

ON SPECIAL FORMS OF T-TENSOR IN FOUR DIMENSIONAL FINSLER SPACES

T. N. Pandey and Rajesh Kumar Gupta

Department of Mathematics & Statistics,

D.D.U. GKP University, Gorakhpur, U.P. India-273009

Email: tnp1952@gmail.com and rajnandiniharshita@gmail.com

Abstract: In this paper, we have studied special form of the T-tensor T_{hijk} of a four-dimensional Finsler Space in terms of main scalars.

2010 Mathematics Subject Classification: 52B40, 53C60

Keywords and Phrases: T-tensors, main scalars, C-reducible, semi C-reducible Finsler Space.

1. Introduction

Let F^n be n-dimensional Finsler space equipped with the metric function $L(x, y)$. $C_{hij}(x, y)$ be the (h) hv-torsion tensor of F^n and its v-covariant derivative denoted by $|_k$ then T-tensor (T_{hijk}) is defined([10],page 188) by

$$T_{hijk} = LC_{hij}|_k + C_{ijk}l_h + C_{hjk}l_i + C_{hik}l_j + C_{hij}l_k \quad (1)$$

where l_i are covariant components of unit vector along the element of support y^i . In the paper [12] Singh and others have studied many special T-tensor[3,7,8,9] of the form

$$A. \quad T_{hijk} = \mu(h_{hi}h_{jk} + h_{hj}h_{ik} + h_{hk}h_{ij}) \quad (2)$$

$$B. \quad T_{hijk} = h_{hi}P_{jk} + h_{hj}P_{ik} + h_{hk}P_{ij} + h_{ik}P_{hj} + h_{ij}P_{hk} + h_{jk}P_{hi} \quad (3)$$

$$C. \quad T_{hijk} = a_h C_{ijk} + a_i C_{hij} + a_j C_{hik} + a_k C_{hij} + h_{hi}Q_{jk} + h_{hj}Q_{ik} \\ + h_{hk}Q_{ij} + h_{ij}Q_{hk} + h_{ik}Q_{hj} + h_{jk}Q_{hi} \quad (4)$$

$$D. \quad T_{hijk} = \rho C_h C_i C_j C_k + a_h C_i C_j C_k + a_i C_h C_j C_k + a_j C_h C_i C_k + a_k C_h C_i C_j \quad (5)$$

$$E. \quad T_{hijk} = \rho C_i C_j C_k \quad (6)$$

where μ is a scalar, P_{ij} and Q_{ij} are certain tensor fields, all are homogeneous of degree (-1) in y^i . Further, a_i is a covariant vector field, homogeneous of degree zero in y^i and ρ is a scalar function positively homogeneous of degree three in y^i .

Some examples of Finsler spaces having special forms of T_{hijk} are given in the paper [2,5,8,12,13,14].

In the present paper, we have studied the four dimensional Finsler space whose T-tensor is of the form (5) and (6). We have written tensors of four-dimensional Finsler space in the Miron's frame in terms of scalars.

2. Scalar Components in Miron Frame

Let F^4 be a four dimensional Finsler space with the fundamental function $L(x, y)$. With the help of $L(x, y)$ four mutually perpendicular unit vectors $\{e_{\alpha}^i\}$, $\alpha = 1, 2, 3, 4$ are constructed as defined by M. Matsumoto and called as Miron's frame ([10]page179), where $e_{(1)}^i = l^i = \frac{y^i}{L}$ is the unit vector along y^i , $e_{(2)}^i = m^i = \frac{C^i}{C}$ is the normalized torsion vector, $e_{(3)}^i = n^i$ and $e_{(4)}^i = p^i$ are constructed by $g_{ij} e_{\alpha}^i e_{\beta}^j = \delta_{\alpha\beta}$, where C is the length of torsion vector $C_i = C_{ijk} g^{jk}$. Greek letters $\alpha, \beta, \gamma, \delta$ varies from 1 to 4 and summation convention is understood for both the Greek and Latin indices.

In Miron's frame an arbitrary tensor field can be expressed by scalar components along the unit vectors $\{e_{\alpha}^i\}$, $\alpha = 1, 2, 3, 4$. For instance, let T_j^i be a tensor field of Type (1, 1) then the scalar components $T_{\alpha\beta}$ of T_j^i are defined ([10] page179) by $T_{\alpha\beta} = T_j^i e_{\alpha i} e_{\beta}^j$ and the tensor components T_j^i are obtained as $T_j^i = T_{\alpha\beta} e_{\alpha}^i e_{\beta j}$.

