Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.18, No.1&2, January-June, 2019, 25-44

EXACT SOLUTION OF MHD FREE CONVECTIVE AND MASS TRANSFER FLOW NEAR A MOVING VERTICAL PLATE IN THE PRESENCE OF HEAT SOURCE/SINK

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Abstract: The heat source/sink effect on free convective incompressible viscous fluid flow and mass transfer in presence of uniform magnetic field applied normal to the infinite vertical plate moving with time dependent velocity has been investigated. The equations of the fluid flow are solved by Laplace transformation method. Solution is obtained in terms of exponential and complementary error function. The expressions of temperature, velocity, nusselt number and skin-friction have been obtained. The effect of various parameters on these expressions has been shown with help of graphs and tables. Movement of the plate with single acceleration and uniform velocity has been discussed.
Keywords: Free-convection, Mass transfer, MHD, Heat Source/Sink, Vertical Plate, Skin-Friction, Nusselt Number.

1. Introduction

The study of convective viscous incompressible MHD flow are being studied now a days due to large number of applications in areas of astrophysics, geophysics, power generators, petroleum industry, boundary layer control energy generators, cooling of reactors etc. This study has many engineering applications as in MHD bearings, MHD pumps etc. It is applied to study the solar and stellar composition in astrophysics and geophysics. The magnetic field effect[4] on free convective flow is studied because of its importance in ionized gases, liquid metals and electrolytes.

Many researchers [20], [11], [19], [16], [1], [3] have investigated on effect of heat and mass transfer of free convective flow with different boundary conditions as constant mass flux, uniform mass diffusion, exponentially accelerated vertical plate, effect of heat sources, transverse magnetic field etc. The study of radiative fluid flows is important in many industrial and environmental processes such as heating and cooling chambers, combustion of fossil fuel, solar power technology, energy processes, evaporation processes from huge open water reservoirs, space vehicle re-entry and astrophysical flows. Some researchers such as Rajesh and Varma [17], Muthucumaraswamy and

Muralidharan [15], Garg et al. [10], Kumar and Varma [13] Santhosha et al. [18] investigated on effect of radiations on MHD flow. Recently Garg et al [7] have investigated on mixed convective MHD flow in hot vertical channel through porous medium with span wise co sinusoidal temperature and thermal radiations. Very recently Garg et al. [8], [9] have investigated on unsteady free convective flow in vertical channel through porous medium under different boundary conditions with thermal radiations. The heat generation/absorption study in moving fluids is applicable in numerous physical problems associated to chemical reactions and surfaces related to high temperature differences by Mohan et al. [14]. Chamkha [2] studied the effect of thermal radiation and buoyancy forces with heat source or sink on hydro magnetic flow over an accelerating permeable surface. Kandasamy et al. [12] studied the heat and mass transfer and chemical reaction effects with heat source on MHD flow over a vertical stretching surface. Very recently Garg et al. [5], [6] studied the effects of heat source on MHD flow past an impulsively started vertical plate with variable heat and mass transfer.

The present investigation is aimed to get the exact solution of free convective unsteady incompressible fluid flow and mass transfer near a moving infinite vertical plate with transverse magnetic field. The governing equations of the flow are solved by using the Laplace transform method and the solutions are expressed in terms of complementary error functions and exponential functions.

2. Mathematical Analysis

We are considering unsteady incompressible free convective flow of a viscous fluid past an infinite non-conducting vertical plate in the presence of uniform transverse magnetic fieldB_o applied in the direction of the fluid flow. The x'-axis is taken along the plate in vertically upward direction and y'-axis is taken normal to it. At time $t' \leq 0$, the plate and fluid are at same temperature in fixed condition T'_∞ and with similar concentrationC'_∞. At time $t' > 0$, the plate is given a velocity $u_o f(t')$ in its own plane along x'-axis. The concentration and temperature of plate is slightly increased to C_w' and T_w' respectively. It is assumed for free convective fluid that physical variables are functions of space coordinate and time only. The flow is described by the following set of equations under above assumptions:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) - \frac{\sigma B_o^2 u'}{\rho} + \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta^*(C' - C'_\infty) \qquad (1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - Q^*(T' - T'_\infty) \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \qquad (3)$$

The initial and boundary conditions are

$$\left.\begin{array}{l} u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \forall\ y' \geq 0, \quad t' \leq 0 \\ u' = u_o f(t'), \quad T' = T'_w, \quad C' = C'_w \quad \forall\ y' = 0, \ t' > 0 \\ u' \to 0, \quad T' \to T'_\infty, \quad C \to C'_\infty \ as \ y' \to \infty, \ t' > 0 \end{array}\right] \qquad (4)$$

Here u', t', ρ, g, T', T'_w, T'_∞, C', C'_w, C'_∞, β, β^*, c_p, k, σ, B_o, ν, Q^* and D are velocity in x' direction, time, density, acceleration due to gravity, temperature of the fluid, the plate temperature, temperature of fluid far away from plate, species concentration, the species concentration near the wall, species concentration in the fluid far away from the plate, the coefficient of volume expansion, the thermal expansion coefficient with concentration, specific heat at constant pressure, thermal conductivity, electrical fluid conductivity, electromagnetic induction, the kinematic viscosity, heat source/sink and chemical molecular diffusivity respectively.

Introducing the following dimensionless variables and parameters to decrease the above equations in dimensionless form:

$$u = \frac{u'}{u_o} \quad y = \frac{y'}{\nu} u_o \quad t = \frac{t' u_o^2}{\nu} \quad H = \frac{Q^* \nu^2}{k u_o^2}$$

$$\theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)} \quad C = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)} \quad G_r = \frac{\nu g \beta (T'_w - T'_\infty)}{u_o^3}, G_m = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_o^3} \tag{5}$$

$$M = \frac{\sigma B_o^2 \nu}{\rho u_o^2} \quad P_r = \frac{\nu \rho c_p}{k} \quad S_c = \frac{\nu}{D}$$

Where H, M, θ, G_r, G_m, Pr and Sc are the heat source parameter, the magnetic field parameter, the dimensionless temperature, thermal Grashof number, mass Grashof number, the Prandtl number and Schmidt number respectively.

Then in view of (5), equations (1), (2) and (3) reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m C - Mu \tag{6}$$

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - H\theta \tag{7}$$

$$S_c \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \tag{8}$$

The corresponding initial and boundary conditions becomes

$$u(y,t) = 0,\ \theta(y,t) = 0,\ C(y,t) = 0\ \forall\ y \text{ and } t \le 0$$

$$u(y,t) = f(t), \theta(y,t) = 1, C(y,t) = 1 \text{ for } y = 0 \text{ and } t > 0 \tag{9}$$

$$u(y,t) \to 0,\ \theta(y,t) \to 0,\ C(y,t) \to 0 \text{ as } y \to \infty \text{ and } t > 0$$

The system of equations of flow (6)-(8), under the boundary conditions (9) consist of the effect of heat source on free convective and mass transfer flow near a moving vertical plate with transverse magnetic field.

