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SIMILARITY SOLUTIONS FOR UNSTEADY FLOW BEHIND AN EXPONENTIAL SHOCK IN A PERFECTLY CONDUCTING DUSTY GAS

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Abstract: In the present paper, self-similar solutions for unsteady one-dimensional isothermal (or adiabatic flow) behind a strong exponential shock driven out by a piston moving with time according to an exponential law is obtained. The medium is assumed to be a perfectly conducting dusty gas under the action of an azimuthal magnetic field. The dust is assumed to consist of small solid particles distributed continuously in a perfect gas and equilibrium flow condition is maintained in the flow field. The viscous-stress and heat conduction of the mixture are negligible solutions are obtained, in both the cases, when the flow between the piston and the shock is isothermal and adiabatic. Effects of variation in the mass concentration of the solid particles in the mixture k_p, the ratio of the density of the solid particles to that of initial density of the gas $G_1$and the strength of initial magnetic field are obtained. Also, a comparison is made between isothermal and adiabatic flows.

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1. Introduction

The study of shock wave has been the subject of the great interest both from physical and mathematical points of view due to its application in a variety of fields such as nuclear science, astrophysics, plasma physics, geophysics and interstellar gas masses.

Since the early work of Taylor [18], a considerable number of publications on the shock wave propagation have appeared in the literature, including reviews and treatises such as those of Sedov [14], Sakurai [13] and Korobeinikov [4]. The pioneering studies of this phenomena (Taylor [18] and Sedov [14]) were based on self-similarity considerations and found in good agreement with experimental results. Sedov [14], Ranga Rao and Ramana [11] and Vishwakarma and Nath [20,21] have pointed out that a limiting case of a self-similar flow-field with a power law shock is the flow-field formed with an exponential shock. Singh and Vishwakarma [15] have obtained solutions for the problem of unsteady self-similar motion of a gas displaced by a piston moving with time according to an exponential law.

The study of shock wave in a mixture of gas and small solid particles (dust particles) is of great interest in several branches of engineering and science (Pai et al. [10], Higashino andSuzuki [3], Nath [7]). Muira and Glass [6] have studied and analytical solution for a planer dusty gas flow with constant velocity of the shock and the piston moving behind it. They neglected the volume occupied by the solid particles mixed into the perfect gas, the dust virtually has a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of the dust upon the shock propagation. Vishwakarma [19]) has considered the non-zero volume fraction of solid particles in the dusty gas, the results reflect the influence of both the decrease of compressibility and the increase of inertia of the medium on the shock propagation (Steiner and Hirschler [16]).

The gas attains high temperature due to passes of shockwaves across it and at such a high temperature the gas gets ionized, hence electromagnetic effects become significant in the study of shock waves. Similarity solutions for the shock wave propagation in magnetogasdynamics have been studied by a number of researchers, for example, Pai [9], Cole and Greifinger [1], Deb Ray [2], Summers [17], Verma et al. [25], Vishwakarma and Yadav [24], Vishwakarma and Singh [23], Nath et al. [8]. Recently, Vishwakarma et al. [22] have obtainedself-similar solution for cylindrical shock waves in a weekly conducting dusty gas (mixture of a perfect gas and small solid particles).

In the present work, we generalize the work of Vishwakarma and Nath [20] to study the propagation of shock waves in the mixture of perfectly conducting dusty gas driven out by a piston in the presence of an azimuthal magnetic field. In order to determine some essential features of shock propagation, small solid particles (dust particles) are considered as a pseudo-fluid and it is assumed that the equilibrium flow conditions is maintained in the flow field and that the viscous stress and heat conduction of the mixture are negligible (Pai et al. [10], Higashino and Suzuki [3]). Also, the mixture of the perfect gas and dust particle are taken to perfectly conducting. It is assumed that the motion of the piston obeys the exponential law (see Ranga Rao and Ramana [11], Vishwakarma and Nath [20, 21])

$$r_p = A \exp(\lambda t) \quad , \quad \lambda > 0 \ , \tag{1}$$

wherer_p is the radius of the piston, A andλ are dimensional constants and$t$ is the time.'A' represents the initial radius of the piston. The law of piston motion (1) implies a boundary condition on the gas speed at the piston which is required for the determination of the problem. It is also assumed that the shock propagation follows the exponential law

$$R = B \exp\ (\lambda t), \tag{2}$$

where R is the radius of the shock and$B$ is a dimensional constant which is to be determined.

As the shock propagates, the temperature behind the shock increases and becomes very large so that intense radiation heat transfer takes place behind the strong shock. This causes the temperature gradient to approach zero throughout the flow field. Due to this reason, the temperature in the flow-field depends only on time t and not on the distance r from the point of explosion and the flow is isothermal as described by Sedov [14], Laumbach and Probstein [5], Sachdev and Ashraf [12] and Zhuravskaya and Levin [27]. With this assumption we obtain, in sections 2 and 3, the similarity solutions. In section 4, we present the solutions for the flow taken to be adiabatic. The effects of variation of mass concentration of solid particles in the mixture (k_p), the ratio of density of solid particles to the initial density of the gas in the mixture (G_1) and the strength of the initial magnetic field(M_A^{-2}) are obtained. Also, comparison between the solutions in the cases of isothermal and adiabatic flows is made.

