Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.17, No.1&2, January-June, 2018, 57-64

BIANCHI TYPE VI_0 INFLATIONARY COSMOLOGICAL MODEL WITH FLAT POTENTIAL AND EXPONENTIAL AVERAGE SCALE FACTOR

Radhika Goyal
Deparment of Mathematics, Biyani Girls College, Jaipur-302039, India
Email: radhika.goyal1982@gmail.com

Abstract: Bianchi Type VI_0 inflationary cosmological model with flat potential and exponential average scalar factor is investigated. The anisotropic parameter (Ã) is not initially zero but tends to zero at late time i. e. The model isotropizes at late time. The model does not lead to FRW model in general but for specific value of $K = 3H_0^2$, it leads to FRW model which is isotropic and homogeneous. The scalar field (Ø) decreases with time & for large value of t, it leads to zero. The deceleration parameter $q = -1$ showes that the model leads to de-Sitter space time. The model has no initial singularity i. e. at t = 0, the slow role parameter (ϵ, δ) are in agreement with Plank results (2013).

Keywords: Bianchi $VI_{0,}$ scale factor, flat potential, isotropy.

1. Introduction

The stage of exponential expansion of universe is termed as inflation. Guth [9] introduced the idea of early inflationary phase in context of grand unified field theory (Zel'dovich [17]) in which symmetry breaking phase transition occur with decrease of temperature at the very early stage of evolution of universe and suggested that rapid expansion is due to false vacuum as confirmed by Cosmic Microwave Background radiation (Bassett et al. [6]). The inflationary scenario explains several mysteries of modern cosmology like flatness problem, horizon problem, homogeneity and isotropy etc. as mentioned by Liddle and Lyth [10].

The inflationary cosmology by Guth [9] is not replacement for the Hot Big Bang model but rather an add-on that occurs at very early times without disturbing any of its success. The universe in smaller scale is neither homogeneous nor isotropic. We expect the universe to have these properties in its early stage. The adequacy of spatially homogeneous and isotropic Fridman-Roberston – Walker (FRW) models for describing the present state of universe, is no basis for expecting suitable for describing the early stages of evolution of universe. Cosmological models which are spatially homogeneous but not isotropic have significant role in the description of the universe at its early stage of evolution.

Rothman & Ellis [12] have pointed out that we can have the solution of isotropy problem if we work with anisotropic space-times and these space-times can be isotropized under very general circumstances. Stein-Schabes [15] has investigated that inflation will take place if effective potential V(Ø) has flat region where Higgs field (Ø) evolves very slowly but the universe expands in an exponential way due to vacuum field energy. Bali and Jain [2], Bali [1], Bali and Swati [4], Singh and Kumar [14] investigated inflationary cosmological models with flat potential under different space times following Rothman & Ellis [12]. Spatially homogeneous and anisotropic Bianchi space-time (I-IX) are considered to study the universe in its early stage of evolution. Among these Bianchi Type VI_0 space-time is of particular interest because this is simple generalization of Bianchi Type I space-time. Barrow [5] pointed out that Bianchi Type VI_0 space-time gives a better explanation of some of the cosmological problems and these can be isotropized in special case. Seeing the importance of these models, various authors viz. Ellis and MacCallum [8], Collins [7], Roy and Singh [13], Bali et al [3] have investigated cosmological models considering Bianchi Type VI_0 space-time in different contexts.

2. Metric and Field Equations

We consider Bianchi Type VI_0 space-time in the form

$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}B^2dy^2 + e^{-2x}C^2dz^2 \tag{1}$$

Where A, Band C are metric potentials and functions of t-alone. We assume the coordinates to be commoving so that $v^1 = 0 = v^2 = v^3,\ v^4 = 1$.

The Lagrangian in which gravity is minimally coupled to the scalar field (Ø) is given by (Stein-Schabes [15] as

$$L = \int \sqrt{-g}\ \{R - \frac{1}{2} g^{ij}\, \partial_i \emptyset \partial_j \emptyset - V(\emptyset)\} d^4x \tag{2}$$

The Einstein's field equations (in gravitational units $8\pi G = c = 1$) in case of massless scalar field with potential V(Ø) is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{3}$$

With

$$T_{ij} = \partial_i \emptyset \partial_j \emptyset - [\frac{1}{2} \partial_k \emptyset \partial^k \emptyset + V(\emptyset)] g_{ij} \tag{4}$$

The conservation relation $T^j_{i;j} = 0$ leads to

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\partial_\mu \emptyset\right) = -\frac{dV(\emptyset)}{d\emptyset} \tag{5}$$

Which leads to

$$\emptyset_{44} + \emptyset_4 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{dV}{d\emptyset} \tag{6}$$

