}{\partial y'}, T' = T_w, C' = C_w \text{ at } y' = 0 \quad \text{where } L_1 = \left\langle \frac{2-r_1}{r_1} \right\rangle L$$

$$u' \rightarrow 0, T' \rightarrow \infty, C' \rightarrow \infty \text{ as } y' \rightarrow \infty \quad (5)$$

Here $v^0, g, \beta, \beta', \nu, K, \sigma, \kappa, \rho, C_p, \mu, Q', T', T_\infty, C', C_\infty, D, K_1, L_1, r_1$

respectively are suction velocity, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of expansion of concentration, kinematic coefficient of viscosity, permeability of medium, electrical conductivity, thermal conductivity, fluid density, specific heat at constant pressure, viscosity of fluid, radiative flux, temperature of the fluid near plate, free stream temperature, species concentration, free stream species concentration, chemical molecular diffusivity, chemical reaction constant, mean free path, the Maxwell's reflexion coefficient.

The fluid under considered is optically thin having relatively low density, so the radiative hear flux given by Cogley et al. [5] is

$$\frac{\partial Q'}{\partial y'} = 4(T' - T_\infty)I' \quad (6)$$

where $I' = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T'} d\lambda$, $e_{b\lambda}$ is the Planck's function and $K_{\lambda w}$ is the absorption coefficient

On introducing the following non-dimensional quantities and parameters as

$$y = \frac{y'v^0}{v}, \quad u = \frac{u'}{v^0}, \quad T = \frac{T' - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C_\infty}{C_w - C_\infty}, \quad M = \frac{\sigma B_0^2 v}{\rho v^0^2},$$

$$K = \frac{K'v^0^2}{v^2}, \quad Gr = \frac{g\beta v(T_w - T_\infty)}{v^0^3}, \quad Gm = \frac{g\beta' v(C_w - C_\infty)}{v^0^3}, \quad Pr = \frac{\mu C_p}{\kappa},$$

$$E = \frac{v^0^2}{C_p(T_w - T_\infty)}, \quad R = \frac{4vI'}{\rho C_p v^0^2}, \quad Sc = \frac{v}{D}, \quad h = \frac{L_1 v^0}{v}, \quad k_1 = \frac{K_1 v^2}{D v^0^2} \quad (7)$$

Equations (2) - (5) reduces to

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left(M + \frac{1}{K}\right)u + GrT + GmC = 0 \quad (8)$$

$$\frac{\partial^2 T}{\partial y^2} + Pr \frac{\partial T}{\partial y} - RPrT + PrE \left(\frac{\partial u}{\partial y}\right)^2 + PrEMu^2 = 0 \quad (9)$$

$$\frac{\partial^2 C}{\partial y^2} + Sc \frac{\partial C}{\partial y} - k_1 C = 0 \quad (10)$$

along with corresponding boundary conditions as

$$u = h \frac{\partial u}{\partial y}, \quad T = 1, \quad C = 1 \quad \text{at } y = 0$$

$$u \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{at } y \rightarrow \infty \quad (11)$$

M is the magnetic number, K is the permeability parameter, Gr is the thermal Grashof number, Gm is the mass Grashof number, Pr is the Prandtl number, R is the radiation parameter, E is the Eckert number, k_1 is the chemical reaction parameter, Sc is the Schmidt number and h is slip flow parameter.

3. Solution of the problem

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we assume the solution for velocity, temperature and concentration in the powers of E (Eckert number). This is possible as $E \ll 1$ for incompressible fluid flow which physically interprets that the main flow is dominated by Joules dissipation. Thus the solution can be represented as

$$\begin{aligned} u(y) &= u_0(y) + Eu_1(y) + O(E^2) \\ T(y) &= T_0(y) + ET_1(y) + O(E^2) \\ C(y) &= C_0(y) + EC_1(y) + O(E^2) \end{aligned} \quad (12)$$

Using equations (12) in equations (8) - (11), we get

$$\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - \left(M + \frac{1}{K}\right)u_0 + GrT_0 + GmC_0 = 0 \quad (13)$$

$$\frac{\partial^2 T_0}{\partial y^2} + Pr \frac{\partial T_0}{\partial y} - RPrT_0 = 0 \quad (14)$$

$$\frac{\partial^2 C_0}{\partial y^2} + Sc \frac{\partial C_0}{\partial y} - k_1 C_0 = 0 \quad (15)$$

With corresponding boundary conditions as

$$\begin{aligned} u_0 &= h \frac{\partial u_0}{\partial y}, T_0 = 1, C_0 = 1 \text{ at } y = 0 \\ u_0 &\rightarrow 0, T_0 \rightarrow 0, C_0 \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \quad (16)$$

and

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - \left(M + \frac{1}{K}\right)u_1 + GrT_1 + GmC_1 = 0 \quad (17)$$

$$\frac{\partial^2 T_1}{\partial y^2} + Pr \frac{\partial T_1}{\partial y} - RPrT_1 + Pr \left(\frac{\partial u_0}{\partial y}\right)^2 + Mu_0^2 = 0 \quad (18)$$

$$\frac{\partial^2 C_1}{\partial y^2} + Sc \frac{\partial C_1}{\partial y} - k_1 C_1 = 0 \quad (19)$$