From the equation, $g_{ij} e_{\alpha}^i e_{\beta}^j = \delta_{\alpha\beta}$, we have

$$g_{ij} = l_i l_j + m_i m_j + n_i n_j + p_i p_j \quad (7)$$

The torsion-tensor C_{ijk} is defined as,

$$C_{ijk} = \frac{1}{2} \frac{\partial g_{jk}}{\partial y^i}$$

satisfies $C_{ijk} l^k = 0$ and is symmetric in i, j, k, and if $C_{\alpha\beta\gamma}$ be scalar components of LC_{ijk} then, we can write

$$LC_{ijk} = C_{\alpha\beta\gamma} e_{\alpha i} e_{\beta j} e_{\gamma k}, \quad (8)$$

Then, the results by Prasad et al.[11], we haave

$$\begin{aligned}
 LC_{ijk} = & C_{222}m_i m_j m_k + C_{333}n_i n_j n_k + C_{444}p_i p_j p_k + C_{233}\pi_{(ijk)}(m_i n_j n_k) \\
 & + C_{244}\pi_{(ijk)}(m_i p_j p_k) + C_{344}\pi_{(ijk)}(n_i p_j p_k) + C_{322}\pi_{(ijk)}(m_i m_j n_k) \\
 & + C_{433}\pi_{(ijk)}(n_i n_j p_k) + C_{422}\pi_{(ijk)}(m_i m_j p_k) + C_{234}\pi_{(ijk)}\{m_i(n_j p_k + n_k p_j)\}
 \end{aligned} \tag{9}$$

where $\pi_{(ijk)}$ denote the cyclic permutation of indices i, j, k , and summation. For instance

$$\pi_{(ijk)}(A_i B_j C_k) = A_i B_j C_k + B_i C_j A_k + C_i A_j B_k.$$

Contracting (8) with g^{jk} , we get $LCm_i = C_{\alpha\beta\gamma}e_{\alpha}^i$. Thus, if we put

$$\begin{aligned}
 C_{222} = H, \quad C_{233} = I, \quad C_{244} = K, \quad C_{333} = J, \quad C_{344} = J', \\
 C_{444} = H', \quad C_{433} = I', \quad C_{234} = K',
 \end{aligned} \tag{10}$$

Then, we have

$$H + I + K = LC, \quad C_{322} = -(J + J'), \quad C_{422} = -(H' + I'), \tag{11}$$

The eight scalars $H, I, J, K, H', I', J', K'$ are called the main scalars of a four dimensional Finsler space.

The v-covariant derivative of the frame field e_{α}^i ([10]page179) is given by

$$Le_{\alpha}^i |_{j} = V_{\alpha)\beta\gamma} e_{\beta}^i e_{\gamma}^j, \tag{12}$$

where $V_{\alpha)\beta\gamma}, \gamma$ being fixed are given by

$$V_{\alpha)\beta\gamma} = \begin{bmatrix} 0 & \delta_{2\gamma} & \delta_{3\gamma} & \delta_{4\gamma} \\ \delta_{2\gamma} & 0 & u_{\gamma} & v_{\gamma} \\ \delta_{3\gamma} & -u_{\gamma} & 0 & w_{\gamma} \\ \delta_{4\gamma} & -v_{\gamma} & -w_{\gamma} & 0 \end{bmatrix} \quad \text{and} \quad \begin{aligned} V_{2)3\gamma} &= -V_{3)2\gamma} = u_{\gamma} \\ V_{2)4\gamma} &= -V_{4)2\gamma} = v_{\gamma} \\ V_{3)4\gamma} &= -V_{4)3\gamma} = w_{\gamma} \end{aligned} \tag{13}$$

Thus in a four dimensional Finsler space there are, three v-connection vectors u_i, v_i, w_i .

i.e.

$$u_i = u_{\gamma} e_{\gamma}^i, \quad v_i = v_{\gamma} e_{\gamma}^i, \quad w_i = w_{\gamma} e_{\gamma}^i \tag{14}$$