Now Einstein's field equation (3) together with (4) for space-time (1) leads to

$$\frac{B_4C_4}{BC}+\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{1}{A^2}=-\frac{1}{2}\phi_4^2+V(\phi) \quad (7)$$

$$\frac{A_4C_4}{AC}+\frac{A_{44}}{A}+\frac{C_{44}}{C}-\frac{1}{A^2}=-\frac{1}{2}\phi_4^2+V(\phi) \quad (8)$$

$$\frac{A_4B_4}{AB}+\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{1}{A^2}=-\frac{1}{2}\phi_4^2+V(\phi) \quad (9)$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{1}{A^2}=\frac{1}{2}\phi_4^2+V(\phi) \quad (10)$$

$$\frac{B_4}{B}-\frac{C_4}{C}=0 \quad (11)$$

Equation (11) leads to

$$B=mC \quad (12)$$

Where m is constant of integration

3. Solution of field equations

To get the deterministic model, we assume that the average scale factor (R) as

$$R^3=ABC=e^{3H_0t} \quad (13)$$

And potential $V(\phi)$ is flat. Thus we have

$$V(\phi)=K \quad (14)$$

Where H_0 is Hubble constant and K is a constant. Now equation (13) using (12) leads to

$$AB^2=me^{3H_0t} \quad (15)$$

Equation (11) leads to

$$\frac{B_{44}}{B}=\frac{C_{44}}{C} \quad (16)$$

From equation (15), we have

$$\frac{A_4}{A}+2\frac{B_4}{B}=3H_0 \quad (17)$$

Equation (7), (10), (11), (16) and (17) lead to

$$\frac{B_{44}}{B}-\frac{B_4^2}{B^2}+3H_0\frac{B_4}{B}=K \quad (18)$$

Thus, we have

$$\left(\frac{B_4}{B}\right)_4+3H_0\left(\frac{B_4}{B}\right)=K \quad (19)$$

Which leads to

$$\frac{B_4}{B}=\frac{K}{3H_0}+Le^{-3H_0t} \quad (20)$$

This leads to

$$B = N\, exp\left[\frac{1}{3H_0}(Kt - Le^{-3H_0 t})\right] \tag{21}$$

where N and L are constants of integration.

Now B = mC leads to

$$C = \frac{N}{m}\; exp\left[\frac{1}{3H_0}(Kt - Le^{-3H_0 t})\right] \tag{22}$$

From equation (15) and (21), we have

$$A = \frac{m e^{3H_0 t}}{N^2 exp\left[\frac{2}{3H_0}(Kt - Le^{-3H_0 t})\right]} \tag{23}$$

After suitable transformation of coordinates, the metric (1) leads to

$$ds^2 = -dt^2 + \frac{e^{6H_0 t}}{exp\left[\frac{4}{3H_0}(Kt - Le^{-3H_0 t})\right]} dX^2 + \; exp\left[\frac{2}{3H_0}(Kt - Le^{-3H_0 t})\right] dY^2$$

$$+ \, exp\left[\frac{2}{3H_0}(Kt - Le^{-3H_0 t})\right] dZ^2 \tag{24}$$

Where $\frac{m}{N^2} x = X, \quad Ny = Y, \quad \frac{N}{m} z = Z$

4. Physical and geometrical aspects

The expansion (θ), shear (σ), Hubble parameter (H), the scalar field (Ø), deceleration parameter (q), anisotropy parameter (Ã), slow roll parameters (ϵ, δ), the third slow parameter (S) for the model (24) are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}$$

$$= 3H_0 \tag{25}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{A_4}{A} - \frac{B_4}{B}\right)$$

$$= \sqrt{3}\left[\left(H_0 - \frac{K}{3H_0}\right) - Le^{-3H_0 t}\right] \tag{26}$$

$$H = \frac{\theta}{3} = H_0 \qquad (27)$$

Equation (6) leads to

$$\emptyset = -\frac{l}{3H_0} e^{-3H_0 t} \tag{28}$$

Where l is constant of integration.

$$q = -\frac{R_{44}/R}{R_4^2/R^2}$$

$= -1$ (29)

Anisotropic parameter (Ã) is defined as

$$\tilde{A} = \frac{1}{3}\left[\left(\frac{H_1}{H} - 1\right)^2 + \left(\frac{H_2}{H} - 1\right)^2 + \left(\frac{H_3}{H} - 1\right)^2\right] \quad (30)$$

Where $H_1 = \frac{A_4}{A},\ H_2 = \frac{B_4}{B},\ H_3 = \frac{C_4}{C}$ (31)

Thus

$$\tilde{A} = 2\left[\frac{1}{3H_0}(K + 3H_0 L e^{-3H_0 t}) - 1\right]^2 \quad (32)$$