3. Solution of the Problem

To find the solution of the equations of flow (6)-(8), under the boundary conditions (9), we have used Laplace Transformation method. The solution of given problem for velocity, temperature and concentration are as follow:

Velocity $u(y,t)$ for $t > 0$ is derived with two cases

Case (i): When Plate moves with uniform velocity

We take $f(t) = H(t)$, where $H(t)$ is Heaviside Unit Function

Then expressions for $u(y,t)$ are as below

$$u(y,t) = \frac{1}{2}(1 - U_1 - U_2)[exp(A_1)erfc(B_1) + exp(-A_1)erfc(B_2)] + 2U_1 \exp(\alpha_1)[\{exp(A_2)erfc(B_3) + exp(-A_2)erfc(B_4)\} - \{exp(A_2)erfc(B_5) + exp(-A_2)erfc(B_6)\}] + 2U_1[exp(A_3)erfc(B_7) + exp(-A_3)erfc(B_8)] + 2U_2 \exp(\alpha_2)[\{exp(A_4)erfc(B_9) + exp(-A_4)erfc(B_{10})\} - \{exp(A_5)erfc(B_{11}) + exp(-A_5)erfc(B_{12})\}] + U_2 erfc(B_{13}) \quad (10)$$

Case (ii): When Plate moves with single acceleration

We take $f(t) = tH(t)$, where $H(t)$ is Heaviside Unit Function

Then expressions for $u(y,t)$ are as below

$$u(y,t) = \frac{1}{2}(C_1 - U_1 - U_2)[exp(A_1)erfc(B_1)] + \frac{1}{2}(C_2 - U_1 - U_2)[exp(-A_1)erfc(B_2)] + 2U_1 \exp(\alpha_1)[\{exp(A_2)erfc(B_3) + exp(-A_2)erfc(B_4)\} - \{exp(A_2)erfc(B_5) + exp(-A_2)erfc(B_6)\}] + 2U_1[exp(A_3)erfc(B_7) + exp(-A_3)erfc(B_8)] + 2U_2 \exp(\alpha_2)[\{exp(A_4)erfc(B_9) + exp(-A_4)erfc(B_{10})\} - \{exp(A_5)erfc(B_{11}) + exp(-A_5)erfc(B_{12})\}] + U_2 erfc(B_{13}) \quad (11)$$

Temperature is given by:

$$\theta(y,t) = \frac{1}{2}[exp(A_3)erfc(B_7) + exp(-A_3)erfc(B_8)] \quad (12)$$

Concentration is given by:

$$C(y,t) = erfc(B_{13}) \quad (13)$$

4. Skin Friction

Skin Friction is derived from velocity field. The effect of t, H, M, Sc and Pr on Skin Friction coefficient in the dimensionless form is given by

(i) When Plate moves with uniform velocity

$$\tau_1 = -\left(\frac{\partial u(y,t)}{\partial t}\right)_{y=0}$$

$$= (1 - U_1 - U_2)\left[\sqrt{M}erf\sqrt{Mt} + \frac{1}{\sqrt{\pi t}}e^{-Mt}\right] + U_1 e^{at}\left[\sqrt{d}erf\sqrt{dt} + \frac{2}{\sqrt{\pi t}}e^{-dt} - \sqrt{d}erf\sqrt{\frac{d}{P_r}t} - \sqrt{\frac{P_r}{\pi t}}e^{-\frac{d}{P_r}t}\right] + U_1\left[\sqrt{H}erf\sqrt{\frac{H}{P_r}t} + \sqrt{\frac{P_r}{\pi t}}\, e^{-\frac{H}{P_r}t}\right] + U_2 e^{bt}\left[\sqrt{bS_c}erf\sqrt{bS_c t} - \sqrt{b}erf\sqrt{bt} - \sqrt{\frac{S_c}{\pi t}}\, e^{-bt/Sc}\right] + U_2\sqrt{\frac{S_c}{\pi t}}. \quad (14)$$

(ii) When Plate moves with single acceleration

$$\tau_2 = -\left(\frac{\partial u(y,t)}{\partial t}\right)_{y=0}$$

$$= \left(\frac{1}{2\sqrt{M}} + t\sqrt{M}\right) erf\sqrt{Mt} + \sqrt{\frac{t}{\pi}}e^{-Mt} - (U_1 + U_2)\left[\sqrt{M}erf\sqrt{Mt} + \frac{1}{\sqrt{\pi t}}e^{-Mt}\right] + U_1 e^{at}\left[\sqrt{d}erf\sqrt{dt} + \frac{2}{\sqrt{\pi t}}e^{-dt} - \sqrt{d}erf\sqrt{\frac{d}{P_r}t} - \sqrt{\frac{P_r}{\pi t}}e^{-\frac{d}{P_r}t}\right] + U_1\left[\sqrt{H}erf\sqrt{\frac{H}{P_r}t} + \sqrt{\frac{P_r}{\pi t}}\, e^{-\frac{H}{P_r}t}\right] + U_2 e^{bt}\left[\sqrt{bS_c}erf\sqrt{bS_c t} - \sqrt{b}erf\sqrt{bt} - \sqrt{\frac{S_c}{\pi t}}\, e^{-bt/Sc}\right] + U_2\sqrt{\frac{S_c}{\pi t}}. \quad (15)$$

5. Nusselt Number

The Nusselt number is derived from temperature field and it is the measure of heat transfer rate. In dimensionless form, Nusselt number is given by

$$N_u = -\left(\frac{\partial \theta(y,t)}{\partial t}\right)_{y=0}$$

$$= \sqrt{H}\, erf\sqrt{\frac{H}{P_r}t} + \sqrt{\frac{P_r}{\pi t}}\, e^{-\frac{H}{P_r}t}. \quad (16)$$

6. Sherwood Number

The Sherwood number is derived from Concentration field and it is the measure of mass transfer rate at plate. In dimensionless form, Sherwood number is given by

$$S_h = -\left(\frac{\partial C(y,t)}{\partial t}\right)_{y=0} = 2\sqrt{\frac{S_c}{\pi t}}. \quad (17)$$

7. Discussion and Results

To analysis the effect of various parameters like thermal Grashof number Gr, Schmidt number Sc, mass Grashof number Gm, Prandtl number Pr, Heat source/sink parameter H, Magnetic field parameter M and time t on Velocity, temperature, concentration distribution, numerical calculations are carried out and these effects are shown by Table (1)-(3) and Figures (1)-(9). Also the variations of Skin Friction, Nusselt number and Sherwood number are explained through Table (4)-(7) and Figures (10)-(16).

Figures (1) and (2) depict the effect of t, M and Sc on velocity of fluid with plate moving with uniform velocity and single acceleration. It is noticed that fluid velocity increases by increasing the Schmidt number and with progression in time. Also velocity decreases by increasing magnetic field parameter. It means the transverse magnetic field induces drag force which causes retardation in velocity of fluid.

