Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.17, No.1&2, January-June, 2018, 29-44

A DISCRETE PARAMETRIC MARKOV-CHAIN MODEL OF TWO UNIT ACTIVE REDUNDANT SYSTEM WITH INSPECTION AND TWO TYPES OF REPAIR

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Abstract: The paper deals with cost benefit analysis of a two identical unit parallel system model. Each unit has two modes- normal (N) and total failure (F). A failed unit is first goes to inspection to decide whether it needs minor or major repair. Two repairmen (skilled and non-skilled) are always available with the system. Skilled repairman is available for major repair whereas non-skilled repairman inspects a failed unit to decide the type of repair and to do minor repair. The failure time, inspection time and both types of repair times are taken as independent random variables of discrete nature having geometric distributions with different parameters.

Keywords: Regenerative point, reliability, MTSF, availability, active redundancy, geometric distribution, Markov-Chain.

1. Introduction

Two unit active redundant systems have been widely studied in the literature of reliability by various authors including [2,4,5,6] due to their prevalence in modern business and industries. To help system designers and maintenance engineers several authors have analyzed two unit redundant system models under different model formulation. Agnihotri and Satsangi [1] analyzed a two non-identical unit parallel system assuming that one of the unit gets priority over the other for repair, inspection and post repair. After repair a unit goes for inspection to decide whether the repair is perfect or not. If the repair is found imperfect it is sent for post-repair. Kumar [7] analyzed a two unit cold standby system with inspection and on-line/off-line repairs of a partially failed unit i.e. a partially failed unit first goes for inspection to identify whether it will be repaired on line or off line with fixed known probabilities p and q respectively. Kumar [8] analyzed a two unit active redundant system model with inspection, two types of repair and post repair. Whenever a unit fails it requires inspection to decide the type of repair (type-1 and type-2) and then accordingly the repair of the failed unit is started. On completion of any type of repair, the unit is finally goes for post repair. Goel et al. [3] analyzed a single server two-unit cold standby system model with slow switch. Each unit consists of n-separately maintained independent components and an inspection facility is considered to detect which component of the unit has failed.

The purpose of the present paper is to analyze a two identical unit system model with inspection, minor and major repair. A failed unit first goes for inspection to detect the fault and to identify whether the failed units needs minor repair or major repair. Two repairmen are always available with the system (skilled and non-skilled). Non-skilled repairman inspects the failed unit and do the minor repair whereas skilled repairman is available to do major repair. The following economic related measures of system effectiveness are obtained by using regenerative point technique –

i) Transition probabilities and mean sojourn times in various states.
ii) Reliability and mean time to system failure.
iii) Point-wise and steady-state availability of the system during time (0, t-1).
iv) Expected busy period of inspector and repairman during time (0, t-1).
v) Net expected profit incurred by the system during a finite and steady-state are obtained.

2. Model Description and Assumptions

1. The system comprises of two identical units in parallel configuration.
2. Each unit has two modes- Normal (N) and total failure (F).
3. A failed unit first goes to inspection to decide whether the failed unit needs minor or major repair. whose probabilities are θ and $\bar{\theta}(=1-\theta)$
4. Two repairmen are always available with the system (skilled and non-skilled). Non-skilled repairman inspects the failed unit and do the minor repair whereas skilled repairman is available to do major repair.
5. After minor or major repair, a unit becomes as good as new.
6. The time to failure, time to inspection and each type of repair time follow geometric distributions with different parameters.

3. Notations and States of the System

a) Notations :

pq^x **:** p.m.f. of failure time of a unit; $p+q=1$.