With corresponding boundary conditions as

$$\begin{aligned} u_1 &= h \frac{\partial u_1}{\partial y}, T_1 = 0, C_1 = 0 \text{ at } y = 0 \\ u_1 &\rightarrow 0, T_1 \rightarrow 0, C_1 \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \quad (20)$$

On solving the Eqs. (13) - (15) and (17) - (19) along with their respective boundary conditions (16) and (20), we get

$$u_0(y) = A_1 [b_1 e^{-a_5 y} - e^{-a_1 y}] + A_2 [b_2 e^{-a_5 y} - e^{-a_3 y}] \quad (21)$$

$$T_0(y) = e^{-a_1 y} \quad (22)$$

$$C_0(y) = e^{-a_3 y} \quad (23)$$

$$u_1(y) = A_{17}e^{-a_5 y} - A_{10}e^{-a_1 y} + A_{11}e^{-2a_1 y} + A_{12}e^{-2a_3 y} + A_{13}e^{-2a_5 y} - A_{14}e^{-a_6 y} + A_{15}e^{-a_7 y} - A_{16}e^{-a_8 y} \quad (24)$$

$$T_1(y) = A_9e^{-a_1 y} - A_3e^{-2a_1 y} - A_4e^{-2a_3 y} - A_5e^{-2a_5 y} + A_6e^{-a_6 y} - A_7e^{-a_7 y} + A_8e^{-a_8 y} \quad (25)$$

$$C_1(y) = 0 \quad (26)$$

On neglecting higher powers of E, complete solutions for velocity, temperature and concentration fields can be written as

$$u(y) = A_1[b_1e^{-a_5 y} - e^{-a_1 y}] + A_2[b_2e^{-a_5 y} - e^{-a_3 y}] + E[A_{17}e^{-a_5 y} - A_{10}e^{-a_1 y} + A_{11}e^{-2a_1 y} + A_{12}e^{-2a_3 y} + A_{13}e^{-2a_5 y} - A_{14}e^{-a_6 y} + A_{15}e^{-a_7 y} - A_{16}e^{-a_8 y}] \quad (27)$$

$$T(y) = e^{-a_1 y} + E[A_9e^{-a_1 y} - A_3e^{-2a_1 y} - A_4e^{-2a_3 y} - A_5e^{-2a_5 y} + A_6e^{-a_6 y} - A_7e^{-a_7 y} + A_8e^{-a_8 y}] \quad (28)$$

$$C(y) = e^{-a_3 y} \quad (29)$$

The validity and correctness of the analysis:

Case I: When $K_1 = 0$, $h = 0$ and without porous medium i.e. with no slip, no chemical reaction and in the absence of porous medium, we get the results obtained by Chaudhay et al. [18] from the Eqs (27) – (29).

Case II: When $h = 0$ and $K_1 = 0$ i.e. with no slip and no chemical reaction, we get the results as studied by Babu and Reddy [3].

Some important physical parameters for the boundary layer flow are given as:

Skin friction coefficient

The skin friction which is the measure of drag at the plate derived from velocity field (27) as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = A_1[-a_5 b_1 + a_1] + A_2[-a_5 b_2 + a_3] + E(-a_5 A_{17} + a_1 A_{10} - 2a_1 A_{11} - 2a_3 A_{12} - 2a_5 A_{13} + a_6 A_{14} - a_7 A_{15} + a_8 A_{16}) \quad (30)$$

Nusselt Number

The Nusselt number represents the measure of rate of heat transfer is derived from temperature field (28) as

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = a_1 + E(a_1 A_9 - 2a_1 A_3 - 2a_3 A_4 - 2a_5 A_5 + a_6 A_6 - a_7 A_7 + a_8 A_8) \quad (31)$$

Sherwood Number

The Sherwood number represents the measure of rate of mass transfer at the plate is derived from concentration field (29) as

$$Sh = - \left(\frac{\partial C}{\partial y} \right)_{y=0} = a_3 \quad (32)$$

4. Results and Discussions

In this section, we have assessed the various parametric effects viz. Prandtl number (Pr), Schmidt number (Sc), thermal Grashoff number (Gr), mass Grashoff number (Gm), permeability parameter (K), magnetic number (M), radiation parameter (R), chemical reaction parameter (k_1) and slip flow parameter (h) on velocity profiles, temperature profiles and concentration profiles through Figs. 1-12. Also while studying one parametric effect, others are kept constant.

In Figs. 1- 4, it is observed that velocity profiles have decreasing pattern with the increase in the parameter values of Pr, Sc, R and M. However the thermal Grashof number, mass Grashof number, Permeability of porous medium has adverse effects on velocity profiles showing increasing behaviour in flow pattern with increasing the values of Gr, Gm and K as observed in Figs. 5-7. But from the Figs.1-9, it is also depicted that there is high tides of velocity near the plate and as we move far away from plate it shows decreasing behaviour approaching to stationary position. Also the chemical reaction effects (k_1) and slip flow effects (h) are studied through the Figs. 8 and 9 which indicate that velocity profiles has increasing pattern with the increase of slip flow parameter and shows the decreasing behaviour with chemical reaction parameter. Also chemical reaction effect and slip flow effect is most significant near plate, thereafter goes on decreasing on moving away from plate. In the Figs. 10 and 11, we have summarized the parametric effects of permeability of porous medium and chemical reaction on temperature profiles. A small increment in the behaviour of temperature profiles with permeability parameter (K) is observed which is being concluded in the Fig. 10 and a small decrement in temperature profiles is seen with the increasing values of chemical reaction parameter (k_1) as shown in the Fig. 11. In Fig. 12 the chemical reaction parameter (k_1) effect on concentration profiles has been concluded. It is noticed that there is a drop in species concentration with increase in k_1 values.