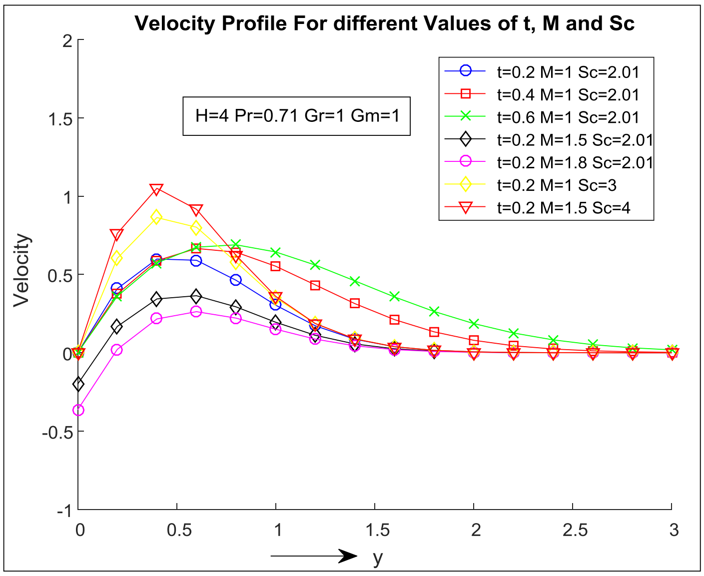

Fig.1 Effect of t, M and Sc on Velocity Profile (For uniform velocity)

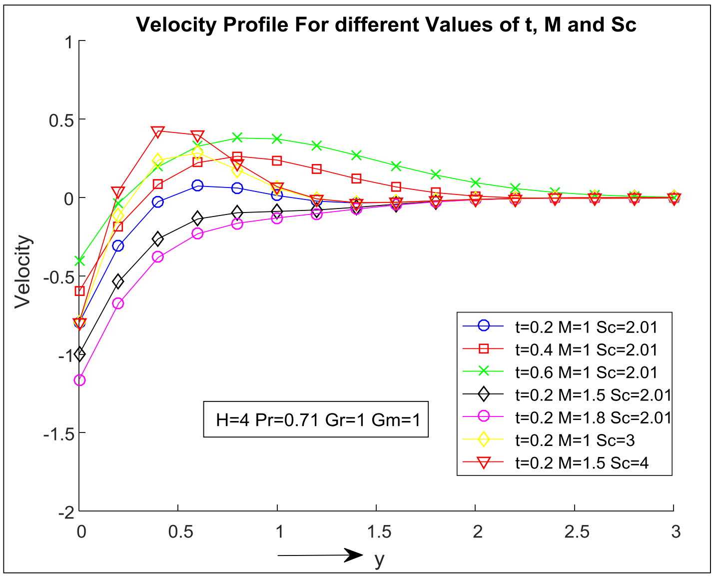

Fig.2 Effect of t, M and Sc on Velocity Profile (For single acceleration)

The effect of Gr, Gm and Pr on velocity profile is shown by figures (3) and (4). It is observed that velocity profile of fluid decreases by increasing thermal Grashof number and Prandtl number whereas it increases with increment in mass Grashof number keeping other parameters as constants.

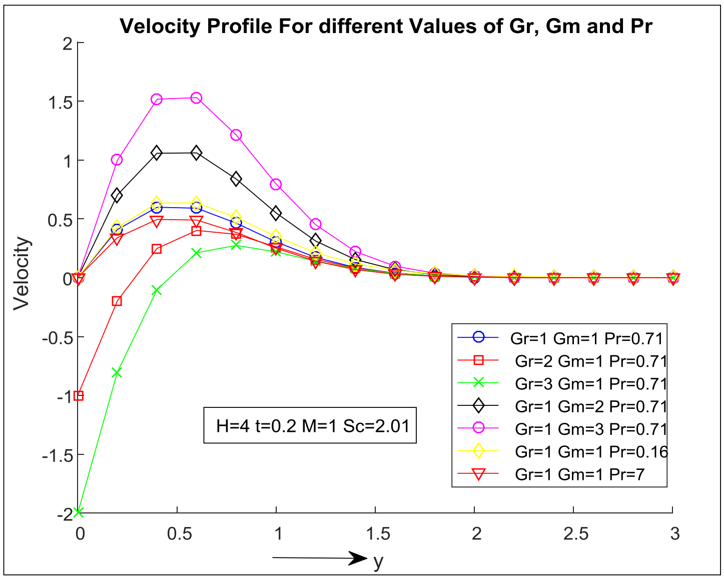

Fig.3 Effect of Gr, Gm and Pr on Velocity Profile (For uniform velocity)

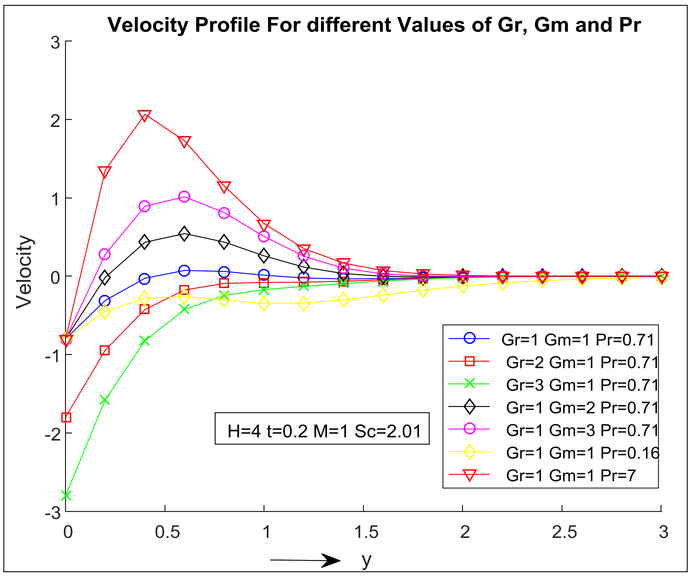

Fig.4 Effect of Gr, Gm and Sc on Velocity Profile (For single acceleration)

The effect of H on velocity of fluid is shown by Table-(1) & (2) in cases when plate is moving with uniform velocity and single acceleration respectively. This effect is depicted graphically by figures (5) & (6). It is noticed that fluid velocity in both cases is high initially without heat source/sink parameter as compared to velocity with heat source/sink parameter but it started decreasing after some distance in case of single acceleration. It is also noticed that there is an increment in velocity with progression in time without heat source/sink parameter. Fig. (8) shows that velocity profile of fluid when plate is moving with uniform velocity increases by increasing heat source/sink parameter. From Fig. (9), it is observed that fluid velocity when plate is moving with single acceleration increases initially with increasing values of heat source parameter but it started decreasing after certain distance keeping time as constant.