$r_i s_i^x$ **:** p.m.f. of repair time by repairman of minor and major types respectively for i=1, 2 and $r_i+s_i=1$.

ab^x	:	p.m.f. of inspection time respectively; $a+b=1$.
$\theta, \bar{\theta}$	:	probability that the failed unit needs minor and major repair respectively; $\theta+\bar{\theta}=1$
$q_{ij}(\cdot), Q_{ij}(\cdot)$	:	p.m.f. and C.d.f. of one step or direct transition time from state S_i to S_j.
p_{ij}	:	steady state transition probability from state S_i to S_j.

$$p_{ij}=Q_{ij}(\infty)$$

$Z_i(t)$	:	probability that the system sojourn in state S_i up to epoch (t-1).
ψ_i	:	mean sojourn time in state S_i.
$*, h$	:	symbol and dummy variable used in geometric transform e. g.

$$GT\left[q_{ij}(t)\right]=q_{ij}^{*}(h)=\sum_{t=0}^{\infty} h^t q_{ij}(t)$$

The transition diagram of the system model is shown in fig. 1.

TRANSITION DIAGRAM

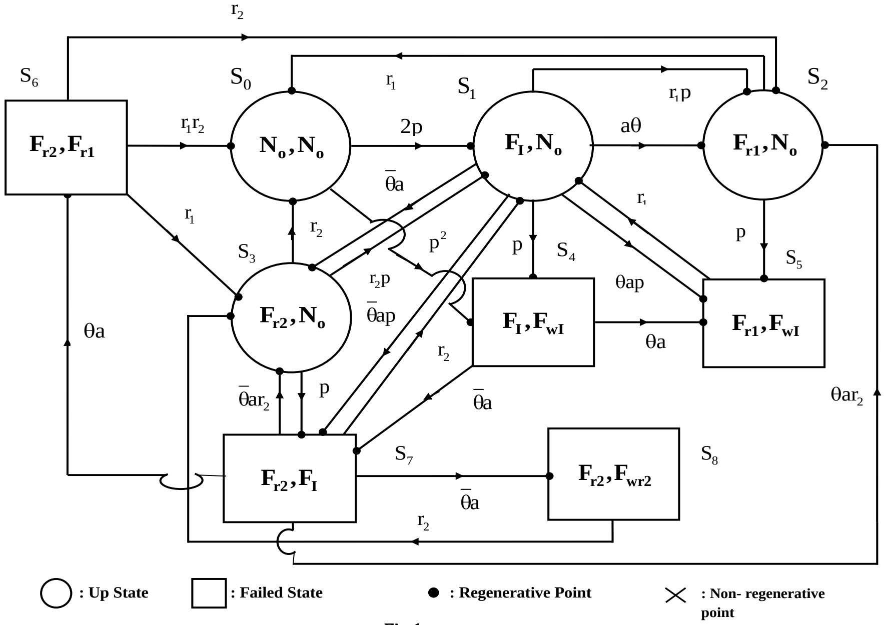

Fig.1

b) Symbols for the states of the system:

N_0 : unit in normal mode and operative.

F_I / F_{wI} : unit is in total failure (F) mode and under inspection/waits for inspection.

F_{r1} / F_{wr1} : unit is in total failure (F) mode and under minor repair/waits for minor repair.

F_{r2} / F_{wr2} : unit is in total failure (F) mode and under major repair/waits for major repair.

With the help of above symbols the possible states of the system are:

$S_0 \equiv (N_o, N_o)$, $S_1 \equiv (F_I, N_o)$ $S_2 \equiv (F_{r1}, N_o)$, $S_3 \equiv (F_{r2}, N_o)$

$S_4 \equiv (F_I, F_{wI})$, $S_5 \equiv (F_{r1}, F_{wI})$, $S_6 \equiv (F_{r2}, F_{r1})$, $S_7 \equiv (F_{r2}, F_I)$

$S_8 \equiv (F_{r2}, F_{wr2})$

4. Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By using simple probabilistic arguments we have,

$$Q_{01}(t) = \frac{2pq}{1-(q)^2}\left[1-(q)^{2(t+1)}\right], \qquad Q_{04}(t) = \frac{p^2}{1-(q)^2}\left[1-(q)^{2(t+1)}\right]$$

$$Q_{12}(t) = \frac{a\theta q}{1-bq}\left[1-(bq)^{t+1}\right], \qquad Q_{13}(t) = \frac{\bar{\theta} aq}{1-bq}\left[1-(bq)^{t+1}\right]$$

$$Q_{14}(t) = \frac{bp}{1-bq}\left[1-(bq)^{t+1}\right], \qquad Q_{15}(t) = \frac{\theta ap}{1-bq}\left[1-(bq)^{t+1}\right]$$