Also the important parametric effects on skin friction, Nusselt number and Sherwood number as calculated in Eqs. (30) - (32) has been studied through the Figs. 13-18.

In Figs. 13-15, skin friction coefficient variation with different parameters against radiation parameter (R) has been plotted. It is observed from the Fig. 13 that the increase in both the Prandtl number and Schmidt number decreases the skin friction at the plate. This observation is physically true in the sense that Prandtl number increase makes the fluid thick and retards the fluid velocity and hence retards the drag at the plate. Fig. 14 indicates that drag at the plate is increased with increase of permeability parameter and decreased with the rise in magnetic number. However the skin friction is reduced at the

plate with the increase of slip parameter and the chemical reaction parameter which is presented through Fig. 15. Figures 16 and 17 represent the variation of Nusselt number V_s radiation parameter (R) with different parameters like slip flow parameter (h), chemical reaction parameter and permeability parameter (K). The rate of heat transfer decreases with the increase in K as indicated in the Fig. 16. Also it is concluded from Fig. 17 that the Nusselt number increases with increase in slip flow parameter (h) and the chemical reaction parameter (k_1). The effect of radiation parameter on Nusselt number can also be observed from Figs. 16-17, which shows that the increase in radiation parameter value accelerates the heat transfer. The variation in Sherwood number with Schmidt number and chemical reaction parameter has been presented through the Fig. 18 which shows that increase in the values of both the parameters increases the rate of mass transfer at the plate.