Table 1: Velocity Results of Fluid without Heat Source/Sink and with Heat Source/Sink Parameter (taking Pr=7 M=1 Sc=2.01 Gr=1 Gm=1 as constants) (Plate moved with Uniform Velocity)

y (t=0.2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=0)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=4)	t (y=0.2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=0)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=4)
0.1	3.6141	-1.5142	0.1944	0.1	2.9514	-0.8648	0.4069
0.2	3.2238	-1.0990	0.3407	0.2	3.2238	-1.0990	0.3407
0.3	2.8344	-0.7550	0.4402	0.3	3.3381	-1.1943	0.3139
0.4	2.4516	-0.4817	0.4950	0.4	3.4037	-1.2437	0.3013
0.5	2.0813	-0.2756	0.5093	0.5	3.4473	-1.2722	0.2952
0.6	1.7305	-0.1301	0.4896	0.6	3.4789	-1.2895	0.2924
0.7	1.4063	-0.0357	0.4446	0.7	3.5032	-1.3002	0.2915
0.8	1.1152	0.0188	0.3840	0.8	3.5228	-1.3068	0.2916
0.9	0.8622	0.0446	0.3170	0.9	3.5390	-1.3106	0.2924
1	0.6494	0.0520	0.2510	1	3.5528	-1.3127	0.2936

Table 2: Velocity Results of Fluid without Heat Source/Sink and with Heat Source/Sink Parameter (taking Pr=7 M=1 Sc=2.01 Gr=1 Gm=1 as constants) (Plate moved with Single Acceleration)

y (t=0.2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=0)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=4)	t (y=0.2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=0)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=2)	$\mathbf{u}(\mathbf{y},\mathbf{t})$ (H=4)
0.1	2.9114	-0.7736	-0.4147	0.1	2.3607	1.7228	1.8937
0.2	2.6174	0.8251	1.3539	0.2	2.6174	0.8251	1.3539
0.3	2.3206	1.8070	1.8977	0.3	2.7709	0.3248	1.0539
0.4	2.0243	2.2026	2.0659	0.4	2.8941	0.0209	0.8798
0.5	1.7327	2.1822	1.9672	0.5	3.0039	-0.1744	0.7767
0.6	1.4517	1.9385	1.7259	0.6	3.1066	-0.3029	0.7179
0.7	1.1877	1.6135	1.4350	0.7	3.2051	-0.3869	0.6890
0.8	0.9474	1.2868	1.1481	0.8	3.3006	-0.4396	0.6813
0.9	0.7360	0.9938	0.8905	0.9	3.3943	-0.4692	0.6893
1	0.5564	0.7465	0.6716	1	3.4865	-0.4813	0.7093

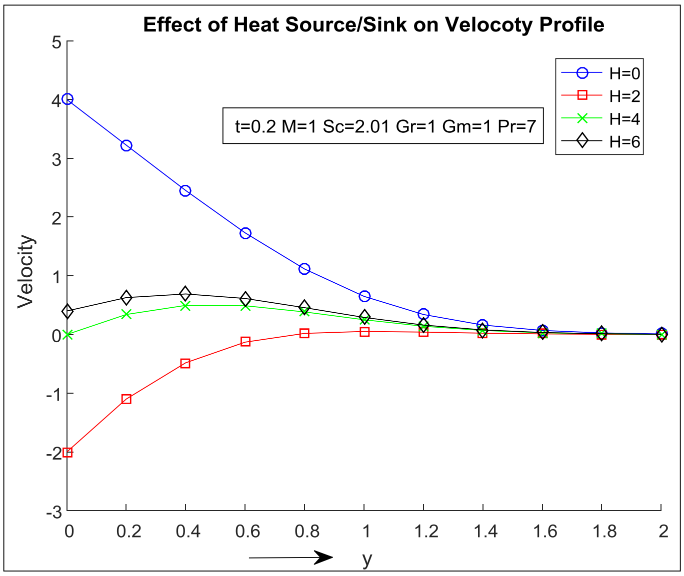

Fig.5 Effect of H on Velocity Profile (For Uniform Velocity)

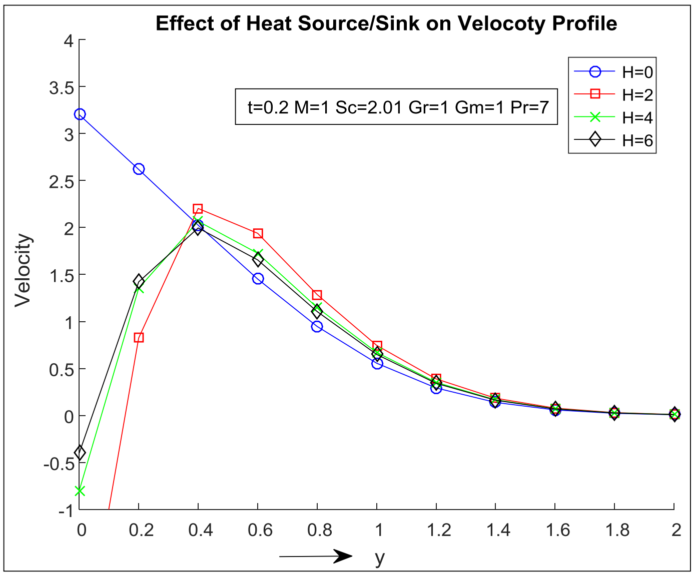

Fig.6 Effect of H on Velocity Profile (For Single Acceleration)

The effect of Pr, H and t on temperature of fluid is shown by Fig. (7). The graph depicts that fluid temperature decreases with increasing value of Prandtl number (Pr) and Heat source/sink parameter (H). Also the fluid temperature increases by progression in time. So viscosity and heat source parameter reduces the fluid temperature and time has opposite effect on it.

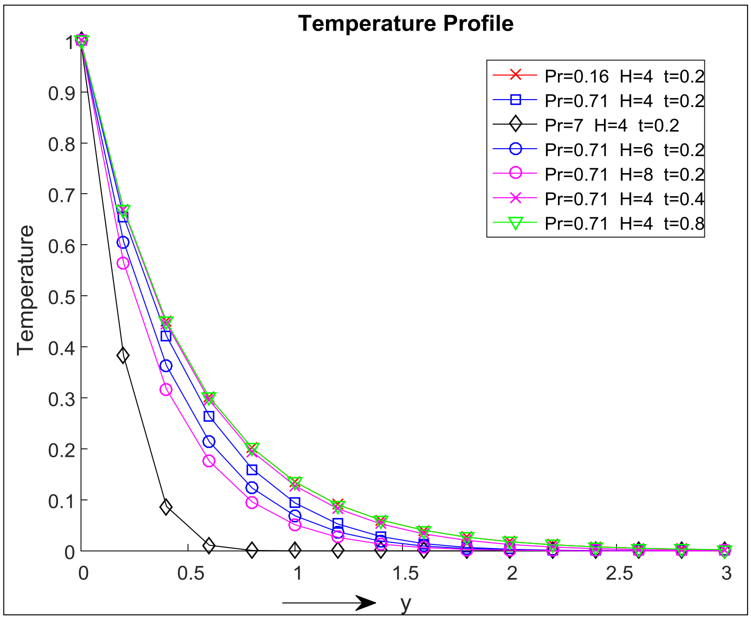

Fig.7 Effect of Pr, H and t on Temperature Profile

The temperature of fluid without heat source parameter and with heat source parameter has been compared by Table-(3) and this comparison has been shown graphically by Fig. (8).