2. Fundamental Equations and Boundary Conditions

The basic equations for one dimensional unsteady and isothermal flow of a perfectly conducting mixture of perfect gas and small solid particles in the presence of an azimuthal magnetic field may be written as (c.f. Pai et al. [10], Whitham [26])

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] = 0 , \tag{3}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + j\frac{\rho u}{r} = 0 , \tag{4}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + (j-1)\frac{h u}{r} = 0 , \tag{5}$$

$$\frac{\partial T}{\partial r} = 0 , \tag{6}$$

where $j = 1$ or 2 corresponds to the cylindrical and spherical symmetry and$p$,ρ, T and $u$are the pressure, density, temperature and the flow velocity of the mixture and t and r are the time and space coordinates respectively.

The equation of state of the mixture of perfect gas and small solid particles can be written as

$$p = \frac{(1-k_p)}{(1-Z)}\rho R^* T , \tag{7}$$

where R^* is the gas constant, $k_p$the mass concentration of solid particles, Z the volume fraction of solid particles and T the temperature in the mixture.

The relation betweenk_p and$Z$ is given by

$$k_p = \frac{Z\rho_{sp}}{\rho}, \tag{8}$$

whereρ_{sp} is the species density of solid particles.In the equilibrium flow, k_p is a constant in the whole flow-field. Therefore

$$\frac{Z}{\rho} = \text{constant}, \tag{9}$$

in the whole flow field. Also, we have the relation

$$Z = \frac{k_p}{(1-k_p)G+k_p}, \tag{10}$$

where $G = \frac{\rho_{sp}}{\rho}$ is the ratio of the density of the solid particles to the density of the perfect gas in the mixture.

The internal energy of the mixture may be written as

$$U_m = \left[k_p C_{sp} + \left(1 - k_p\right)C_v\right]T = C_{vm}T, \tag{11}$$

where C_{sp} is the specific heat of solid particles, C_v the specific heat of gas at constant volume and C_{vm} the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is

$$C_{pm} = k_p C_{sp} + \left(1 - k_p\right)C_p, \tag{12}$$

where C_p is the specific heat of the gas at constant pressure.

The ratio of specific heats of the mixture is given by (Pai [9])

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{1+\frac{\delta\beta'}{\gamma}}{1+\delta\beta'} \quad , \tag{13}$$

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{k_p}{1-k_p}$ and $\beta' = \frac{C_{sp}}{C_v}$. Now,

$$C_{pm} - C_{vm} = \left(1 - k_p\right)\left(C_p - C_v\right) = (1 - k_p)R^*. \tag{14}$$

The internal energy per unit mass of the mixture is, therefore, given by

$$U_m = \frac{p(1-Z)}{\rho(\Gamma-1)}. \tag{15}$$

The equilibrium speed of sound ‘a’ in the mixture is given by

$$a^2 = \frac{\Gamma p}{\rho(1-Z)} \, . \tag{16}$$

We consider that a shock is propagating into the perfectly conducting dusty gas (mixture of a perfect gas and small solid particles) of constant density at rest in the presence of azimuthal magnetic field.

The flow variables immediately ahead of the shock front are

$$u = u_1 = 0 \, , \tag{17}$$

$$\rho = \rho_1 = \text{constant} \, , \tag{18}$$

$$h = h_1 = LR^{-k} \quad , \tag{19}$$

$$p = p_1 = \frac{(1-m)\ \mu L^2}{2m\ R^{2m}} + c, \quad (0 < m < 1), \tag{20}$$

where R is the shock radius and subscript '1' denotes the conditions immediately ahead of the shock, L and m are constants.

The shock jump conditions are derived from the laws of conservation of mass, magnetic flux, momentum and energy under the assumption that the shock front is a discontinuous surface with no thickness, propagating with velocity$U\left(=\frac{dR}{dt}\right)$ into the dusty gas at rest.

The shock conditions are as follows

$$\rho_1 U = \rho_2 (U - u_2) \ , \tag{21}$$

$$h_1 U = h_2 (U - u_2) \ , \tag{22}$$

$$p_1 + \rho_1 U^2 + \frac{1}{2}\mu h_1^2 = p_2 + \rho_2 (U - u_2)^2 + \frac{1}{2}\mu h_2^2 \ , \tag{23}$$

$$U_{m_1} + \frac{p_1}{\rho_2} + \frac{U^2}{2} + \frac{\mu h_1^2}{\rho_1} = U_{m_2} + \frac{p_2}{\rho_2} + \frac{(U-u_2)^2}{2} + \frac{\mu h_2^2}{\rho_2} \ , \tag{24}$$

$$\frac{Z_1}{\rho_1} = \frac{Z_2}{\rho_2}, \tag{25}$$

where suffix '2' refers to the values just behind the shock front.