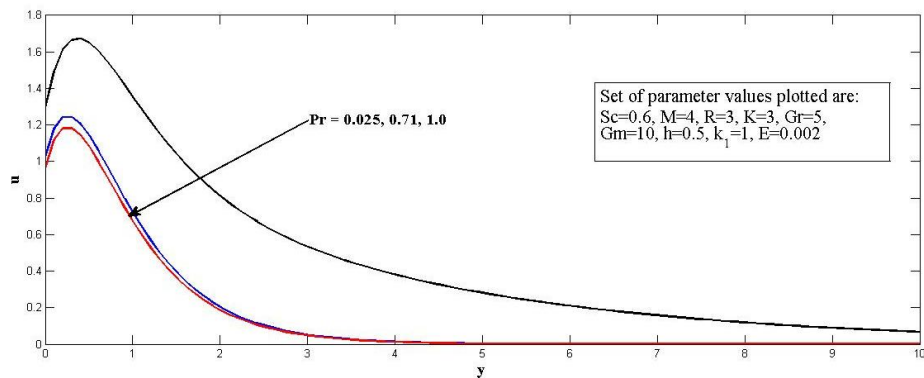


Fig.1. Effects of Prandtl number (Pr) on velocity profiles

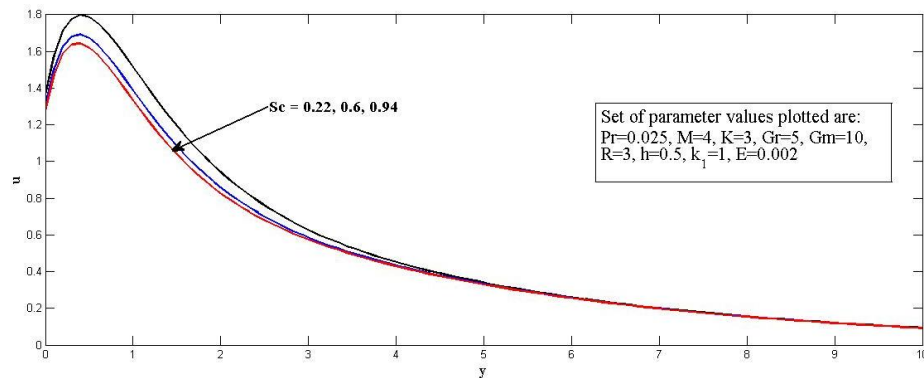


Fig.2. Effects of Schmidt number (Sc) on velocity profiles

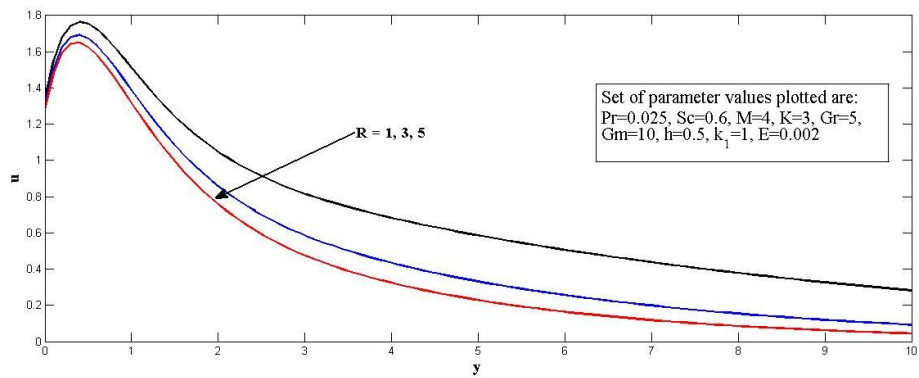


Fig.3. Effects of radiation parameter (R) on velocity profiles

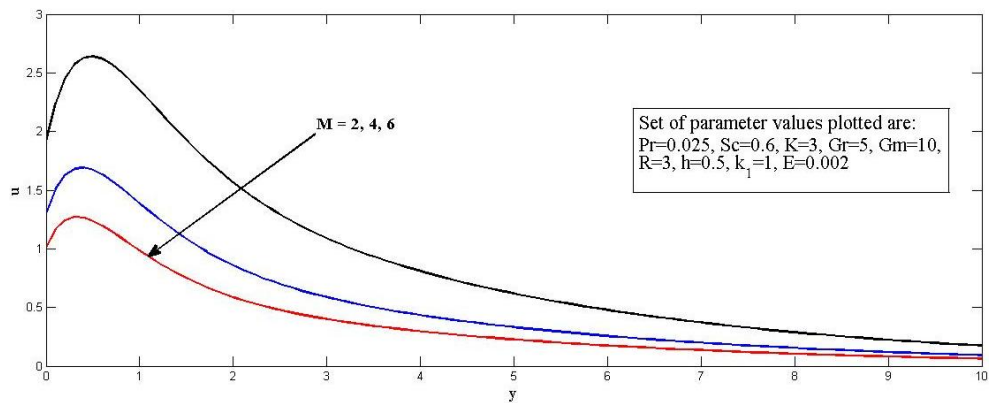


Fig.4. Effects of magnetic number (M) on velocity profiles

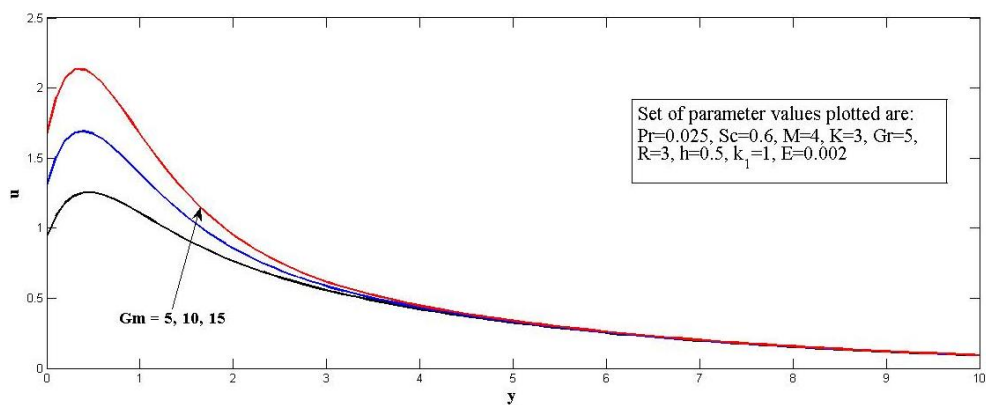


Fig.5. Effects of mass Grashof number (Gm) on velocity profiles