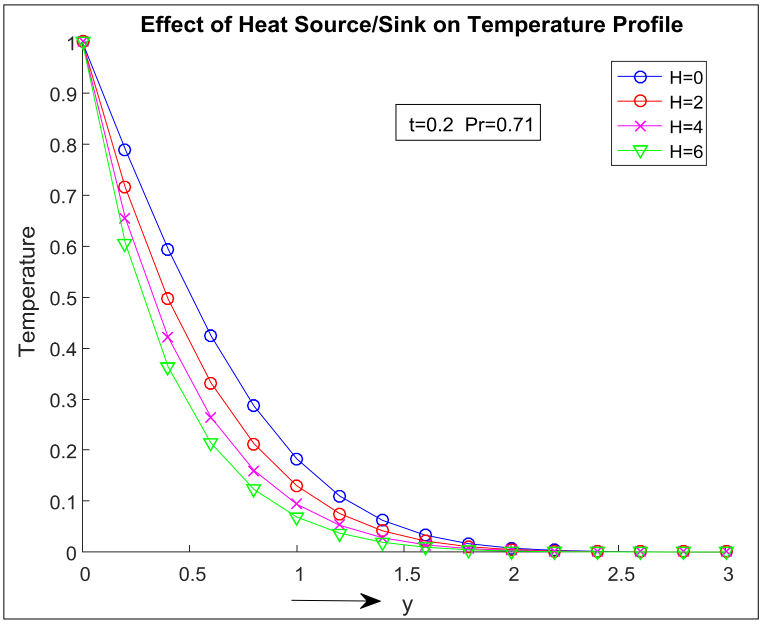

Fig.8 Effect of H on Temperature Profile

Table 3: Temperature Results of Fluid without Heat Source/Sink and with Heat Source/Sink Parameter (taking Pr=0.71)

y (t=0.1)	$\theta(y,t)$ (H=0)	$\theta(y,t)$ (H=2)	$\theta(y,t)$ (H=4)	t (y=0.1)	$\theta(y,t)$ (H=0)	$\theta(y,t)$ (H=2)	$\theta(y,t)$ (H=4)
0.1	0.8506	0.8192	0.7658	0.1	0.8506	0.8192	0.7912
0.2	0.7063	0.6588	0.5800	0.2	0.8940	0.8485	0.8110
0.3	0.5719	0.5191	0.4335	0.3	0.9134	0.8583	0.8159
0.4	0.4511	0.3999	0.3189	0.4	0.9249	0.8626	0.8176
0.5	0.3462	0.3009	0.2305	0.5	0.9328	0.8649	0.8182
0.6	0.2583	0.2207	0.1633	0.6	0.9387	0.8661	0.8185
0.7	0.1872	0.1577	0.1132	0.7	0.9432	0.8669	0.8186
0.8	0.1317	0.1096	0.0766	0.8	0.9469	0.8673	0.8187
0.9	0.0899	0.0740	0.0506	0.9	0.9499	0.8676	0.8187
1	0.0595	0.0486	0.0325	1	0.9525	0.8678	0.8187

It is observed that fluid temperature is high without heat source parameter but as we introduce heat source parameter, the temperature of fluid started decreasing with increasing value of heat source parameter.

Fig. (9) depicts the concentration profile for Schmidt number Sc and time t. The graph clearly shows that concentration of plate decreases with increasing Schmidt number. The same graph also shows that concentration of fluid increases with progression in time.

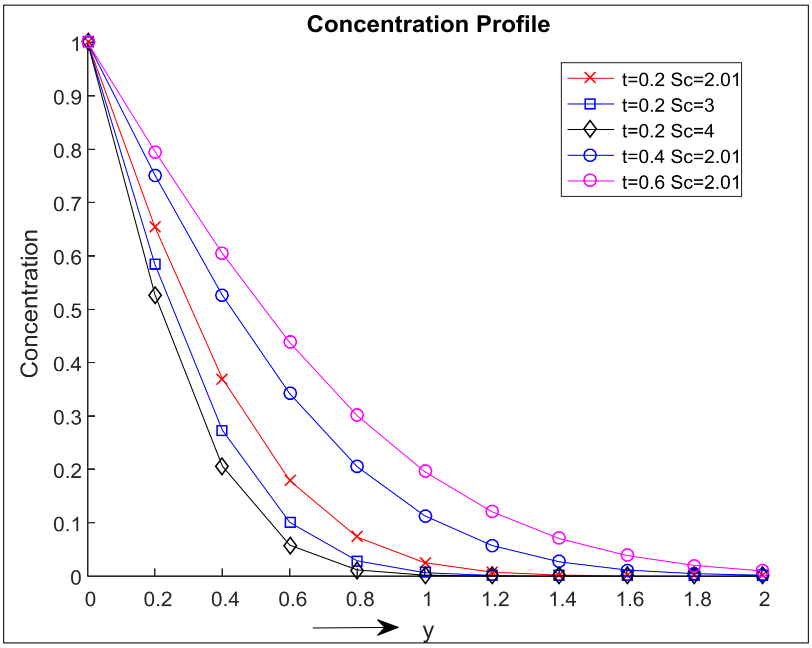

Fig.9 Effect of time and Schmidt number on Concentration profile

The effect of different parameters on Skin Friction is shown by Table-(4) and Figures (10) & (11). It is observed that Skin friction increases with time, Heat source parameter, Prandtl number, Schmidt number whereas it decreases by increment in values of Gr and Gm

Table-4: Variations of Parameters on Skin Friction

t	H	M	Pr	Sc	Gr	Gm	τ_1	τ_2
0.2	**2**	**1**	**0.71**	**2.01**	**1**	**1**	-0.9557	-1.9239
0.4	2	1	0.71	2.01	1	1	-0.4519	-0.8736
0.6	2	1	0.71	2.01	1	1	-0.1693	-0.2565
0.2	**4**	1	0.71	2.01	1	1	-0.2505	-1.2187
0.2	**6**	1	0.71	2.01	1	1	-0.099	-1.0672
0.2	2	**1.5**	0.71	2.01	1	1	-1.2471	-2.3157
0.2	2	**1.8**	0.71	2.01	1	1	-3.5649	-4.6921
0.2	2	1	**0.16**	2.01	1	1	-0.9975	-1.9657
0.2	2	1	**7**	2.01	1	1	-0.8458	-1.814
0.2	2	1	0.71	**3**	1	1	-0.9375	-1.9057
0.2	2	1	0.71	**4**	1	1	-0.9221	-1.8903
0.2	2	1	0.71	2.01	**2**	1	-2.2326	-3.2008
0.2	2	1	0.71	2.01	**3**	1	-3.5096	-4.4777
0.2	2	1	0.71	2.01	1	**2**	-2.1403	-3.1085
0.2	2	1	0.71	2.01	1	**3**	-3.3249	-4.2931