Equation (12) in view of equation (13) may be explicitly written as

$$Ll_i |_{j} = m_i m_j + n_i n_j + p_i p_j \quad Lm_i |_{j} = -l_i m_j + n_i u_j + p_i v_j \tag{15}$$

$$Ln_i |_{j} = -l_i n_j - m_i u_j + p_i w_j, \quad Lp_i |_{j} = -l_i p_j - m_i v_j - n_i w_j.$$

Since m_i , n_i , p_i are homogeneous functions of degree zero in y^i , we have

$$Lm_i |_j l^j = Ln_i |_j l^j = Lp_i |_j l^j = 0,$$

which in view of equation (14) and (15) gives, $u_1 = 0, v_1 = 0, w_1 = 0$. Therefore, we have the following lemma.

Lemma (2.1) In a four-dimensional Finsler space F^4 , the first scalar components u_1, v_1, w_1 of the v_i -connection vector u_i, v_i, w_i vanishes identically, that is u_i, v_i, w_i are orthogonal to l^i .

The scalar derivative of the adopted components $T_{\alpha\beta}$ of T_j^i is defined ([10], 27.13', page 180).

$$T_{\alpha\beta;\gamma} = L(\partial_k T_{\alpha\beta}) e_{\gamma}^k + T_{\mu\beta} V_{\mu\alpha\gamma} + T_{\alpha\mu} V_{\mu\beta\gamma} \quad (16)$$

Thus $T_{\alpha\beta;\gamma}$ are adopted components of $LT_j^i |_k$, i.e.

$$LT_j^i |_k = T_{\alpha\beta;\gamma} e_{\alpha}^i e_{\beta}^j e_{\gamma}^k \quad (17)$$

If the tensor field T_j^i is positively homogeneous of degree zero in y^i , then $T_{\alpha\beta}$ is also positively homogeneous of degree zero in y^i , so equation (16) gives

$$T_{\alpha\beta;1} = T_{\mu\beta} V_{\mu\alpha 1} + T_{\alpha\mu} V_{\mu\beta 1},$$

which in view of (13) and lemma (7) gives $T_{\alpha\beta;1} = 0$. Therefore, we have the following proposition.

Proposition (2.1). Let T_j^i be a tensor of (1,1) type of a four-dimensional Finsler space positively homogeneous of degree zero in y^i and $T_{\alpha\beta}$ be the scalar component of T_j^i then, $T_{\alpha\beta;1} = 0$.

Now, let T_j^i be positively homogeneous of degree r in y^i and $T_{\alpha\beta}$ be the scalar components of $L^{-r} T_j^i$, then

$$L(L^{-r} T_j^i) |_k = T_{\alpha\beta;\gamma} e_{\alpha}^i e_{\beta}^j e_{\gamma}^k = L^{-r+1} T_j^i |_k - r L^{-r}$$

$T_j^i e_{1k}$, which implies

$$L^{-r+1} T_j^i |_k = (T_{\alpha\beta;\gamma} + r T_{\alpha\beta} \delta_{1\gamma}) e_{\alpha}^i e_{\beta}^j e_{\gamma}^k. \quad (18)$$

Hence we have

Proposition (2.2). If the tensor field T_j^i is positively homogeneous of degree r in y^i

and $T_{\alpha\beta}$ be the scalar components of $L^{-r}T_j^i$, then the scalar components of $L^{-r+1}T_j^i|_k$ are given by $T_{\alpha\beta;\gamma} + rT_{\alpha\beta}\delta_{1\gamma}$.

The T- tensor T_{hijk} of special form[1]:

$$T_{hijk} = \rho C_h C_i C_j C_k + a_h C_i C_j C_k + a_i C_h C_j C_k + a_j C_h C_i C_k + a_k C_h C_i C_j$$

The scalar components of LT_{hijk} are given by [12].

$$LT_{hijk} = [C_{\alpha\beta\gamma;\delta} + C_{\beta\gamma\delta}\delta_{1\alpha} + C_{\alpha\gamma\delta}\delta_{1\beta} + C_{\alpha\beta\delta}\delta_{1\gamma}]e_{(\alpha)h}e_{(\beta)i}e_{(\gamma)j}e_{(\delta)k} \quad (19)$$

where the semicolon denotes the v -scalar derivative. We shall use the following relations, which have been obtained in the paper [11].