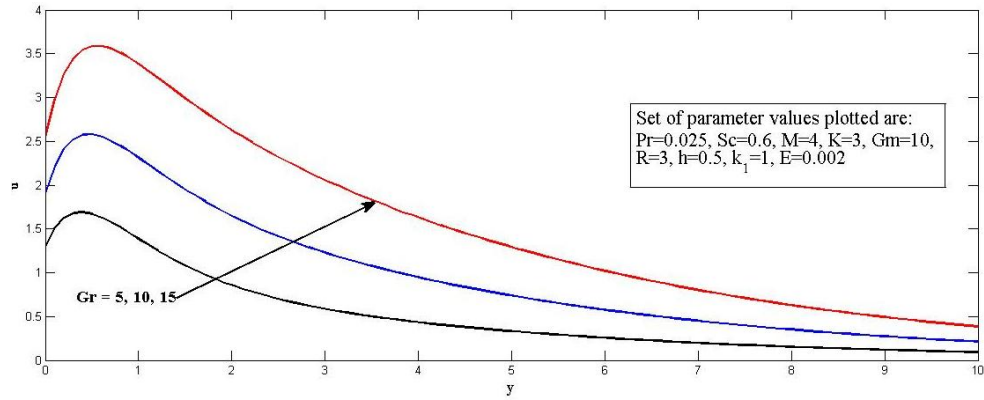


Fig.6. Effects of thermal Grashof number (Gr) on velocity profiles

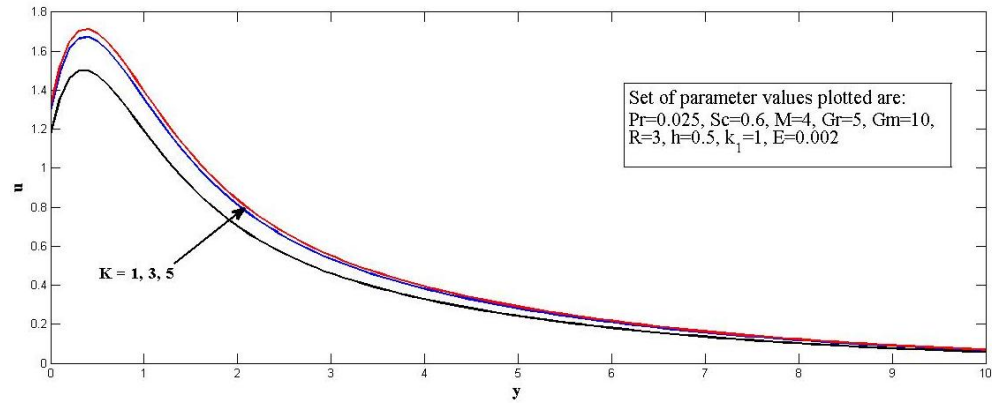


Fig.7. Effects of permeability parameter (K) on velocity profiles

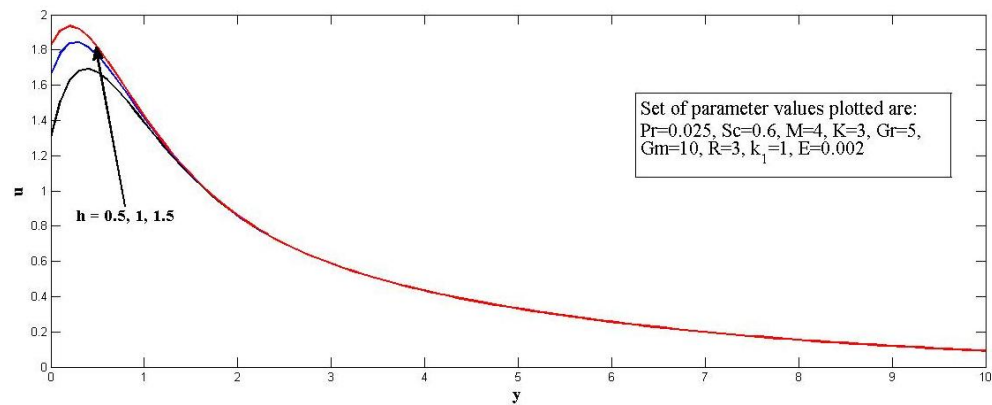


Fig.8. Effects of slip flow parameter (h) on velocity profiles

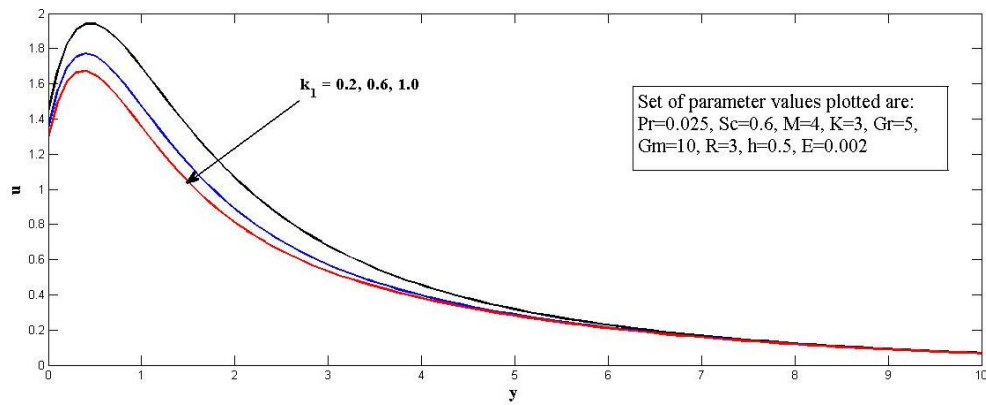


Fig.9. Effects of chemical reaction parameter (k_1) on velocity profiles

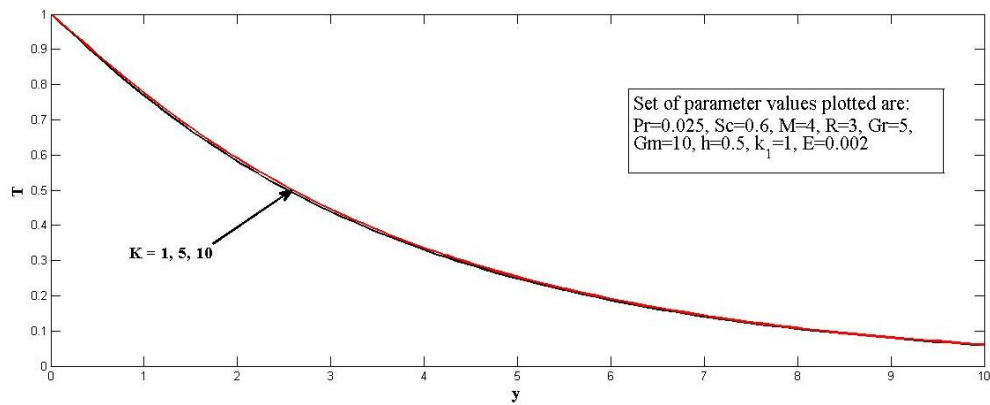


Fig.10. Effects of permeability parameter (K) on temperature profiles