$$Q_{17}(t) = \frac{\bar{\theta} ap}{1-bq}\left[1-(bq)^{t+1}\right], \qquad Q_{20}(t) = \frac{qr_1}{1-qs_1}\left[1-(qs_1)^{t+1}\right]$$

$$Q_{21}(t) = \frac{pr_1}{1-qs_1}\left[1-(qs_1)^{t+1}\right], \qquad Q_{25}(t) = \frac{ps_1}{1-qs_1}\left[1-(qs_1)^{t+1}\right]$$

$Q_{30}(t)=\frac{qr_2}{1-qs_2}\left[1-(qs_2)^{t+1}\right]$, $Q_{31}(t)=\frac{pr_2}{1-qs_2}\left[1-(qs_2)^{t+1}\right]$

$Q_{37}(t)=\frac{ps_2}{1-qs_2}\left[1-(qs_2)^{t+1}\right]$, $Q_{45}(t)=\theta\left[1-b^{t+1}\right]$

$Q_{47}(t)=\bar{\theta}\left[1-b^{t+1}\right]$, $Q_{51}(t)=\left[1-s_1^{t+1}\right]$

$Q_{60}(t)=\frac{r_1r_2}{1-s_1s_2}\left[1-(s_1s_2)^{t+1}\right]$, $Q_{62}(t)=\frac{r_2s_1}{1-s_1s_2}\left[1-(s_1s_2)^{t+1}\right]$

$Q_{63}(t)=\frac{r_1s_2}{1-s_1s_2}\left[1-(s_1s_2)^{t+1}\right]$, $Q_{71}(t)=\frac{r_2b}{1-bs_2}\left[1-(bs_2)^{t+1}\right]$

$Q_{72}(t)=\frac{\theta ar_2}{1-bs_2}\left[1-(bs_2)^{t+1}\right]$, $Q_{73}(t)=\frac{\bar{\theta}ar_2}{1-bs_2}\left[1-(bs_2)^{t+1}\right]$

$Q_{76}(t)=\frac{\theta as_2}{1-bs_2}\left[1-(bs_2)^{t+1}\right]$, $Q_{78}(t)=\frac{\bar{\theta}as_2}{1-bs_2}\left[1-(bs_2)^{t+1}\right]$

$Q_{83}(t)=1-s_2^{t+1}$ (1-25)

The steady state transition probabilities from state S_i to S_j can be obtained from (1-26) by taking $t\to\infty$, as follows:

$p_{01}=\frac{2pq}{1-(q)^2}$, $p_{04}=\frac{p^2}{1-(q)^2}$, $p_{12}=\frac{\theta aq}{1-bq}$, $p_{13}=\frac{\bar{\theta}aq}{1-bq}$

$p_{14}=\frac{bp}{1-bq}$, $p_{15}=\frac{\theta ap}{1-bq}$, $p_{17}=\frac{\bar{\theta}ap}{1-bq}$, $p_{20}=\frac{qr_1}{1-qs_1}$

$p_{21}=\frac{pr_1}{1-qs_1}$, $p_{25}=\frac{ps_1}{1-qs_1}$, $p_{30}=\frac{qr_2}{1-qs_2}$, $p_{31}=\frac{pr_2}{1-qs_2}$

$p_{37}=\frac{ps_2}{1-qs_2}$, $p_{45}=\theta$, $p_{47}=\bar{\theta}$, $p_{51}=1$

$p_{60}=\frac{r_1r_2}{1-s_1s_2}$, $p_{62}=\frac{r_2s_1}{1-s_1s_2}$, $p_{63}=\frac{r_1s_2}{1-s_1s_2}$, $p_{71}=\frac{br_2}{1-bs_2}$

$p_{72}=\frac{\theta ar_2}{1-bs_2}$, $p_{73}=\frac{\bar{\theta}ar_2}{1-bs_2}$, $p_{76}=\frac{\theta as_2}{1-bs_2}$, $p_{78}=\frac{\bar{\theta}as_2}{1-bs_2}$