$$\begin{aligned} C_{1\beta\gamma;\delta} &= -C_{\beta\gamma\delta} \\ C_{222;\delta} &= H_{;\delta} + 3(J+J')u_{\delta} + 3(H'+I)v_{\delta} \\ C_{223;\delta} &= -(J+J')_{;\delta} + (H+2I)u_{\delta} - 2K'v_{\delta} + (H'+I)w_{\delta} \\ C_{233;\delta} &= I_{;\delta} - (3J+2J')u_{\delta} - I'v_{\delta} - 2K'w_{\delta} \\ C_{244;\delta} &= K_{;\delta} - J'u_{\delta} - (3H'+2I')v_{\delta} + 2K'w_{\delta} \\ C_{333;\delta} &= J_{;\delta} + 3Iu_{\delta} - 3I'w_{\delta} \\ C_{344;\delta} &= J'_{;\delta} + Ku_{\delta} + 2K'v_{\delta} + (2I'-H')w_{\delta} \\ C_{444;\delta} &= H'_{;\delta} + 3Kv_{\delta} + 3J'w_{\delta} \\ C_{433;\delta} &= I'_{;\delta} + 2K'u_{\delta} + Iv_{\delta} + (J-2J')w_{\delta} \\ C_{422;\delta} &= -(H'+I')_{;\delta} - 2K'u_{\delta} + (H-2K)v_{\delta} - (J+J')w_{\delta} \\ C_{234;\delta} &= K'_{;\delta} - (H'+2I')u_{\delta} - (J+2J')v_{\delta} + (I-K)w_{\delta} \end{aligned} \quad (20)$$

where u_{δ}, v_{δ} and w_{δ} are the scalar components of v -connection vector u_i, v_i and w_i respectively.

Let a_{α} be the scalar components of La_i .

$$La_i = a_\alpha e_{\alpha i} \quad (21)$$

We assume that the T-tensor of the space F^4 have the form (11) then in view of the relation (19)

$$LT_{hijk} = C^3[CL\rho\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma}\delta_{2\delta} + a_\alpha\delta_{2\beta}\delta_{2\gamma}\delta_{2\delta} + a_\beta\delta_{2\alpha}\delta_{2\gamma}\delta_{2\delta} \\ + a_\gamma\delta_{2\alpha}\delta_{2\beta}\delta_{2\delta} + a_\delta\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma}]e_{(\alpha)h}e_{(\beta)i}e_{(\gamma)j}e_{(\delta)k} \quad (22)$$

Comparing (19) and (22) we get

$$C_{\alpha\beta\gamma;\delta} + C_{\beta\gamma\delta}\delta_{1\alpha} + C_{\alpha\gamma\delta}\delta_{1\beta} + C_{\alpha\beta\delta}\delta_{1\gamma} = C^4L\rho\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma}\delta_{2\delta} + C^3(a_\alpha\delta_{2\beta}\delta_{2\gamma}\delta_{2\delta} \\ + a_\beta\delta_{2\alpha}\delta_{2\gamma}\delta_{2\delta} + a_\gamma\delta_{2\alpha}\delta_{2\beta}\delta_{2\delta} + a_\delta\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma}) \quad (23)$$

Since T_{hijk} is an indicatory tensor, from (17) it follows that $a_i y^i = 0$

Which in view of (19) gives $a_1 = 0$ thus we have from equation (18)

$$H_{;\delta} + 3(J+J')u_\delta + 3(H'+I')v_\delta = C^3(L\rho C + 3a_2)\delta_{2\delta} + C^3a_\delta \quad (a)$$

$$I_{;\delta} - (3J+2J')u_\delta - I'v_\delta - 2K'w_\delta = 0 \quad (b)$$

$$-(J+J')_{;\delta} + (H-2I)u_\delta - 2K'v_\delta + (H'+I')w_\delta = C^3a_3\delta_{2\delta} \quad (c)$$

$$K_{;\delta} - J'u_\delta - (3H'+2I')v_\delta + 2K'w_\delta = 0 \quad (d)$$

$$J_{;\delta} + 3Iu_\delta - 3I'w_\delta = 0 \quad (e) \quad (24)$$

$$J'_{;\delta} + Ku_\delta + 2K'v_\delta + (2I'-H')w_\delta = 0 \quad (f)$$

$$H'_{;\delta} + 3Kv_\delta + 3J'w_\delta = 0 \quad (g)$$

$$I'_{;\delta} + 2K'u_\delta + Iv_\delta + (H-2K)v_\delta - (J-2J')w_\delta = 0 \quad (h)$$

$$-(H'+I')_{;\delta} - 2K'u_\delta + (H-2K)v_\delta - (J+J')w_\delta = C^3a_4\delta_{2\delta} \quad (i)$$

$$K'_{;\delta} - (H'+2I')u_\delta - (J+2J')v_\delta + (I-K)w_\delta = 0 \quad (j)$$