Using the strong shock approximations ($p_1 \cong 0, U_{m_1} \cong 0$), we have from above relations

$$u_2 = (1 - \beta) U \ , \tag{26a}$$

$$\rho_2 = \frac{\rho_1}{\beta}, \tag{26b}$$

$$h_2 = \frac{h_1}{\beta}, \tag{26c}$$

$$Z_2 = \frac{Z_1}{\beta}, \tag{26d}$$

$$p_2 = \left[(1 - \beta) + \frac{1}{2} M_A^{-2} \left(1 - \frac{1}{\beta^2}\right)\right] \rho_1 U^2. \tag{26e}$$

From equation (23), the following relation giving the density ratioβ $(0 < \beta < 1)$ is derived,

$$\beta^3 (\Gamma + 1) - \beta^2 \{(M_A^{-2} + 1)\Gamma + 2Z_1 - 1\} + \beta \{Z_1 + \Gamma - 2\} M_A^{-2} + Z_1 M_A^{-2} = 0, \tag{27}$$

where$M_A$is the Alfven Mach number such that$M_A^{-2} = U^2 / \frac{\mu\, h_1^2}{\rho_1}$.

The expression for the initial volume fraction of the solid particles is given by

$$Z_1 = \frac{k_p}{(1-k_p)G_1 + k_p}. \tag{28}$$

Equation (28) gives three different values ofβ out of which only one lies in the required range$0 < \beta < 1$ satisfying the physical limits of the considered problem andG_1 is the ratio of the density of the solid particles to the initial density of the gas.

Equation (26) together with equation of state (27) gives

$$\frac{p}{p_2} = \frac{(1-Z_2)}{(1-Z)}\frac{\rho}{\rho_2}. \tag{29}$$

3. Similarity Solution

The similarity transformations for the problem under consideration are taken

$$u = UV(\eta),\ \ \rho = \rho_1 D(\eta),\ \ p = \rho_1 U^2 P(\eta),\ \ Z = Z_1 D(\eta),\ \ \mu^{\frac{1}{2}} h = \rho_1^{\frac{1}{2}} UH(\eta)\ , \tag{30}$$

where V, D, P and $H$are the functions of the non-dimensional (similarity variable) η $(= \frac{r}{R})$ only. The variableη assumes the value '1' at the shock front andη_p on the piston. Equations (1), (2), (4) and (30) yield a relation between A and B in the form

$$A = B\ \eta_p. \tag{31}$$

Equation (29) with the aid of equations (30) yields a relation between$P$ and D in the form

$$P(\eta) = \frac{(\beta - Z_1)\left[(1-\beta)\ +\ \frac{M_A^{-2}}{2}\left(1-\frac{1}{\beta^2}\right)\right]\ D(\eta)}{(1-Z_1 D(\eta))}. \tag{32}$$

The conservation equations (3)-(6) can be transformed into the following system of ordinary differential equations with respect toη

$$(V-\eta)\frac{dD}{d\eta} + D\frac{dV}{d\eta} = -\frac{jDV}{\eta}, \tag{33}$$

$$(V-\eta)\frac{dH}{d\eta} + H\frac{dV}{d\eta} = -\frac{[(j-1)V+\eta]}{\eta}H, \tag{34}$$

$$(V-\eta)D\frac{dV}{d\eta} + \frac{P}{D(1-Z_1 D)}\frac{dD}{d\eta} + H\frac{dH}{d\eta} = -\left[\frac{H^2+VD\eta}{\eta}\right]. \tag{35}$$

Solving the above three equations, we get

$$\frac{dV}{d\eta} = \frac{PjV + (1-Z_1 D)2\eta H^2 + (j-2)(1-Z_1 D)VH^2 - (V-\eta)(1-Z_1 D)VD\eta}{\eta[(1-Z_1 D)(V-\eta)^2 D - P - (1-Z_1 D)H^2]}\ , \tag{36}$$

$$\frac{dD}{d\eta} = -\frac{D(1-Z_1 D)[(V-\eta)VjD - 2H^2 - VD\eta]}{\eta[(1-Z_1 D)(V-\eta)^2 D - P - (1-Z_1 D)H^2]}\ , \tag{37}$$

$$\frac{dH}{d\eta} = -\frac{H[(\eta+(j-1)V)(1-Z_1 D)(V-\eta)D + P - (1-Z_1 D)H^2 - (1-Z_1 D)VD\eta]}{\eta[(1-Z_1 D)(V-\eta)^2 D - P - (1-Z_1 D)H^2]}\ . \tag{38}$$

Using the self-similarity transformations (30), the shock conditions (26) take the form

$$V(1) = (1-\beta), \tag{39a}$$

$$D(1) = \frac{1}{\beta}, \tag{39b}$$

$$H(1) = \frac{1}{\beta}M_A^{-1}, \tag{39c}$$

$$P(1) = \left[(1-\beta) + \frac{M_A^{-2}}{2}\left(1 - \frac{1}{\beta^2}\right)\right], \quad (39d)$$

At the inner boundary surface (or piston) of the flow-field behind the shock, the condition is that the velocity of the surface is equal to the fluid velocity at the surface. The kinematic condition, from (30), at the inner boundary surface can be written as

$$V(\eta_p) = \eta_p. \quad (40)$$

Now, the differential equations (36)-(38) can be integrated, numerically, with the boundary conditions (39d) to obtain the solutions for the isothermal flow behind the shock surface.