$p_{83}=1$

We observe that the following relations hold-

$p_{51} = p_{83} = 1$, $p_{01} + p_{04} = 1$, $p_{12} + p_{13} + p_{14} + p_{15} + p_{17} = 1$

$p_{20} + p_{21} + p_{25} = 1$, $p_{30} + p_{31} + p_{37} = 1$, $p_{45} + p_{47} = 1$

$p_{60} + p_{62} + p_{63} = 1$ $p_{71} + p_{72} + p_{73} + p_{76} + p_{78} = 1$ (26-33)

5. Mean Sojourn Times

Let T_i be the sojourn time in state S_i (i=0-8) then ψ_i mean sojourn time in state S_i is given by

$$\psi_i = \sum_{t=1}^{\infty} P\left[T \geq t\right]$$

In particular,

$$\psi_0 = \frac{q^2}{1-q^2}, \quad \psi_1 = \frac{bq}{1-bq}, \quad \psi_2 = \frac{qs_1}{1-qs_1}, \quad \psi_3 = \frac{qs_2}{1-qs_2}$$

$$\psi_4 = \frac{b}{1-b}, \quad \psi_5 = \frac{s_1}{r_1}, \quad \psi_6 = \frac{s_1 s_2}{1-s_1 s_2}, \quad \psi_7 = \frac{bs_2}{1-bs_2}$$

$$\psi_8 = \frac{s_2}{r_2} \qquad (34\text{-}42)$$

6. Methodology For Developing Equations

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the System

Here we define $R_i(t)$ as the probability that the system does not fail up to epochs 0, 1, 2,.., (t-1) when it is initially started from up state S_i. To determine it, we regard the failed state S_4, S_5, S_6, S_7 and S_8 as absorbing state. Now, the expression for $R_i(t)$; i=0, 1, 2, 3; we have the following set of convolution equations.

$$R_0(t) = q^{2t} + \sum_{u=0}^{t-1} q_{01}(u) R_1(t-1-u)$$

$$= Z_0(t) + q_{01}(t-1) © R_1(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{12}(t-1)©R_2(t-1) + q_{13}(t-1)©R_3(t-1)$$

$$R_2(t) = Z_2(t) + q_{20}(t-1)©R_0(t-1) + q_{21}(t-1)©R_1(t-1)$$

$$R_3(t) = Z_3(t) + q_{30}(t-1)©R_0(t-1) + q_{31}(t-1)©R_1(t-1) \quad (43\text{-}46)$$

Where,

$Z_1(t) = q^t b^t$, $Z_2(t) = q^t s_1^t$, $Z_3(t) = s_2^t q^t$

b) Availability of the System

Let $A_i(t)$ be the probability that the system is up at epoch (t-1), when it initially started from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i(t)$; i=0 to 8.

$$A_0(t) = Z_0(t) + q_{01}(t-1)©A_1(t-1) + q_{04}(t-1)©A_4(t-1)$$

$$A_1(t) = Z_1(t) + q_{12}(t-1)©A_2(t-1) + q_{13}(t-1)©A_3(t-1) + q_{14}(t-1)©A_4(t-1)$$
$$+q_{15}(t-1)©A_5(t-1) + q_{17}(t-1)©A_7(t-1)$$

$$A_2(t) = Z_2(t) + q_{20}(t-1)©A_0(t-1) + q_{21}(t-1)©A_1(t-1) + q_{25}(t-1)©A_5(t-1)$$

$$A_3(t) = Z_3(t) + q_{30}(t-1)©A_0(t-1) + q_{31}(t-1)©A_1(t-1) + q_{37}(t-1)©A_7(t-1)$$

$$A_4(t) = q_{45}(t-1)©A_5(t-1) + q_{47}(t-1)©A_7(t-1)$$

$$A_5(t) = q_{51}(t-1)©A_1(t-1)$$

$$A_6(t) = q_{60}(t-1)©A_1(t-1) + q_{62}(t-1)©A_2(t-1) + q_{63}(t-1)©A_3(t-1)$$

$$A_7(t) = q_{71}(t-1)©A_2(t-1) + q_{72}(t-1)©A_2(t-1) + q_{73}(t-1)©A_2(t-1)$$
$$+q_{76}(t-1)©A_6(t-1) + q_{78}(t-1)©A_8(t-1)$$

$$A_8(t) = q_{83}(t-1)©A_3(t-1) \quad (44\text{-}52)$$

Where,

The values of $Z_i(t)$; i=0 to 3 are same as given in section 6(a).