Adding eq. (c),(e),(f), we get

$$3Iu_\delta - 3I'w_\delta + (H-2I)u_\delta - 2K'v_\delta + (H'+I')w_\delta + Ku_\delta + 2K'v_\delta + (2I'-H')w_\delta = C^3a_3\delta_{2\delta} \\ (H+I+K)u_\delta - I'w_\delta = C^3a_3\delta_{2\delta}$$

$$LCu_{\delta} = C^3 a_3 \delta_{2\delta}$$

$$LCu_2 = C^3 a_3$$

Adding eq. (g),(h),(i), we get

$$3Kv_{\delta} + 3J'w_{\delta} + 2K'u_{\delta} + Iv_{\delta} + (J-2J')w_{\delta} - 2K'u_{\delta} + (H-2K)v_{\delta} - (J+J')w_{\delta} = C^3 a_4 \delta_{2\delta}$$

$$(H+I+K)v_{\delta} = C^3 a_4 \delta_{2\delta}$$

Adding eq. (a),(b),(d) we get

$$(H+I+K)_{;\delta} = C^3 (L\rho C + 3a_2) \delta_{2\delta} + C^3 a_2$$

$$(LC)_{;\delta} = C^3 (L\rho C + 3a_2) + C^3 a_2$$

$$\left(\frac{(LC)_{;\delta}}{C^3} - LC\rho\right) = a_2$$

$$(3J+2J')u_{\delta} = I_{;\delta} - I'v_{\delta} - 2K'w_{\delta} + K_{;\delta} - (3H'+2I')v_{\delta} + 2K'w_{\delta}$$

$$(H+I+K)_{;\delta} = C^3 (L\rho C + 3a_2) \delta_{2\delta} + C^3 a_{\delta}$$

$$a_1 = 0, a_2 = \frac{1}{4} \left[\frac{(LC)_{;2}}{C^3} - LC\rho \right], a_3 = \frac{L}{C^2} u_2, a_4 = \frac{L}{C^2} v_2 \quad (25)$$

Hence, we have the following.

Theorem (2.1) If the T-tensor T_{hijk} in a four-dimensional Finsler Space is of the form (5), then, the scalar components a_1, a_2, a_3, a_4 of La_{α} , satisfy the relation (25). Further, u_3, u_4 and v_3, v_4 of u_i and v_i -connection vectors of v-connection vanishes also.

Now, we solve the above equation for w_{δ} , the scalar components of w_i -connection vector. From equation (22)

$$I_{;\delta} - (3J+2J')u_{\delta} - I'v_{\delta} - 2K'w_{\delta} = 0$$

$$-(J+J')_{;\delta} + (H-2I)u_{\delta} - 2K'v_{\delta} + (H'+I')w_{\delta} = C^3 a_3 \delta_{2\delta}$$

$$K_{;\delta} - J'u_{\delta} - (3H'+2I')v_{\delta} + 2K'w_{\delta} = 0$$

$$J_{;\delta} + 3Iu_{\delta} - 3I'w_{\delta} = 0$$

$$J'_{;\delta} + Ku_{\delta} + 2K'v_{\delta} + (2I' - H')w_{\delta} = 0$$

$$H'_{;\delta} + 3Kv_{\delta} + 3J'w_{\delta} = 0$$

$$I'_{;\delta} + 2K'u_{\delta} + Iv_{\delta} + (J - 2J')w_{\delta} = 0$$

$$-(H' + I')_{;\delta} - 2K'u_{\delta} + (H - 2K)v_{\delta} - (J + J')w_{\delta} = C^3 a_4 \delta_{2\delta}$$

$$K'_{;\delta} - (H' + 2I)u_{\delta} - (J + 2J')v_{\delta} + (I - K)w_{\delta} = 0$$