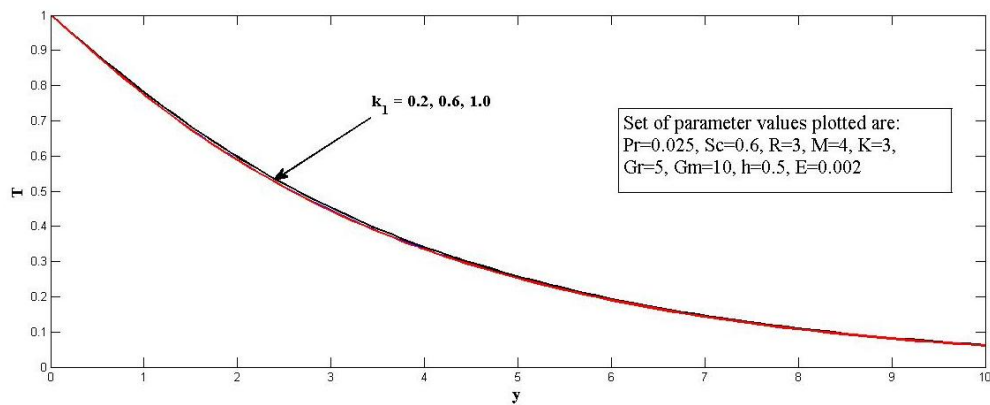


Fig.11. Effects of chemical reaction parameter (k_1) on temperature profiles

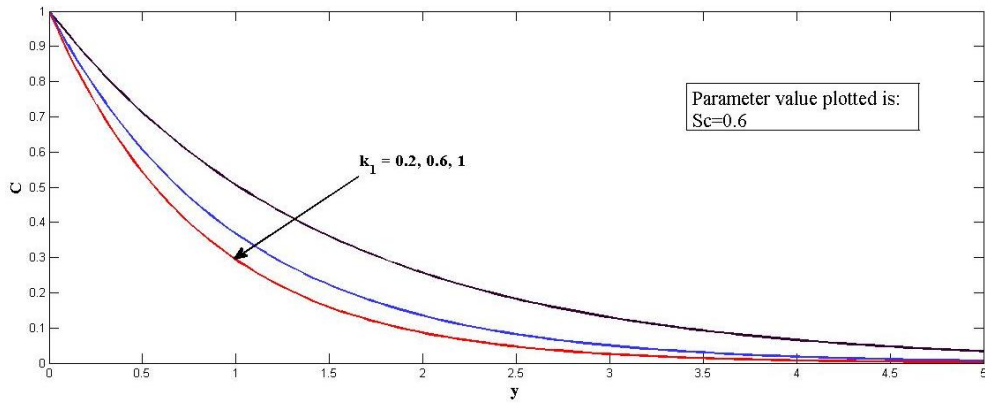


Fig.12. Effects of chemical reaction parameter (k_1) on concentration profiles

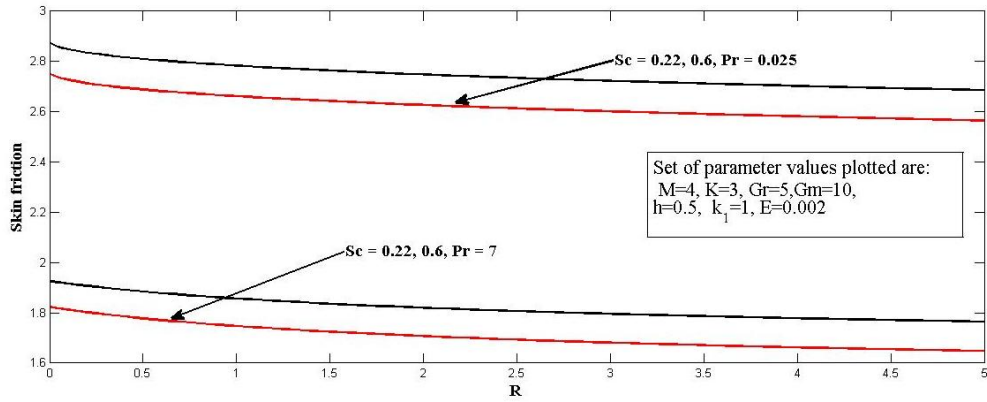


Fig.13. Variation of Skin friction with Schmidt number (Sc) and Prandtl number (Pr)

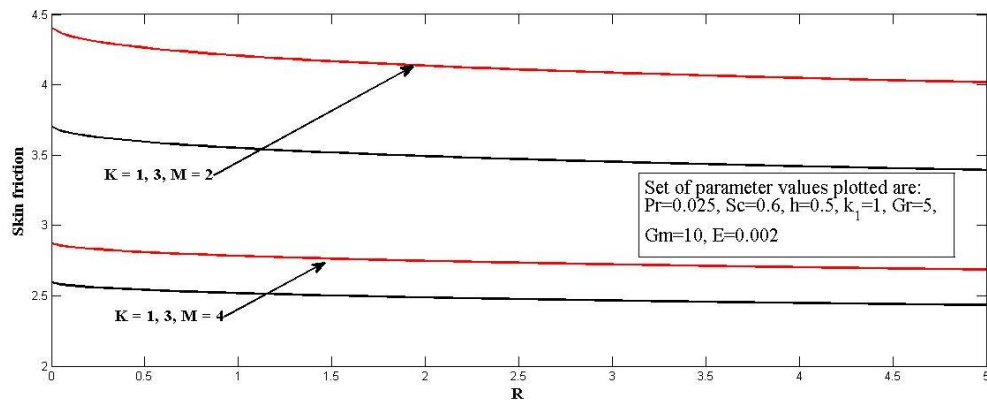


Fig.14. Variation of Skin friction with permeability parameter (K) and magnetic parameter (M)