Now we calculate slow roll parameters (ϵ, δ) to examine whether these parameters are in excellent agreement with Plank [11] results. For the model (24) the average scale factor (R) is given by

$$R = e^{H_0 t} \quad (33)$$

Slow roll parameter as given by Unnikrishan and Sahni [16] as

$$\epsilon = -\frac{H_4}{H} = 0 \quad (34)$$

And

$$\delta = \epsilon - \frac{\epsilon_4}{2H_0} = 0 \quad (35)$$

Thus slow roll parameters (ϵ, δ) corresponds to

$\epsilon \ll 1, \delta \ll 1$

Now we discus third slow role parameter (S) as given by canonical scalar field (Ø) with the Lagrangian

$$L(\emptyset, X) = X - V(\emptyset) \quad (36)$$

Where $X = \frac{\emptyset_4^2}{2}$ (37)

For a general $L(\emptyset, X)$, the third slow roll parameter (S) is defined as

$$S = \frac{(C_s)_4}{HC_S} \quad (38)$$

Where C_S is the speed of the scalar field given as

$$C_S^2 = \frac{\partial L / \partial X}{\frac{\partial L}{\partial X} + 2X\frac{\partial^2 L}{\partial X^2}} \quad (39)$$

Which leads to

$C_S^2 = 1$

Thus S = 0

Thus for a canonical scalar field, the value of S is identically zero.

5. Conclusion

We find that spatial volume increases exponentially which represents inflationary scenario of the universe. Since in general $\sigma/\theta \neq 0$ so model represents anisotropic space-time but as $t \to \infty$ and for specific value $K = 3H_0$, the model isotropizes. The deceleration parameter q = -1, shows that model leads to de-Sitter space time. The anisotropy parameter Ã is not zero initially zero i. e. anisotropy is initially large but disappears for large values of t i. e. Model isotropizes at late time and $K = 3H_0$. In general $\sigma \neq 0$ so model does not leads to FRW model immediately but for large values of t, it leads to FRW model because shear is zero in FRW model. The scalar field (Ø) decreases with time and for large values of time, it vanishes. The slow roll parameter (ϵ, δ) are in agreement with Plank [11] results.

Acknowledgment: The author is extremely thankful to Prof. Raj Bali, CSIR Emeritus Scientist for helpful discussion and suggestions.

References

[1] Bali, R. (2011). "Inflationary scenario in Bianchi Type I space-time", Int. J. Theor. Phys. **50**, 3043.

[2] Bali. R. And Jain, V. C. (2002). "Bianchi Type I inflationary Universe in General Relativity", Pramana-J. Phys. **59**, 1-7.

[3] Bali, R., Pradhan, A. and Hassan, A. (2008). "Bianchi Type VI_0 magnetized barotropic bulk viscous fluid massive string universe in General Relativity", Int. J. Theor. Phys. **47**, 2594-2604.

[4] Bali, R. and Swati (2015). "Bianchi Type VIII Inflationary Universe with massless scalar field in General Relativity", Prespace time journal, **6**, 679-683.

[5] Barrow, J. D. (1984). "Helium formation in cosmologies with anisotropic curvature", Roy. Astron Society **211**, 221-227.

[6] Basset, B. A., Tsujikawa, S. And Wans, D. (2006). "Inf;ation Dynamics and Reheating", Rev. Modern Phys. **78**, 537-589.

[7] Collins, C. B. (1971). "More qualitative cosmology", Comm. in Math. Phys., **23**, 137-158.

[8] Ellis, G. F. R. And MacCallum, M. A. H. (1969). "A class of homogeneous cosmological models", Comm. in Math. Phys., **12**, 108-141.

[9] Guth, A. H. (1981). "Inflationary universe: A possible solution to the horizon and flatness problems", Phys. Rev. D, **23**, 347-356.

[10] Liddle, A. R. And Lyth, D. H. (1992). “COBE gravitational waves, inflation and extendedinflation”, Phys. Lett. B 291, 391-398.

[11] P. A. R. Ade et al. (2013). Plank Collaboration “Plank 2013 results XXII constrains on inflation”, arXiv, 1303, 5082.

[12] Rothman, T. and Ellis, G. F. R. (1986). “Can inflation occur in anisotropic cosmologies?”, Phys. Lett. B.180, 19-24.

[13] Roy, S. R. and Singh, J. P. (1983). “Some Bianchi TypeVI_0 cosmological models with free gravitational field of magnetic type”, Acta Physical Austriaca, Vol. 55, 57-66.

[14] Singh, C. P. and Kumar, S. (2007). “Bianchi Type III inflationary modelswith constant deceleration parameter in GeneralRelativity.” Pramana- J.Phy. 68, 707-720.

[15] Stein-Schabes, J. A. (1987). “Inflation in spherically symmetric inhomogeneous models”, Phys. Rev. D. 35, 2345-2351.

[16] Unnikrishnan, S. And Sahni, V. (2013). “Resurrecting power law inflation in the light of plank results”, J. Cosmol. Astropart. Phys. 10, 063.

[17] Zel’ dovich, Ya, B. (1962). “The equation of state at altra high densities and its relativistic limitations”, Soviet. Phys. JETP 14, 1143-1147.