Solving all these equation, we get the values of w_{δ} .

$$\begin{aligned} w_{\delta} &= \frac{1}{2K'} [I'_{;\delta} - (3J + 2J') \frac{C^3 a_3 \delta_{2\delta}}{LC} - I' \frac{C^3 a_4 \delta_{2\delta}}{LC}] \\ &= \frac{1}{(H' + I')} [C^3 a_3 + (J + J')_{;\delta} - (H - 2I) \frac{C^3 a_3 \delta_{2\delta}}{LC} + 2K' \frac{C^3 a_4 \delta_{2\delta}}{LC}] \\ &= \frac{1}{2K'} [-K'_{;\delta} + J' \frac{C^3 a_3 \delta_{2\delta}}{LC} + (3H' + 2I') \frac{C^3 a_4 \delta_{2\delta}}{LC}] \\ &= \frac{1}{3I'} [J'_{;\delta} + 3I \frac{C^3 a_3 \delta_{2\delta}}{LC}] \\ &= -\frac{1}{(2I' - H')} [J'_{;\delta} + K \frac{C^3 a_3 \delta_{2\delta}}{LC} + 2K' \frac{C^3 a_4 \delta_{2\delta}}{LC}] \\ &= -\frac{1}{3J'} [H'_{;\delta} + 3K \frac{C^3 a_4 \delta_{2\delta}}{LC}] \\ &= -\frac{1}{(J - 2J')} [I'_{;\delta} + 2K' \frac{C^3 a_3 \delta_{2\delta}}{LC} + I \frac{C^3 a_4 \delta_{2\delta}}{LC}] \\ &= \frac{1}{(J + J')} [-C^3 a_4 \delta_{2\delta} - (H' + I')_{;\delta} - 2K' \frac{C^3 a_3 \delta_{2\delta}}{LC} + (H - 2K) \frac{C^3 a_4 \delta_{2\delta}}{LC}] \\ &= \frac{1}{(I - K)} [-K'_{;\delta} + (H' + 2I) \frac{C^3 a_3 \delta_{2\delta}}{LC} + (J + 2J') \frac{C^3 a_4 \delta_{2\delta}}{LC}] \end{aligned} \tag{26}$$

Theorem (2.2) If the T-tensor T_{hijk} in a four-dimensional Finsler Space is of the form (5) then, the scalar components a_1, a_2, a_3, a_4 of La_α satisfy the relation (25). Further, scalars u_3, u_4 and v_3, v_4 of u_i and v_i - connection vectors of v-connection vanishes. Also, the first component of w_i -connection vector vanishes and equation (26) gives values of w_2, w_3 and w_4 .

3. T-tensor T_{hijk} of Special Form

$$T_{hijk} = \rho C_h C_i C_j C_k$$

The T-tensor T_{hijk} is symmetric in all its four indices, T_{ijk} is a symmetric tensor in all its indices. Since C_h is non-vanishing vector, we may suppose $C_1 \neq 0$, then $T_{1ijk} = T_{i1jk}$ implies that $C_1 T_{ijk} = C_i T_{1jk}$, which gives

$$T_{ijk} = C_i T_{jk} \tag{27}$$

where

$$T_{jk} = T_{1jk} / C_1$$

In view of relation (27), the identity $T_{1jk} = T_{j1k}$ gives $C_1 T_{jk} = C_j T_{1k}$, which gives $T_{jk} = C_j T_k$, where $T_k = T_{1k} / C_1$.

Since T_{jk} is symmetric tensor, we have

$$T_{1k} - T_{k1} = C_1 T_k - C_k T_1 = 0$$

Thus $T_k = \rho C_k$

where, $\rho = T_1 / C_1$.

Hence, we get equation (12)

The scalar components of LT_{hijk} are given by

$$LT_{hijk} = [C^4 L\rho \delta_{2\alpha} \delta_{2\beta} \delta_{2\gamma}] e_{(\alpha)h} e_{(\beta)i} e_{(\gamma)j} e_{(\delta)k} \tag{28}$$

Using equation (18), u_δ, v_δ and w_δ are the scalar components of v-connection vector u_i, v_i and w_i respectively.

Using equation (19), (20).