4. Adiabatic Flow

In this part, we presentthe self-similar solution for adiabatic flow behind the strong shock driven by a spherical (or cylindrical) piston moving according to the exponential law (1), in the perfectly conducting dusty gas in the presence of azimuthal magnetic field.

For adiabatic flow, equation (6) is replaced by

$$\frac{\partial U_m}{\partial t} + u\frac{\partial U_m}{\partial r} - \frac{p}{\rho^2}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] = 0\ . \quad (41)$$

The equations (3)-(5) and (40) can be transformed, using equation (3.1), into

$$D(V-\eta)\frac{dV}{d\eta} + \frac{dP}{d\eta} + H\frac{dH}{d\eta} = -\frac{[H^2 + VD\eta]}{\eta}\ , \quad (42)$$

$$(V-\eta)\frac{dD}{d\eta} + D\frac{dV}{d\eta} = -j\frac{DV}{\eta}\ , \quad (43)$$

$$(V-\eta)\frac{dH}{d\eta} + H\frac{dV}{d\eta} = -H\frac{[(j-1)V+\eta]}{\eta}\ , \quad (44)$$

$$(V-\eta)(1-Z_1 D)\frac{dP}{d\eta} - \frac{(V-\eta)}{D}P\Gamma\frac{dD}{d\eta} = -2P(1-Z_1 D)\ , \quad (45)$$

Solving equations (42)-(45) for $\frac{dV}{d\eta}$, $\frac{dD}{d\eta}$, $\frac{dP}{d\eta}$ and $\frac{dH}{d\eta}$, we have

$$\frac{dV}{d\eta} = \frac{jPV\Gamma + 2P\eta(1-Z_1D) + 2\eta H^2(1-Z_1D) + (j-2)VH^2(1-Z_1D) - VD\eta(V-\eta)(1-Z_1D)}{\eta[D(V-\eta)^2(1-Z_1D) - P\Gamma - H^2(1-Z_1D)]}, \quad (46)$$

$$\frac{dD}{d\eta} = -\frac{D(1-Z_1D)[jVD(V-\eta)^2 - (2H^2 + V\eta D)(V-\eta) + 2P\eta]}{\eta(V-\eta)[D(V-\eta)^2(1-Z_1D) - P\Gamma - H^2(1-Z_1D)]}, \quad (47)$$

$$\frac{dP}{d\eta} = -\frac{P[\{jV\Gamma + 2\eta(1-Z_1D)\}D(V-\eta)^2 - 2\Gamma H^2(V-\eta) - 2\eta H^2(1-Z_1D) - V\eta D\Gamma(V-\eta)]}{\eta(V-\eta)[D(V-\eta)^2(1-Z_1D) - P\Gamma - H^2(1-Z_1D)]}, \quad (48)$$

$$\frac{dH}{d\eta} = -\frac{[j\Gamma PV + \{(j-1)V+\eta\}D(1-Z_1D)(V-\eta)^2 - \{(j-1)V+\eta\}P\Gamma + 2P\eta(1-Z_1D) - (V-\eta)(VD\eta + H^2)(1-Z_1D)]}{[D(V-\eta)^2(1-Z_1D) - P\Gamma - H^2(1-Z_1D)]} \times \frac{H}{\eta(V-\eta)}. \quad (49)$$

The transformed shock conditions (39) and the kinematic conditions (40) at the piston will be same as in the case of isothermal flow. The ordinary differential equations (46)-

(49) with boundary conditions (39) can now be numerically integrated to obtain the solution for the adiabatic flow behind the shock surface.

5. Results and Discussion

The flow variables V, D, P and Hagainst the similarity variable η are obtained by numerical integration of differential equations (36)-(38) for isothermal flow and equations (46)-(49) for adiabatic case with the boundary conditions (39) using fourth-order Runga-kutta algorithm. For the purpose of numerical calculations the value of constant parameters are taken as (Pai et al.[9], Miura and Glass [6], Vishwakarma [19])$j = 2$, $\gamma = \frac{5}{3}$,$\beta' = 0.25$, $k_p = 0, 0.2, 0.4$, $G_1 = 1, 100$ and$M_A^{-2} = 0, 0.005, 0.01$. The value$j = 2$ corresponds to the spherical shock$k_p = 0$ to the dust-free case (perfect gas) and$M_A^{-2} = 0$to the non-magnetic case. Figures (1)-(4) and figures (5)-(6) show the variation of the flow variables$V$, D, $P$and H with respect to η in isothermal and adiabatic cases respectively. Table 1 shows the values of the density ratio across the shock front β and the position of piston η_p at various values ofk_p , G_1 and M_A^{-2}.