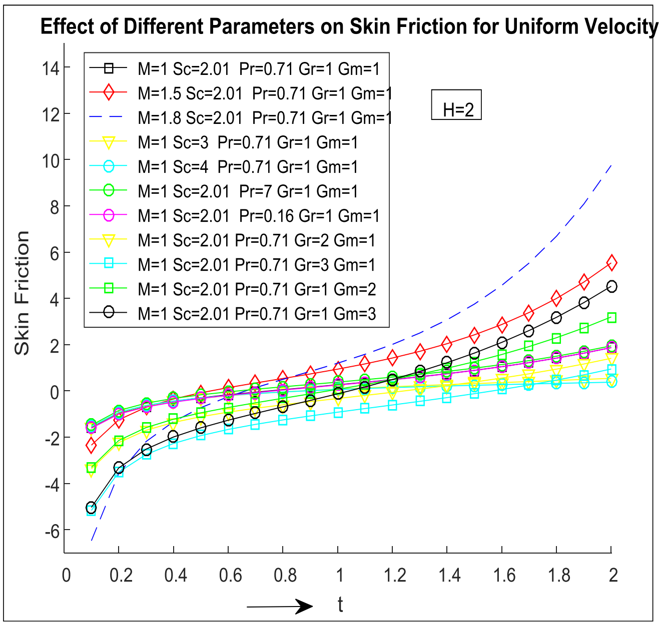

Fig.10 Effect of Various Parameters on Skin-Friction (For Uniform Velocity)

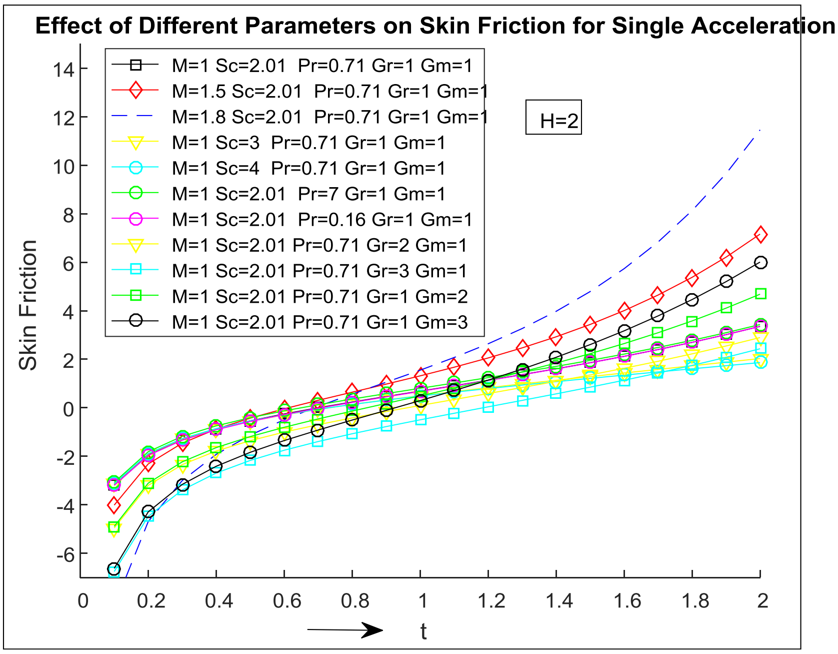

Fig.11 Effect of Various Parameters on Skin-Friction (For Single Acceleration)

The comparison of values of Skin Friction without heat source parameter and with heat source/sink parameter is represented by Table-(5) and Figures (12) & (13). It is noticed that value of Skin Friction is greater without heat source parameter in both cases when plate is moving with uniform velocity and single acceleration.

Table-5: Effect of Heat Source Parameter on Skin Friction

Time(t)	H=0		H=2		H=4		H=6	
	τ_1	τ_2	τ_1	τ_2	τ_1	τ_2	τ_1	τ_2
0.1	1.7457	0.1542	-1.4823	-3.0733	-0.4054	-1.9964	-0.1895	-1.7805
0.2	1.2181	0.2499	-0.8458	-1.8140	-0.1554	-1.1236	-0.0159	-0.9841
0.3	0.9916	0.3451	-0.5313	-1.1777	-0.0194	-0.6658	0.0855	-0.5610
0.4	0.8664	0.4447	-0.3245	-0.7462	0.0789	-0.3428	0.1631	-0.2585
0.5	0.7923	0.5503	-0.1686	-0.4106	0.1605	-0.0815	0.2310	-0.0109
0.6	0.7503	0.6630	-0.0403	-0.1275	0.2345	0.1473	0.2954	0.2082
0.7	0.7313	0.7835	0.0727	0.1249	0.3060	0.3582	0.3598	0.4120
0.8	0.7310	0.9125	0.1774	0.3590	0.3787	0.5599	0.4269	0.6084
0.9	0.7468	1.0508	0.2789	0.5829	0.4539	0.7579	0.4985	0.8024
1	0.7777	1.1991	0.3807	0.8020	0.5348	0.9561	0.5762	0.9976

It is also noticed that the Skin friction increases with increasing value of heat source parameter which means the shear stress on plates moving with uniform velocity and single acceleration increases by increasing heat source/sink parameter.

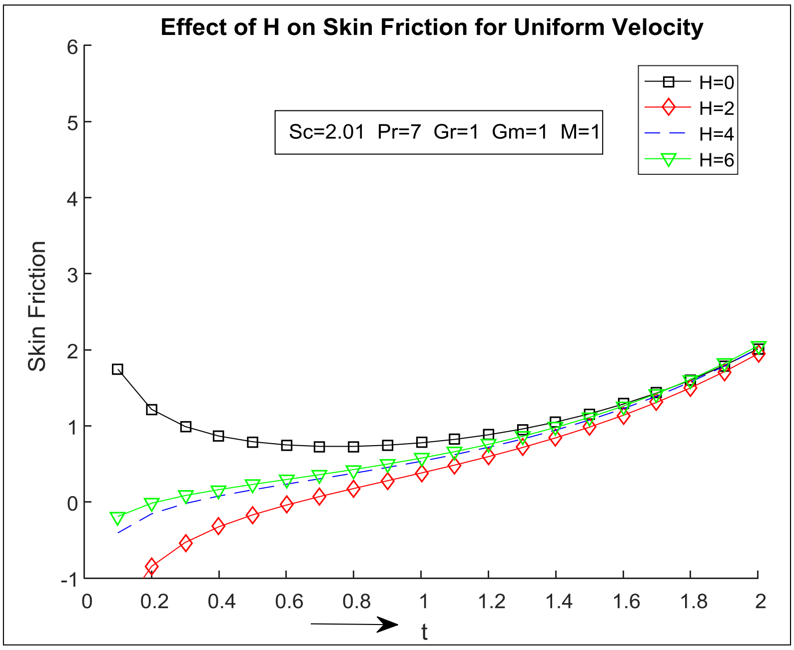

Fig.12 Effect of Heat Source/Sink Parameter on Skin-Friction (For Uniform Velocity)