Comparing equation (19) and (28) we have

$$C_{\alpha\beta\gamma;\delta} + C_{\beta\gamma\delta}\delta_{1\alpha} + C_{\alpha\gamma\delta}\delta_{1\beta} + C_{\alpha\beta\delta}\delta_{1\gamma} = C^4 L\rho\delta_{2\alpha}\delta_{2\beta}\delta_{2\gamma}\delta_{2\delta}$$

Since T_{hijk} is an indicatory tensor, from (5) It follows that $a_i y^i = 0$

which in view of (25) give $a=0$ thus we have from equation (18)

$$H_{;\delta} + 3(J+J')u_{\delta} + 3(H'+I')v_{\delta} = C^4 L\rho\delta_{2\delta} \quad (a)$$

$$I_{;\delta} - (3J+2J')u_{\delta} - I'v_{\delta} - 2K'w_{\delta} = 0 \quad (b)$$

$$-(J+J')_{;\delta} + (H-2I)u_{\delta} - 2K'v_{\delta} + (H'+I')w_{\delta} = 0 \quad (c)$$

$$K_{;\delta} - J'u_{\delta} - (3H'+2I')v_{\delta} + 2K'w_{\delta} = 0 \quad (d)$$

$$J_{;\delta} + 3Iu_{\delta} - 3I'w_{\delta} = 0 \quad (e)$$

$$J'_{;\delta} + Ku_{\delta} + 2K'v_{\delta} + (2I'-H')w_{\delta} = 0 \quad (f) \quad (29)$$

$$H'_{;\delta} + 3Kv_{\delta} + 3J'w_{\delta} = 0 \quad (g)$$

$$I'_{;\delta} + 2K'u_{\delta} + Iv_{\delta} + (H-2K)v_{\delta} - (J-2J')w_{\delta} = 0 \quad (h)$$

$$-(H'+I')_{;\delta} - 2K'u_{\delta} + (H-2K)v_{\delta} - (J+J')w_{\delta} = 0 \quad (i)$$

$$K'_{;\delta} - (H'+2I')u_{\delta} - (J+2J')v_{\delta} + (I-K)w_{\delta} = 0 \quad (j)$$

Adding equation (a), (b), and (d) we get

$$H_{;\delta} + 3(J+J')u_{\delta} + 3(H'+I')v_{\delta} + I_{;\delta} - (3J+2J')u_{\delta} - I'v_{\delta}$$

$$-2K'w_{\delta} + K_{;\delta} - J'u_{\delta} - (3H'+2I')v_{\delta} + 2K'w_{\delta} = C^4 L\rho\delta_{2\delta}$$

$$(H+I+K)_{;\delta} = C^4 L\rho\delta_{2\delta}$$

$$(LC)_{;\delta} = C^4 L\rho\delta_{2\delta}$$

Adding equation (c), (e), (f) we have

$$-J_{;\delta} - J'_{;\delta} + Hu_{\delta} - 2Iu_{\delta} - 2K'v_{\delta} + H'w_{\delta} + I'w_{\delta} + J_{;\delta}$$

$$+3Iu_{\delta} - 3I'w_{\delta} + J'_{;\delta} + Ku_{\delta} + 2K'v_{\delta} + 2I'w_{\delta} - H'w_{\delta} = 0$$

$$(H + I + K)u_{\delta} = 0 \quad (LC)u_{\delta} = 0; \quad u_{\delta} = 0, \quad (30)$$

Adding equation (g), (h), (i) we have

$$\begin{aligned} H'_{;\delta} + 3Kv_{\delta} + 3J'w_{\delta} + I'_{;\delta} + 2K'u_{\delta} + Iv_{\delta} + Jw_{\delta} - 2J'w_{\delta} \\ - H'_{;\delta} - I'_{;\delta} - 2K'u_{\delta} + Hv_{\delta} - 2Kv_{\delta} - Jw_{\delta} - J'w_{\delta} = 0 \\ (H + I + K)v_{\delta} = 0 \\ (LC)v_{\delta} = 0 \quad v_{\delta} = 0 \end{aligned} \quad (31)$$

From equation (30) & (31)

$$H_{;\delta} + 3(J + J')u_{\delta} + 3(H' + I')v_{\delta} = C^4 L\rho\delta_{2\delta}$$

Putting the value of u_{δ}, v_{δ} in above equation, we have

$$H_{;\delta} = C^4 L\rho\delta_{2\delta} \quad (32)$$

Thus, we have

$$u_{\delta} = 0 \quad v_{\delta} = 0 \quad \text{and} \quad H_{;\delta} = C^4 L\rho\delta_{2\delta} \quad (33)$$

Hence, we have the following.