c) Busy Period of Inspector

Let $B_i^I(t)$ be the probability that the inspector is busy in the inspection of a failed unit at epoch t-1, when it initially started from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $B_i^I(t)$; i=0 to 8.

$$B_0^I(t) = q_{01}(t-1)©B_1^I(t-1) + q_{04}(t-1)©B_4^I(t-1)$$

$$B_1^I(t) = Z_1(t) + q_{12}(t-1)©B_2^I(t-1) + q_{13}(t-1)©B_3^I(t-1) + q_{14}(t-1)©B_4^I(t-1)$$

$$+q_{15}(t-1)©B_5^I(t-1) + q_{17}(t-1)©B_7^I(t-1)$$

$$B_2^I(t) = q_{20}(t-1)©B_0^I(t-1) + q_{21}(t-1)©B_1^I(t-1) + q_{25}(t-1)©B_5^I(t-1)$$

$$B_3^I(t) = q_{30}(t-1)©B_0^I(t-1) + q_{31}(t-1)©B_1^I(t-1) + q_{37}(t-1)©B_7^I(t-1)$$

$$B_4^I(t) = Z_4(t) + q_{45}(t-1)©B_5^I(t-1) + q_{47}(t-1)©B_7^I(t-1)$$

$$B_5^I(t) = q_{51}(t-1)©B_1^I(t-1)$$

$$B_6^I(t) = q_{60}(t-1)©B_0^I(t-1) + q_{62}(t-1)©B_2^I(t-1) + q_{63}(t-1)©B_3^I(t-1)$$

$$B_7^I(t) = Z_7 + q_{71}(t-1)©B_1^I(t-1) + q_{72}(t-1)©B_2^I(t-1) + q_{73}(t-1)©B_3^I(t-1)$$

$$+q_{76}(t-1)©B_6^I(t-1) + q_{78}(t-1)©B_8^I(t-1)$$

$$B_8^I(t) = q_{83}(t-1)©B_3^I(t-1) \qquad (53\text{-}61)$$

Where,

The values of $Z_1(t)$ is same as given in section 6(a), $Z_4(t) = b^t$ and $Z_7(t) = b^t s_2^t$.

d) Busy Period of Repairman

Let $B_i^{r1}(t)$ and $B_i^{r2}(t)$ be the respective probabilities that the repairman is busy in the minor and major repair of a failed unit at epoch t-1, when it initially started from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $B_i^j(t)$; i=0 to 8. The dichotomous variable δ takes value 1 and 0 respectively for $j = r_1$ and r_2.

$$B_0^j(t) = q_{01}(t-1)©B_1^j(t-1) + q_{04}(t-1)©B_4^j(t-1)$$

$$B_1^j(t) = q_{12}(t-1)©B_2^j(t-1) + q_{13}(t-1)©B_3^j(t-1) + q_{14}(t-1)©B_4^j(t-1)$$

$$+q_{15}(t-1)©B_5^j(t-1) + q_{17}(t-1)©B_7^j(t-1)$$

$$B_2^j(t) = (1-\delta)Z_2(t) + q_{20}(t-1)©B_0^j(t-1) + q_{21}(t-1)©B_1^j(t-1)$$