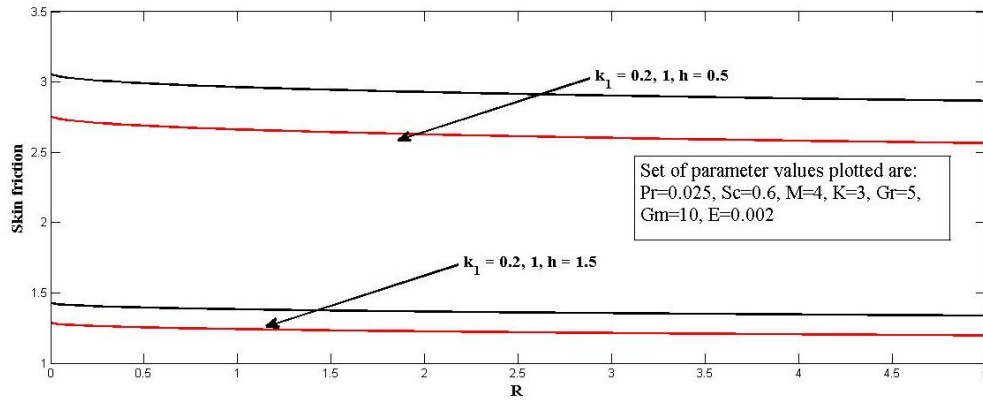


Fig.15. Variation of Skin friction with slip flow parameter (h) and chemical reaction parameter (k_1)

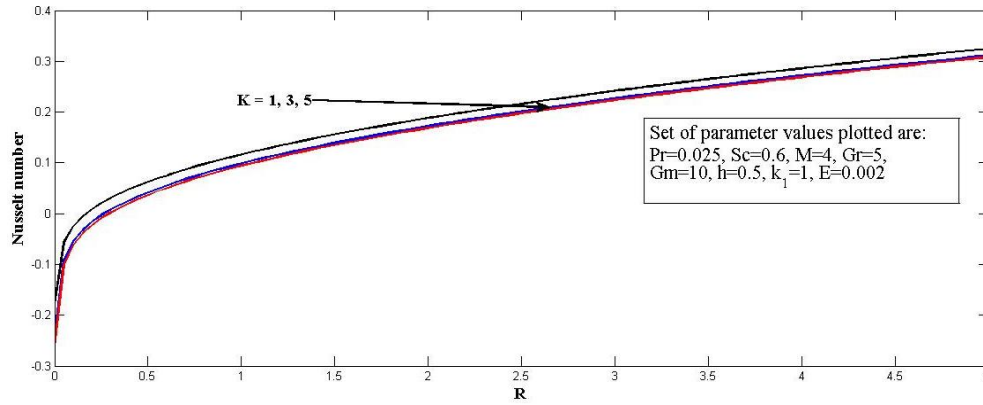


Fig.16. Variation of Nusselt number (Nu) with permeability parameter (K)

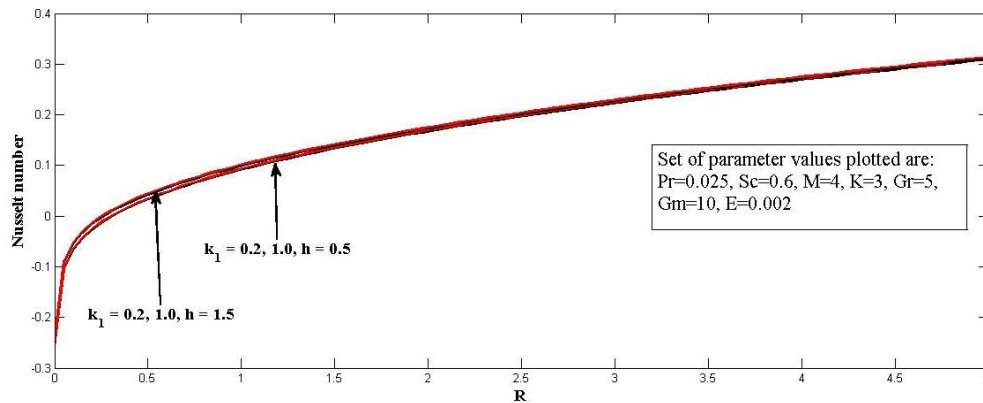


Fig.17. Variation of Nusselt number (Nu) with slip flow parameter (h) and chemical reaction parameter (k_1)