Theorem(3.1). If the T-tensor T_{hijk} in a four-dimensional Finsler Space is of the form (5), then, the scalar components v_4 of the v -connection vector vanishes u_{δ}, v_{δ} are given by (30).

Now we solve the above eq. for w_{δ} the scalar components of w -connection vector from eq. (29)

From equations (9) using sub equation (b),(c),(d),(e),(f),(g),(h),(i),(j) we have the values of w_{δ} .

$$\begin{aligned} w_{\delta} &= -\frac{(J + J')_{;\delta}}{(H' + I')} = -\frac{I_{;\delta}}{2K'} = -\frac{K_{;\delta}}{2K'} = \frac{J_{;\delta}}{3I'} = -\frac{J'_{;\delta}}{(2I' - H')} \\ &= -\frac{H'_{;\delta}}{3J'} = -\frac{I'_{;\delta}}{(J - 2J')} = -\frac{(H' + I')_{;\delta}}{(J + J')} = -\frac{K'_{;\delta}}{(I - K)} \end{aligned} \quad (34)$$

Theorem(3.2). If the T-tensor T_{hijk} in a four-dimensional Finsler Space is of the form (5), then, the scalar components v_4 of the v-connection vector vanishes u_δ, v_δ and given by (33). If we put $\delta=1$ then w_δ is zero if we again put $\delta=2,3,4$ we find the values of w_2, w_3, w_4 .

Acknowledgement: The authors are thankful to the Referee for valuable comments and suggestions.

References

- [1] Gupta, M. K. And Pandey, P. N. (2014). Four-Dimensional Finsler Spaces with Tensors of Some Special Forms, Journal of Rajasthan Academy of Physical Sciences, Vol.18, 219-229.
- [2] Ikeda, F. (1979). On the T-Tensor T_{ijkl} of Finsler Spaces, *Tensor, N.S.*, 33, 203-209.
- [3] Ikeda, F. (1998). Landsberg Spaces Satisfying the T-Condition, *Balkan J.Geom. Appl.*, 3 (1), 23-28.
- [4] Ikeda, F. (1999). On S4-Like Finsler Spaces with T-Conditions, *Tensor, N.S.*, 61, 171- 208.
- [5] Matsumoto, M. (1972). On C-Reducible Finsler Spaces. *Tensor N.S.*, 24, 29-31.
- [6] Matsumoto, M. (1973). A Theory of Three-Dimensional Finsler Spaces In Terms of Scalars, *Demonst. Math.*, 6, 223-251.
- [7] Matsumoto, M. (1975). On Three Dimensional Finsler Spaces Satisfying the T and B^p Condition, *Tensor, N. S.*, 29, 13-20.
- [8] Matsumoto, M. and Shibata, C. (1979). On Semi C-Reducible, T-Tensor=0 and S4 – Likeness of Finsler Spaces, *J.Math. Kyoto Univ.*, 19 (2), 301-314.
- [9] Matsumoto, M. and Numata, S. (1980). On Semi, C-Reducible Finsler Spaces with Constant Coefficients and C2-Like Finsler Spaces. *Tensor, N. S.* 34, 218-222.
- [10] Matsumoto, M. (1986). Foundations of Finsler Geometry and Special Finsler Spaces. *Kaiseisha Press. Saikawa Otsu 520 Japan.*
- [11] Prasad, B.N., Pandey, T.N. and Singh, M. K. (2010). On Four Dimensional Finslr Spaces Satisfying T-Conditions, *Journal of tensor society of India*, 4, 21-31.
- [12] Singh, U.P., Agarawal, P.N., and Prasad, B.N. (1982). On Finsler Spaces with T-Tensor of Some Special Forms. *Indian J. Pure Appl. Math.*, 13, 172-182.
- [13] Singh, U.P., Agarawal, P.N., and Prasad B.N. (1982). On A Special Form of T-Tensor And T2-Like Finsler Spaces. 13(9), 997-1005.
- [14] Tiwari B. (2012). On Spesial Reducible Finsler Spaces. *Balkan Society of Geometers Vol.14*, 132-137.