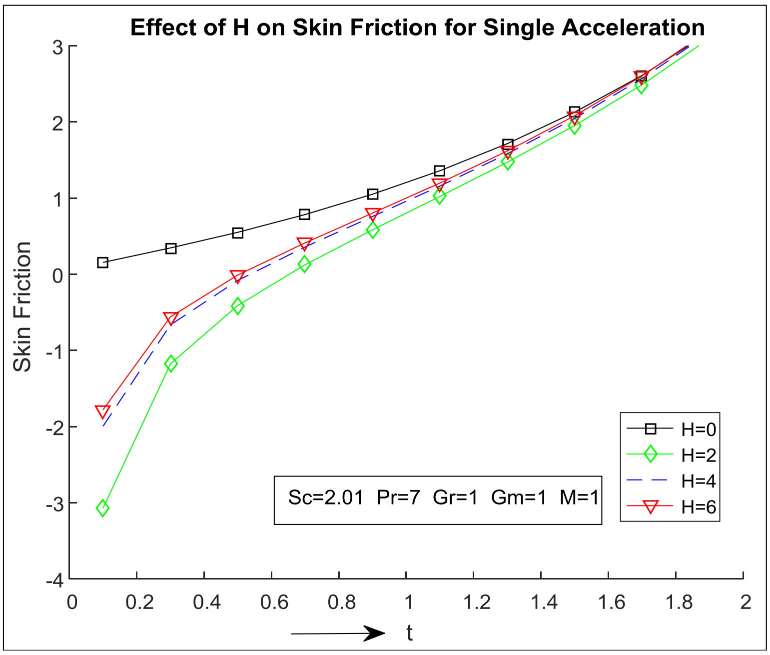

Fig.13 Effect of Heat Source/Sink Parameter on Skin-Friction (For Single Acceleration)

The effect of H and Pr on Nusselt number is depicted by Table-(6). From the table, it is depicted the Nusselt number is low without heat source parameter as compared to with heat source parameter. Table indicates that heat transfer rate is higher by increasing values of heat source parameter and Prandtl Number but time has opposite effect on it. These effects are presented graphically by Figures (14) & (15).

Table-6: Values of Nusselt Number without Heat Source Parameter and with Heat Source Parameter

Pr →	0.16			0.71			7		
Time(t) ↓	H=0	H=2	H=4	H=0	H=2	H=4	H=0	H=2	H=4
0.1	0.7136	1.4577	2.0079	1.5033	1.9080	2.2789	4.7203	4.8546	4.9875
0.2	0.5046	1.4198	2.0003	1.0630	1.6114	2.0779	3.3378	3.5267	3.7121
0.3	0.4120	1.4152	2.000	0.8679	1.5133	2.0282	2.7253	2.9556	3.1796
0.4	0.3568	1.4144	2.000	0.7517	1.4693	2.0114	2.3602	2.6249	2.8800
0.5	0.3192	1.4143	2.000	0.6723	1.4467	2.0050	2.1110	2.4056	2.6870
0.6	0.2913	1.4142	2.000	0.6137	1.4341	2.0023	1.9271	2.2483	2.5525
0.7	0.2697	1.4142	2.000	0.5682	1.4268	2.0011	1.7841	2.1295	2.4537
0.8	0.2523	1.4142	2.000	0.5315	1.4223	2.0005	1.6689	2.0365	2.3786
0.9	0.2379	1.4142	2.000	0.5011	1.4195	2.0002	1.5734	1.9616	2.3198
1	0.2257	1.4142	2.000	0.4754	1.4177	2.0001	1.4927	1.9000	2.2729