It is found that an increase in the value ofk_p

I. decreases the shock strength (increases the value ofβ) whenG_1 , but in the case of$G_1 = 100$ the shock strength increases (the value of βdecreases) (see table 1);
II. increases the distance of piston from the shock front when $G_1 = 1$, and decreases it when $G_1 = 100$ (see table 1);
III. decreases the flow variables V, D and H at any point in the flow-field behind the shock when $G_1 = 1$ and increases these when $G_1 = 100$ (see figures 1, 2 and 4 for isothermal flow and 5, 6 and 8 for adiabatic flow); and
IV. decreases the pressure P at any point in the flow-field $G_1 = 1$ and increases it when $G_1 = 100$, in both, the isothermal and adiabatic flows in the non-magnetic field. In magnetic case, the pressure P decreases when $G_1 = 1$and increases it at higher value of $G_1 = 100$ when the flow is isothermal; whereas in adiabatic flow, the pressure P decreases when $G_1 = 1$ but it shows somewhat different behaviour in the dust-free case ($k_p = 0$) and in dusty case when $G_1 = 100$ (see figures 3 for isothermal flow and 7 for adiabatic flow).

This shows that an increase in k_p decreases the shock strength when $G_1 = 1$ and increases it when $G_1 = 100$. Physical interpretations of these effects are as follows:

In the mixture, small solid particles of density equal to that of the perfect gas occupy a significant part of the volume which lowers the compressibility of the medium at $G_1 = 1$. Also, the compressibility of the mixture is reduced by an increase in k_p which causes an increase in the distance between the shock front and the piston, a decrease in the shock strength, and the above nature of the flow variables. In the mixture at $G_1 = 100$, small solid particles of density equal to 100 times that of the perfect gas occupy a very small portion of the volume, and therefore compressibility is not lowered much; the inertia of the medium is increased significantly due to dust load. An increase ink_p, from 0.1 to 0.4

in the mixture for $G_1 = 100$, means that the perfect gas constituting 90% of the total mass and occupying 99.89% of the total volume now constitutes 60% of the total mass and occupies 99.34% of the total volume. Due to this reason, the density of the perfect gas in mixture is highly decreased which overcomes the effect of incompressibility of the mixture and finally causes a small decrease in the distance between piston and shock front, an increase in the shock strength, and the above behavior of flow variables.

Effects of increase in the value of G_1 are

I. to decrease β (i.e. to increase the shock strength)(see table 1);

II. to decrease the distance of piston from the shock front;

III. to increase the flow variables V, D and H in the flow-field (see figures 1, 2 and 4 for isothermal flow and 5, 6 and 8 for adiabatic flow); and

IV. to increase the pressure P at any point in the flow-field when the flow is isothermal. In adiabatic flow, the pressure P increases in the flow-field behind the shock in non-magnetic case ($M_A^{-2} = 0$); whereas in magnetic case ($M_A^{-2} = 0.005, 0.01$) the gradient of pressure P is constant when $G_1 = 1$ but at higher value of $G_1 (= 100)$ it shows somewhat different behavior.

These effects may be physically interpreted as follows:

Due to increase in G_1 (at constant k_p), there is high decrease in Z_1, i.e. the volume fraction of solid particles in the mixture becomes comparatively very small. This effect induces comparatively more compression of the mixture in the region between shock and piston, which displays the above effect.

An increase in the value of the parameter for strength of the magnetic field M_A^{-2}

I. increases the value of β i.e. decreases the shock strength (see table 1);

II. decreases η_p (i.e. increases the distance of the piston from the shock front) in all the cases in adiabatic flow; whereas in isothermal flow, it decreases in all cases except for $k_p = 0.3, 0.4$ when $G_1 = 1$, where the effect is small and is of opposite nature (see table 1);

III. decreases the velocity V and density D at any point behind the shock front (see figures 1 and 2 for isothermal flow and 5 and 6 for adiabatic flow);

IV. decreases pressure P at any point in the flow-field behind the shock in isothermal flow; whereas in adiabatic flow it decreases when $G_1 = 1$ but it decreases in dust-free case ($k_p = 0$) and in the dusty case when $G_1 = 100$ (see figures 3 and 7 for adiabatic and isothermal flows respectively); and

V. increases the flow variable H at any point in the flow-field behind the shock in isothermal flow (see figure 3); whereas in adiabatic flow, it increases when

$G_1 = 1$ but it decreases in dust-free case ($k_p = 0$) and in the dusty case when $G_1 = 100$ (see figure 7).

6. Comparison Between Isothermal and Adiabatic Flows

I. From table 1 it is clear that η_p (position of the piston surface) in the adiabatic flow is greater than that in the isothermal flow. Physically, it means that the gas is more compressed in the adiabatic flow in comparison to that in isothermal flow. Thus the strength of the shock is higher in adiabatic flow than that in the isothermal flow.