$$+q_{25}(t-1)©B_5^j(t-1)$$

$$B_3^j(t) = \delta Z_3(t) + q_{30}(t-1)©B_0^j(t-1) + q_{31}(t-1)©B_1^j(t-1)$$

$$+q_{37}(t-1)©B_7^j(t-1)$$

$$B_4^j(t) = q_{45}(t-1)©B_5^j(t-1) + q_{47}(t-1)©B_7^j(t-1)$$

$$B_5^j(t) = (1-\delta)Z_5(t) + q_{51}(t-1)©B_1^j(t-1)$$

$$B_6^j(t) = Z_6(t) + q_{60}(t-1)©B_0^j(t-1) + q_{62}(t-1)©B_2^j(t-1)$$

$$+q_{63}(t-1)©B_3^j(t-1)$$

$$B_7^j(t) = \delta Z_7(t) + q_{71}(t-1)©B_1^j(t-1) + q_{72}(t-1)©B_2^j(t-1) + q_{73}(t-1)©B_3^j(t-1)$$

$$+q_{76}(t-1)©B_6^j(t-1) + q_{78}(t-1)©B_8^j(t-1)$$

$$B_8^j(t) = \delta Z(t) + q_{83}(t-1)©B_3^j(t-1)$$

(62-70)

Where,

The values of $Z_i(t)$; i=2, 3, 5, 6, 7 are same as given in section 6(a) and (b), $Z_5(t) = s_1^t$ $Z_6(t) = s_1^t s_2^t$ and $Z(t) = s_2^t$.

7. Analysis of Reliability and MTSF

Taking geometric transform of (43-46) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h) = \frac{N_1(h)}{D_1(h)} \tag{71}$$

Where,

$$N_1(h) = \left[1 - h^2 q_{12}^* q_{21}^* - h^2 q_{13}^* q_{31}^*\right] Z_0^* + h q_{01}^* Z_1^* + h^2 q_{01}^* q_{12}^* Z_2^* + h^2 q_{01}^* q_{13}^* Z_3^*$$

$$D_1(h) = 1 - h^2 q_{12}^* q_{21}^* - h^2 q_{13}^* q_{31}^* - h^3 q_{01}^* q_{12}^* q_{20}^* - h^3 q_{01}^* q_{13}^* q_{30}^*$$

Collecting the coefficient of h^t from expression (77), we can get the reliability of the system $R_0(t)$. The MTSF is given by -

$$E(T) = \lim_{h \to 1} \sum_{t=1}^{\infty} h^t R(t) = \frac{N_1(1)}{D_1(1)} - 1 \qquad (72)$$

$$N_1(1) = [1 - p_{12}p_{21} - p_{13}p_{31}]\Psi_0 + \Psi_1 + p_{12}\Psi_2 + p_{13}\Psi_3$$

$$D_1(1) = 1 - p_{12}(p_{21} + p_{20}) - p_{13}(p_{31} + p_{30})$$

8. Availability Analysis

On taking geometric transform of (44-52) and simplifying the resulting equations for we get,

$$A_0^*(h) = \frac{N_2(h)}{D_2(h)} \qquad (73)$$

Where,

$$N_2(h) = \begin{vmatrix} Z_0 & -hq_{01}^* & 0 & 0 & -hq_{04}^* & 0 & 0 & 0 & 0 \\ Z_1 & 1 & -hq_{12}^* & -hq_{13}^* & -hq_{14}^* & -hq_{15}^* & 0 & -hq_{17}^* & 0 \\ Z_2 & -hq_{21}^* & 1 & 0 & 0 & -hq_{25}^* & 0 & 0 & 0 \\ Z_3 & -hq_{31}^* & 0 & 1 & 0 & 0 & 0 & -hq_{37}^* & 0 \\ 0 & 0 & 0 & 0 & 1 & -hq_{45}^* & 0 & -hq_{47}^* & 0 \\ 0 & -hq_{51}^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -hq_{62}^* & -hq_{63}^* & 0 & 0 & 1 & 0 & 0 \\ 0 & -hq_{71}^* & -hq_{72}^* & -hq_{73}^* & 0 & 0 & -hq_{76}^* & 1 & -hq_{78}^* \\ 0 & 0 & 0 & -hq_{83}^* & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