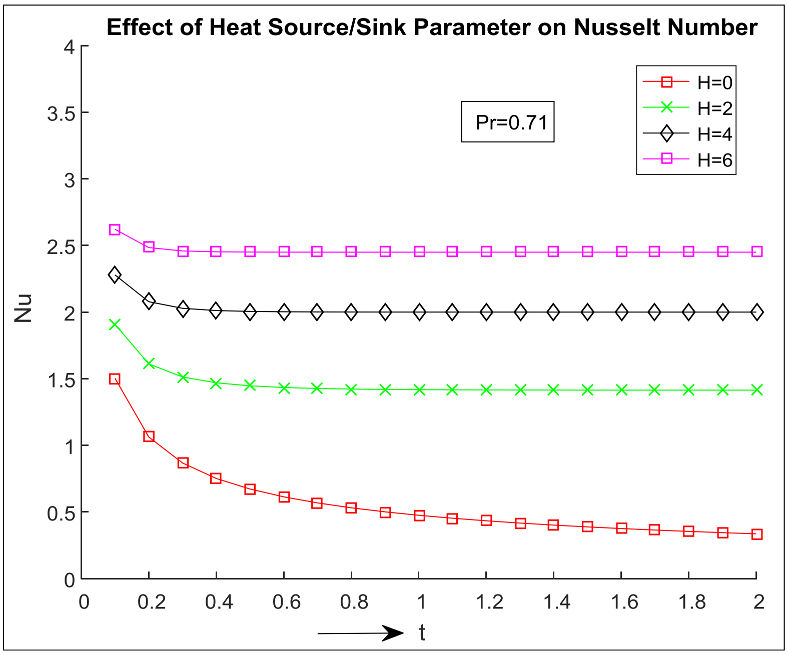

Fig.14 Effect of H on Nusselt Number

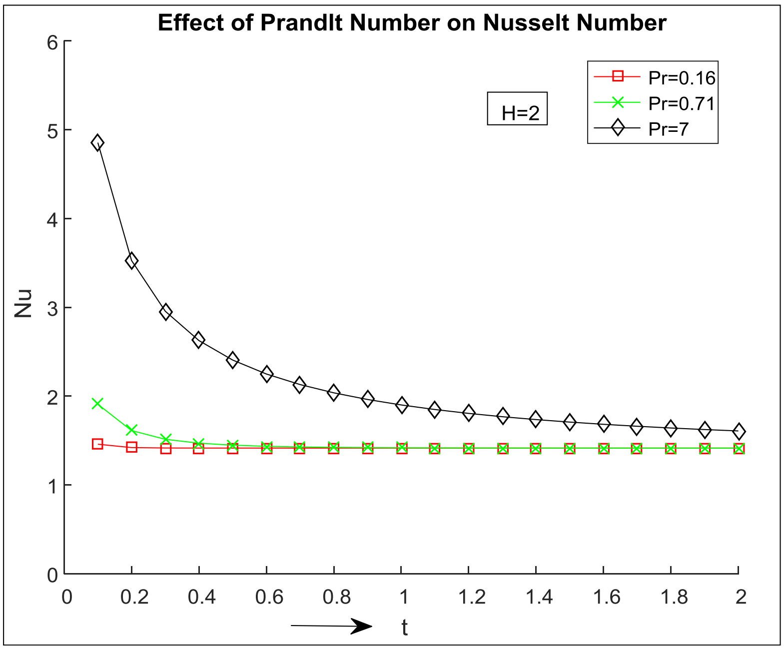

Fig.15 Effect of Pr on Nusselt Number

Table-7: Variations of Sherwood Number

Time(t)/Sc → ↓	2.01	3	4
0.1	5.0589	6.1804	7.1365
0.2	3.5772	4.3702	5.0463
0.3	2.9207	3.5682	4.1203
0.4	2.5294	3.0902	3.5682
0.5	2.2624	2.7640	3.1915
0.6	2.0653	2.5231	2.9135
0.7	1.9121	2.3360	2.6973
0.8	1.7886	2.1851	2.5231
0.9	1.6863	2.0601	2.3788
1	1.5998	1.9544	2.2568

Table-(7) represents the variations of Sherwood number for different values of t and Sc. It is observed that value of Sherwood number decreases with progression in time whereas it increases by increasing the Schmidt number. So rate of mass transfer slowed down with progression in time. Also increased value of Schmidt number fastens the mass transfer rate. These effects are shown graphically by Fig. (16).

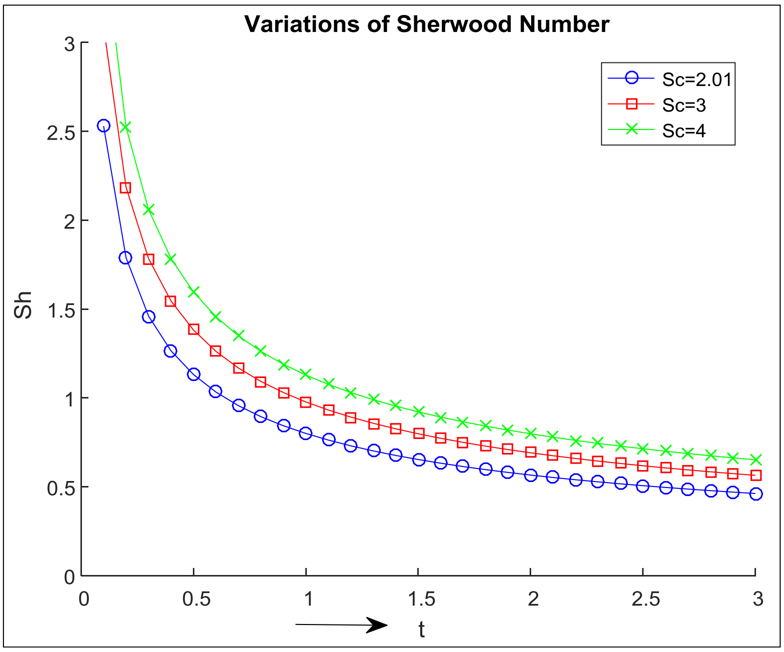

Fig.16 Effect of t and Pr on Sherwood Number

Das [4] studied the same problem with radiation effects instead of effects of heat source/ sink. It is noticed that results of temperature, concentration, Nusselt number are exactly same but results of velocity, skin-friction and Sherwood number are slightly different at some parameters. Velocity profile and Sherwood number show opposite results with Das [4] at parameters Sc. Also Skin-friction decreases with progression in time in our result. Our results of effects of heat source/sink are also similar with Mohan et al. [14] and Santhosha et al. [18].

8. Conclusions

The study of incompressible unsteady free convective and mass transfer flow of fluid near a moving infinite vertical plate with transverse magnetic field can be concluded as:

(i) Mass diffusivity, time and mass buoyancy forces accelerate the fluid motion whereas magnetic field, thermal buoyancy forces and viscosity tends to retard the fluid motion. Also fluid velocity is greater without heat source/sink parameter.

(ii) Viscosity and heat source parameter reduces the fluid temperature whereas increasing time increases the fluid temperature.

(iii) Concentration in the fluid increases with progression in time whereas decreases with increasing value of Schmidt number. It means species concentration increases with mass diffusivity and time.

(iv) Shear stress increases with time, Heat source/ parameter, viscosity and mass diffusivity whereas thermal buoyancy forces and mass buoyancy forces decrease it. Also shear stress is greater without heat source parameter.

(v) The heat transfer rate is increased with increasing heat source parameter. Also heat transfer rate is less without heat source parameter. Also Progression in time reduces the heat transfer rate.

(vi) Time reduces the rate of mass transfer whereas mass diffusivity increases it.

10. Appendix

$a = \frac{M-H}{P_r - 1} \quad b = \frac{M}{S_c - 1} \quad c = M - H \quad d = M + a$

$\alpha_1 = at \quad \alpha_2 = bt \quad U_1 = \frac{G_r}{c} \quad U_2 = \frac{G_m}{M}$

$A_1 = y\sqrt{M} \quad A_2 = y\sqrt{d} \quad A_3 = y\sqrt{H} \quad A_4 = y\sqrt{bS_c}$

$C_1 = t + \frac{y}{2\sqrt{M}} \quad C_2 = t - \frac{y}{2\sqrt{M}} \quad B_1 = \frac{y}{2\sqrt{t}} + \sqrt{Mt} \quad B_2 = \frac{y}{2\sqrt{t}} - \sqrt{Mt}$

$B_3 = \frac{y}{2\sqrt{t}} + \sqrt{dt} \quad B_4 = \frac{y}{2\sqrt{t}} - \sqrt{dt} \quad B_5 = \frac{y}{2\sqrt{t}}\sqrt{P_r} + \sqrt{\frac{d}{P_r}t} \quad B_6 = \frac{y}{2\sqrt{t}}\sqrt{P_r} - \sqrt{\frac{d}{P_r}t}$

$B_7 = \frac{y}{2\sqrt{t}}\sqrt{P_r} + \sqrt{\frac{H}{P_r}t} \quad B_8 = \frac{y}{2\sqrt{t}}\sqrt{P_r} - \sqrt{\frac{H}{P_r}t} \quad B_9 = \frac{y}{2\sqrt{t}} + \sqrt{bS_c t} \quad B_{10} = \frac{y}{2\sqrt{t}} - \sqrt{bS_c t}$

$B_{11} = \frac{y}{2\sqrt{t}}\sqrt{S_c} + \sqrt{bt} \quad B_{12} = \frac{y}{2\sqrt{t}}\sqrt{S_c} - \sqrt{bt} \quad B_{13} = \frac{y}{2\sqrt{t}}\sqrt{S_c}$

Acknowledgements

The authors are very grateful to IKG Punjab Technical University, Kapurthala (Punjab), for providing the platform to carry on research work. Also the authors are thankful to the learned refree for valuable comments which lead to a definite improvement in the paper.

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