II. In adiabatic flow at $k_p = 0$ (dust free case) the pressure P is almost constant in non-magnetic case ($M_A^{-2} = 0$); whereas in magnetic case ($M_A^{-2} = 0.005, 0.01$) P decreases very rapidly near the piston (see figure 8). Same behaviour is also obtained at $G_1 = 100$. In isothermal flow, the gradient of the pressure$P$ is constant in all cases.

III. Figure 6 shows that there is unbounded density distribution near the piston, in some cases, when the flow is adiabatic. This is quite expected and may be explained as follows. First of all Sedov [14] (see Ranga Rao and Ramana [11] and Vishwakarma and Nath [21]) pointed out that a limiting case, as $n \to \infty$ of a flow-field with power law shock

$$R \sim t^{n+1}\ , \tag{50}$$

is the flow-field formed with an exponential shock described by equation (2) for such flow with the power lawshock, in the adiabatic case, it can be seen from the asymptotic form of the adiabatic integral (see Vishwakarma and Nath [20])

$$\left(\frac{D}{(1-Z_1 D)}\right)^{\Gamma} D^{\frac{2n}{(n+1)(j+1)}} = \frac{p\,\eta^{\frac{2}{(n+1)}}}{M}(V-n-1)^{-\frac{2n}{(n+1)(j+1)}}\ , \tag{51}$$

that the density tends to infinity for$n > 0$ as the piston is approached, provided $(1 - Z_1 D)$ does not tend to zero. The density distribution exhibits this nature in the cases of non-magnetic dust-free gas ($M_A^{-2} = 0, k_p = 0$) and dusty gas with higher value of ($M_A^{-2} = 0, G_1 = 100$). Same behavior is also seen in magnetic cases. When $G_1 = 1$, this behavior of density is removed. In this case, this is perhaps due to the fact that the expression$(1 - Z_1 D)$ in (51) tends to zero as the piston is approached for$n > 0$. This phenomenon can be physically interpreted as follows. In the case of non-magnetic or magnetic dust-free gas or in the case of dusty gas with higher value of $G_1 = 1$, the path of the piston converges with the path of the particle immediately head, thus compressing the gas to infinite density; whereas in the case of$G_1 = 1$, the path of the piston is almost parallel to the path of the particle immediately ahead, the above behavior of the density distribution is removed.

It may therefore be concluded that the presence of dust particles with lower values of G_1 removes the nature of unbounded density distribution near the piston.

In the case of isothermal flow, the density is finite at the piston for all values of k_p and G_1. This seems to be necessary because with an unbounded density near the piston the temperature there approaches zero violating the basic assumption of zero temperature gradient throughout the flow.Therefore, it may be observed that the assumption of zero temperature gradient brings a profound change in the density distribution as compared to that of the adiabatic flow (except the case when G_1=1).

IV. In adiabatic flow at $G_1 = 100$ the flow variable H increases rapidly near the piston (see figure 8); whereas in isothermal flow at the flow variable H is almost constant (see figure 4).

6. Conclusion

In the present work, we have investigated the self-similar solution for the flow behind a strong exponential shock wave propagating in a perfectly conducting dusty gas in presence of an azimuthal magnetic field. The shock is driven by a piston moving with time according to an exponential law. On the basis of this work, one may draw the following conclusion:

I. An increase in the mass concentration of solid particles (k_p) decreases the shock strength at lower values of G_1 and increases it at its higher values. Also for $G_1 = 1$ it decreases the velocity, density and magnetic field at any point in the flow-field behind the shock; whereas for $G_1 = 100$, it increases these flow variables.

II. An increase in k_p decreases the pressure at any point in the flow-field when $G_1 = 1$ and increases it when $G_1 = 100$ in, both, the magnetic and non-magnetic cases except in the adiabatic flow with magnetic field and $G_1 = 100$.

III. An increase in the value of the ratio of the density of solid particles and the initial density of the perfect gas in the mixture(G_1) increases the shock strength. Also, it increases the velocity, density and magnetic field at any point in the flow-field behind the shock.

IV. An increase in G_1 increases the pressure P when $M_A^{-2} = 0$,in both, the isothermal and adiabatic flows. Also, in the magnetic case, the pressure P increases at any point in the flow-field when the flow is isothermal; whereas in adiabatic flow, it exhibits somewhat different behavior.

V. The presence of magnetic field decreases the velocity and density at any point in the flow-field behind the shock.

VI. Also, the presence of magnetic field decreases the pressure P at any point in the flow-field behind the shock in isothermal flow; whereas in adiabatic flow it decreases when G_1 but it shows somewhat different behaviour in the dust-free case ($k_p = 0$) and in dusty case with $G_1 = 100$.

VII. In general, the presence of magnetic field decreases the shock strength. This effect of magnetic field on shock strength, in both cases (isothermal flow and adiabatic flow) decreases significantly by increasing k_p at $G_1 = 1$; whereas at $G_1 = 100$ the effect of magnetic field on the shock strength is almost not if influenced bike by increasingk_p.