and

$$D_2(h) = \begin{vmatrix} 1 & -hq_{01}^* & 0 & 0 & -hq_{04}^* & 0 & 0 & 0 & 0 \\ 0 & 1 & -hq_{12}^* & -hq_{13}^* & -hq_{14}^* & -hq_{15}^* & 0 & -hq_{17}^* & 0 \\ -hq_{20}^* & -hq_{21}^* & 1 & 0 & 0 & -hq_{25}^* & 0 & 0 & 0 \\ -hq_{30}^* & -hq_{31}^* & 0 & 1 & 0 & 0 & 0 & -hq_{37}^* & 0 \\ 0 & 0 & 0 & 0 & 1 & -hq_{45}^* & 0 & -hq_{47}^* & 0 \\ 0 & -hq_{51}^* & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -hq_{60}^* & 0 & -hq_{62}^* & -hq_{63}^* & 0 & 0 & 1 & 0 & 0 \\ 0 & -hq_{71}^* & -hq_{72}^* & -hq_{73}^* & 0 & 0 & -hq_{76}^* & 1 & -hq_{78}^* \\ 0 & 0 & 0 & -hq_{83}^* & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

The steady state availabilities of the system due to operation of unit -

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{h\to 1}(1-h)\frac{N_2(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$A_0 = -\frac{N_2(1)}{D_2'(1)} \tag{74}$$

Where,

$$N_2(1) = u_0\psi_0 + u_1\psi_1 + u_2\psi_2 + u_3\psi_3$$

and

$$D_2'(1) = -\left[u_0\psi_0 + u_1(\psi_1 + p_{14}\psi_4 + p_{15}\psi_5) + u_2\psi_2 + u_3\psi_3 + u_6\psi_6 + u_7(\psi_7 + p_{78}\psi_8)\right]$$

where,

$u_i = U_i^*(0)$ and $U_i^*(h)$; i=0, 1,…, 7 are the minors of the elements of first column of $D_2(h)$.

Now the expected up time of the system due to operative unit up to epoch (t-1) are given by

$$\mu_{up}(t) = \sum_{x=0}^{t-1} A_0(x)$$

so that

$$\mu_{up}^{*}(h)=\frac{A_0^{*}(h)}{(1-h)} \tag{75}$$

9. Busy Period Analysis

a) Busy Period of Inspector

On taking geometric transform of (53-61) and simplifying the resulting equations for we get,

$$B_0^{I*}(h)=\frac{N_3(h)}{D_2(h)} \tag{76}$$

Where,

$$N_3(h)=hq_{01}^{*}\left[U_1^{*}\left(Z_1^{*}+hq_{14}^{*}Z_4^{*}\right)+U_7^{*}Z_7^{*}\right]$$

and $D_2(h)$ is same as in availability analysis.

In the long run the respective probabilities that the inspector is busy in the inspection of failed unit given by-

$$B_0^{I}=\lim_{t\to\infty}B_o^{I}(t)=\lim_{h\to 1}(1-h)\frac{N_4(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^{I}=-\frac{N_4(1)}{D_2'(1)} \tag{77}$$

Where,

$$N_3(1)=u_1(\psi_1+p_{14}\psi_4)+u_7\psi_7$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period of inspector is in the inspection of a failed unit up to epoch (t-1) is given by -

$$\mu_b^{I}(t)=\sum_{x=0}^{t-1}B_0^{I}(x)$$

So that,

$$\mu_b^{I*}(h) = \frac{B_0^{I*}(h)}{(1-h)} \tag{78}$$

b) Busy Period of Repairman

On taking geometric transform of (62-70) and simplifying the resulting equations for $j = r_1$ and r_2. We get,

$$B_0^{r_1*}(h) = \frac{N_4(h)}{D_2(h)} \quad \text{and} \quad B_0^{r_2*}(h) = \frac{N_5(h)}{D_2(h)} \tag{79-80}$$

Where,

$$N_4(h) = h\left[U_2^* Z_2^* + U_5^* Z_5^* + U_6^* Z_6^*\right]$$

$$N_5(h) = h\left[U_3^* Z_3^* + U_6^* Z_6^* + U_7^*\left(Z_7^* + hq_{78}^* Z_8^*\right)\right]$$

and $D_2(h)$ is same as in availability analysis.