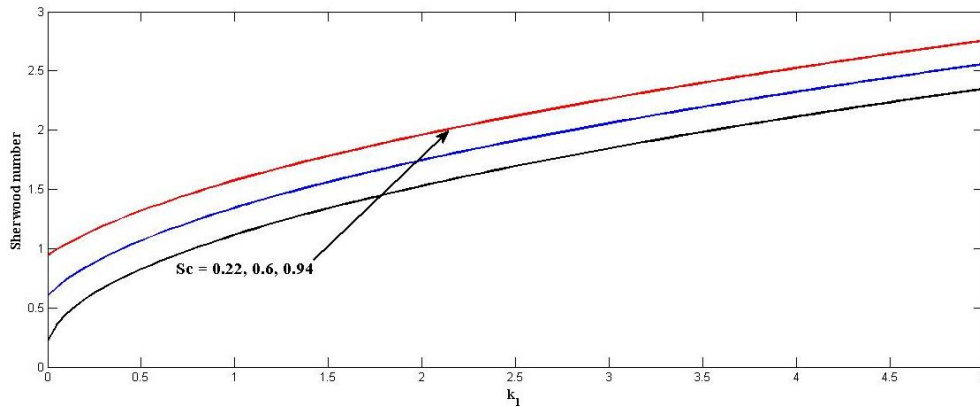


Fig.18. Variation of Sherwood number (Sh) with chemical reaction parameter (k_1) and Schmidt number (Sc) Conclusion

The results discussed in earlier section for the radiative MHD mixed convection flow from an infinite vertical plate under chemical reaction effects with ohmic heating and viscous dissipation on the account of slip boundary conditions are concluded as

- There is significant effect of permeability parameter on velocity profiles
- Prandtl number increase causes reduction in velocity patterns of fluid under slip flow which is due to increased viscosity of fluid that results into the thickness of fluid. This reduces the fluid velocity.
- The presence of radiation effect and magnetic field causes the reduction in fluid velocity. However the magnetic parameter is more dominant on fluid velocity.
- The chemical reaction process significantly slower down the velocity of fluid whereas the slip at the plate accelerates the flow.
- The species concentration is reduced in the presence of chemical reaction and molecular diffusion whereas it has adverse effects on rate of mass transfer at the plate
- Also it is found that the increase in slip flow parameter and chemical reaction parameter enhances the rate of heat transfer in the vicinity of the plate.

Appendix

$$a_1 = \frac{Pr + \sqrt{Pr^2 + 4RPr}}{2}$$

$$a_2 = \frac{-Pr + \sqrt{Pr^2 + 4RPr}}{2}$$

$$a_3 = \frac{Sc + \sqrt{Sc^2 + 4k_1}}{2}$$

$$a_4 = \frac{-Sc + \sqrt{Sc^2 + 4k_1}}{2}$$

$$a_5 = \frac{1 + \sqrt{1 + 4\left(M + \frac{1}{K}\right)}}{2}$$

$$a_5' = \frac{-1 + \sqrt{1 + 4\left(M + \frac{1}{K}\right)}}{2}$$

$$a_6 = a_5 + a_1,$$

$$a_7 = a_3 + a_1$$

$$a_8 = a_5 + a_3$$

$$\begin{aligned}
b_1 &= \frac{1+ha_1}{1+ha_5} & b_2 &= \frac{1+ha_3}{1+ha_5} & A_1 &= \frac{Gr}{a_1^2-a_1-\left(M+\frac{1}{K}\right)} \\
A_2 &= \frac{Gm}{a_3^2-a_3-\left(M+\frac{1}{K}\right)} & A_3 &= \frac{PrA_1^2a_1^2+MA_1^2}{4a_1^2-2a_1Pr-RPr} & A_4 &= \frac{PrA_2^2a_3^2+MA_2^2}{4a_3^2-2a_3Pr-RPr} \\
A_5 &= \frac{(Pr a_5^2+M)(A_1b_1+A_2b_2)^2}{4a_5^2-2a_5Pr-RPr} & A_6 &= \frac{2A_1(Pr a_5a_1+M)(b_1+A_2b_2)}{(a_5+a_1)^2-(a_5+a_1)Pr-RPr} \\
A_7 &= \frac{2A_1A_2(Pr a_3a_1+M)}{(a_3+a_1)^2-(a_3+a_1)Pr-RPr} & A_8 &= \frac{2A_2(Pr a_5a_3+M)(b_2+A_1b_1)}{(a_3+a_5)^2-(a_3+a_5)Pr-RPr} \\
A_9 &= A_3 + A_4 + A_5 - A_6 + A_7 - A_8 & A_{10} &= \frac{Gr A_9}{a_1^2-a_1-\left(M+\frac{1}{K}\right)} \\
A_{11} &= \frac{Gr A_3}{4a_1^2-2a_1-\left(M+\frac{1}{K}\right)} & A_{12} &= \frac{Gr A_4}{4a_3^2-2a_3-\left(M+\frac{1}{K}\right)} & A_{13} &= \frac{Gr A_5}{4a_5^2-2a_5-\left(M+\frac{1}{K}\right)} \\
A_{14} &= \frac{Gr A_6}{a_6^2-a_6-\left(M+\frac{1}{K}\right)} & A_{15} &= \frac{Gr A_7}{a_7^2-a_7-\left(M+\frac{1}{K}\right)} & A_{16} &= \frac{Gr A_8}{a_8^2-a_8-\left(M+\frac{1}{K}\right)} \\
A_{17} &= \frac{1+ha_1}{1+ha_5} A_{10} - \frac{1+2ha_1}{1+ha_5} A_{11} - \frac{1+2ha_3}{1+ha_5} A_{12} - \frac{1+2ha_5}{1+ha_5} A_{13} + \frac{1+ha_6}{1+ha_5} A_{14} \\
&\quad - \frac{1+ha_7}{1+ha_5} A_{15} - \frac{1+ha_8}{1+ha_5} A_{16}
\end{aligned}$$

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