Table 1. Variation of the density ratio β $(= \frac{\rho_1}{\rho_2})$ across the shock front and the position of the piston surface η_p for different values of k_p, G_1 and M_A^{-2} with $\gamma = \frac{5}{3}$.

β						
	$G_1 = 1$			$B_1 = 100$		
k_p	$M_A^{-2} = 0$	$M_A^{-2} = 0.005$	$M_A^{-2} = 0.01$	$M_A^{-2} = 0$	$M_A^{-2} = 0.005$	$M_A^{-2} = 0.01$
0.0	0.25000	0.25557	0.26104	0.25000	0.25557	0.26104
0.1	0.32040	0.32318	0.32599	0.24574	0.25145	0.25704
0.2	0.39105	0.39251	0.39400	0.24070	0.24658	0.25233
0.3	0.46198	0.46278	0.46359	0.23459	0.24076	0.24670
0.4	0.53333	0.53376	0.53420	0.22737	0.23370	0.23988
ηp (Isothermal Flow)						
0.0	0.92989	0.92789	0.92599	0.92989	0.92789	0.92599
0.1	0.90090	0.90035	0.89978	0.93091	0.92881	0.92695
0.2	0.86943	0.86938	0.86932	0.93211	0.93004	0.92808
0.3	0.83492	0.83506	0.83519	0.93355	0.93144	0.92944
0.4	0.79661	0.79681	0.79701	0.93531	0.93314	0.93110
ηp (Adiabatic Flow)						
0.0	0.94503	0.94174	0.93890	0.94503	0.94174	0.93890
0.1	0.91454	0.91346	0.91238	0.94604	0.94271	0.93983
0.2	0.88163	0.88117	0.88071	0.94722	0.94383	0.94091
0.3	0.86525	0.84556	0.84535	0.94860	0.94515	0.94218
0.4	0.80614	0.80606	0.80598	0.95023	0.94671	0.94368

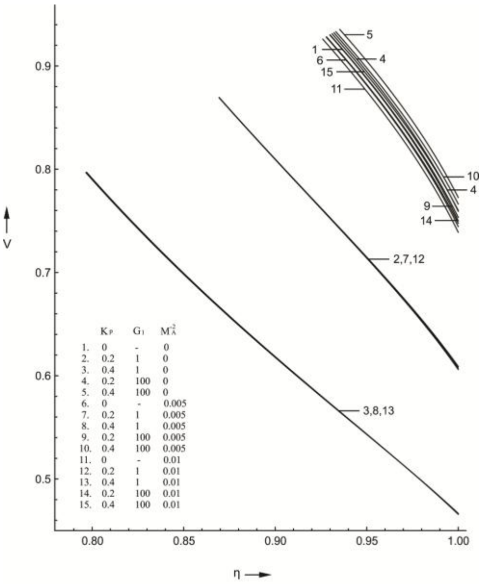

Figure 1: Velocity distribution in the region behind the shock front for isothermal flow.

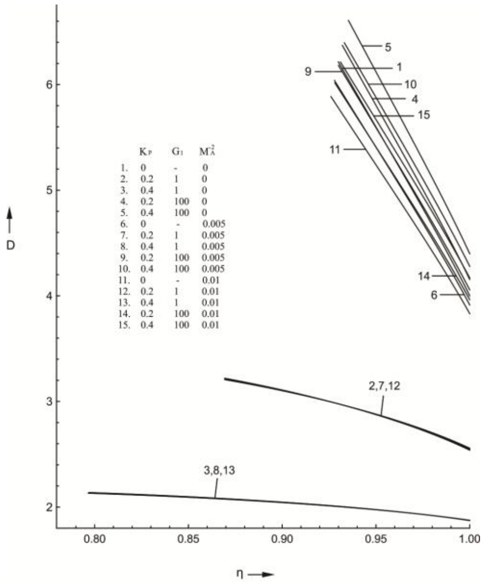

Figure 2: Density distribution in the region behind the shock front for isothermal flow.

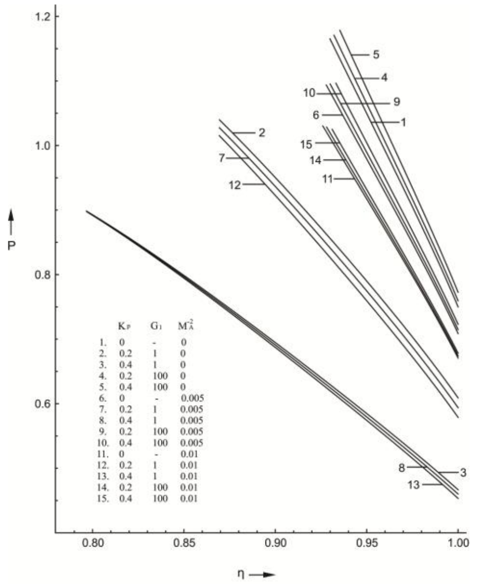

Figure 3: Pressure distribution in the region behind the shock front for isothermal flow.

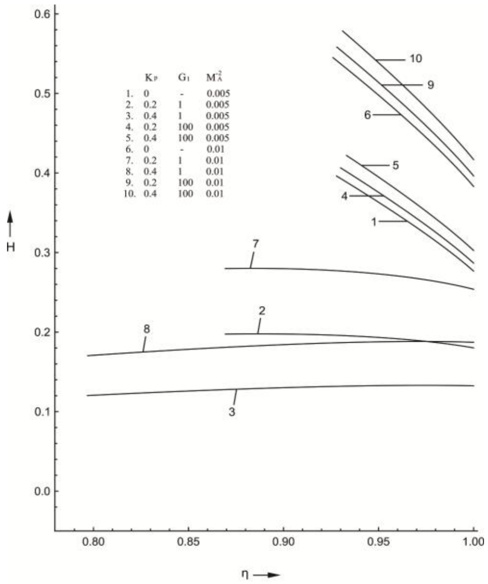

Figure 4: Magnetic field distribution in the region behind the shock front for isothermal flow.

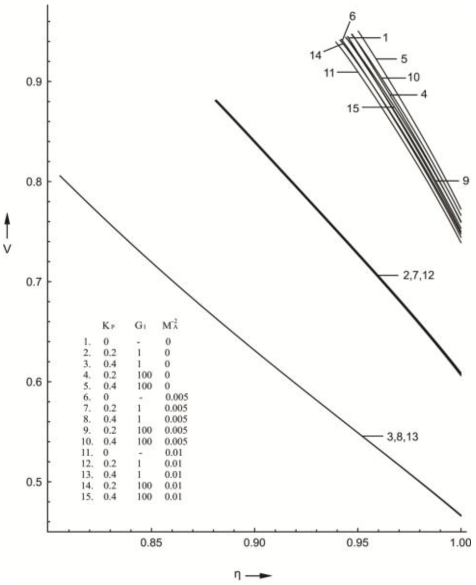

Figure 5: Velocity distribution in the region behind the shock front for adiabatic flow.

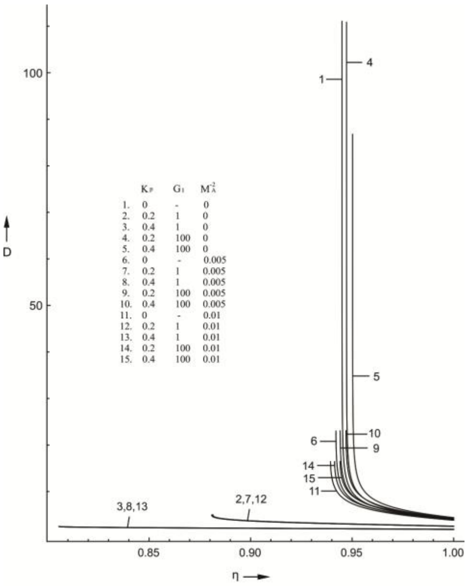

Figure 6: Density distribution in the region behind the shock front for adiabatic flow.

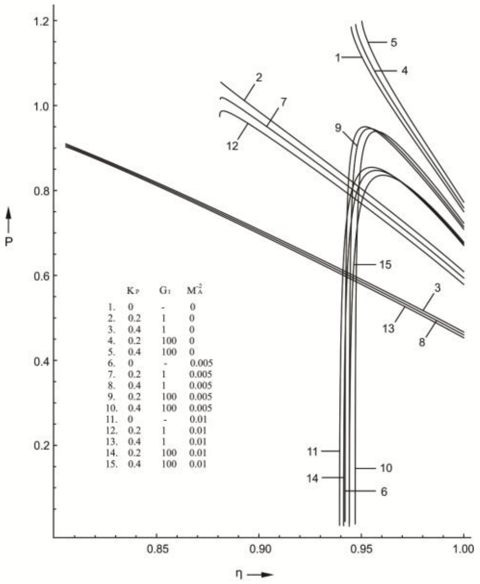

Figure 7: Pressure distribution in the region behind the shock front for adiabatic flow.

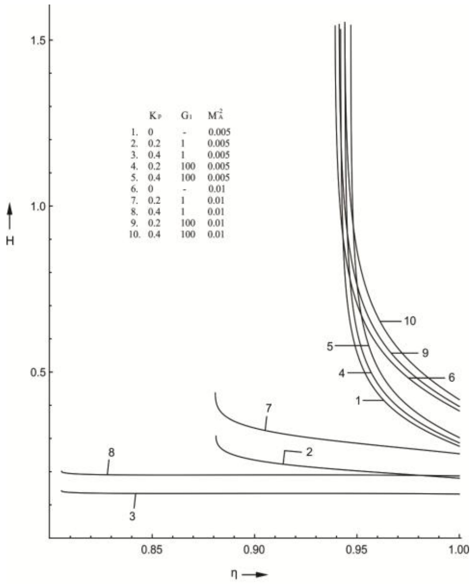

Figure 8: Magnetic field distribution in the region behind the shock front for adiabatic flow.

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