In the long run the respective probabilities that the repairman is busy in the minor repair and major repair of a failed unit are given by-

$$B_0^{r_1} = \lim_{t\to\infty} B_o^{r_1}(t) = \lim_{h\to 1}(1-h)\frac{N_4(h)}{D_2(h)}$$

$$B_0^{r_2} = \lim_{t\to\infty} B_o^{r_2}(t) = \lim_{h\to 1}(1-h)\frac{N_5(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^{r_1} = -\frac{N_4(1)}{D_2'(1)} \quad \text{and} \quad B_0^{r_2} = -\frac{N_5(1)}{D_2'(1)} \tag{81-82}$$

Where,

$$N_4(1) = u_2\psi_2 + u_5\psi_5 + u_6\psi_6$$

$$N_5(1) = u_3\psi_3 + u_6\psi_6 + u_7\left(\psi_7 + p_{78}\psi_8\right)$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period of the repairman in minor repair and major repair of a failed unit up to epoch (t-1) are respectively given by-

$\mu_b^{r_1}(t) = \sum_{x=0}^{t-1} B_0^{r_1}(x)$, $\mu_b^{r_2}(t) = \sum_{x=0}^{t-1} B_0^{r_2}(x)$

So that,

$$\mu_b^{r_1 *}(h) = \frac{B_0^{r_1 *}(h)}{(1-h)}, \qquad \mu_b^{r_2 *}(h) = \frac{B_0^{r_2 *}(h)}{(1-h)} \qquad (83\text{-}84)$$

10. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section. Let us consider,

$K_0 =$ revenue per-unit time by the system due to operative unit.

$K_1 =$ cost per-unit time when inspector is busy in the inspection of failed unit.

$K_2 =$ cost per-unit time when repairman is busy in the minor repair of a failed unit.

$K_3 =$ cost per-unit time when repairman is busy in the major repair of a failed unit.

Then, the net expected profit incurred up to epoch (t-1) is given by

$$P(t) = K_0 \mu_{up}(t) - K_1 \mu_b^I(t) - K_2 \mu_b^{r_1}(t) - K_3 \mu_b^{r_2}(t) \qquad (85)$$

The expected profit per unit time in steady state is given by-

$$P = \lim_{t\to\infty} \frac{P(t)}{t} = \lim_{h\to 1}(1-h)^2 P^*(h)$$

$$= K_0 \lim_{h\to 1}(1-h)^2 \frac{A_0^*(h)}{(1-h)} - K_1 \lim_{h\to 1}(1-h)^2 \frac{B_0^{I*}(h)}{(1-h)} - K_2 \lim_{h\to 1}(1-h)^2 \frac{B_0^{r_1 *}(h)}{(1-h)}$$

$$-K_3 \lim_{h\to 1}(1-h)^2 \frac{B_0^{r_2 *}(h)}{(1-h)}$$

$$= K_0 A_0 - K_1 B_0^I - K_2 B_0^{r_1} - K_3 B_0^{r_2} \qquad (86)$$

11. Graphical Representation

The curves for MTSF and profit function have been drown for different values of failure parameters. Fig. 2 depicts the variation in MTSF with respect to failure rate (p) for different values of major repair rate (r_2) of a unit and inspection rate (θ) when values of other parameters are kept fixed as $r_1 = 0.9$ and $a = 0.8$. From the curves we conclude that expected life of the system decrease with increase in p. Further, increases as the values of r_2 and θ increases.

Similarly, Fig. 3 reveals the variations in profit (P) with respect to p for varying values of r_2 and θ, when other parameters are kept fixed as $r_1 = 0.9$, $a = 0.8$, $K_0 = 100$, $K_1 = 100$, $K_2 = 90$ and $K_3 = 120$. From the figure it is clearly observed from the smooth curves, that the system is profitable if the value of parameter p is greater than 0.06, 0.099 and 0.115 respectively for $r_2 = 0.01$, 0.02 and 0.03 for fixed value of $\theta = 0.6$. From dotted curves, we conclude that system is profitable only if value of parameter p is greater than 0.05, 0.09 and 0.11 respectively for $r_2 = 0.1$, 0.2 and 0.3 for fixed value of $\theta = 0.4$.

Behavior of MTSF with respect to p, r_2 and θ_1

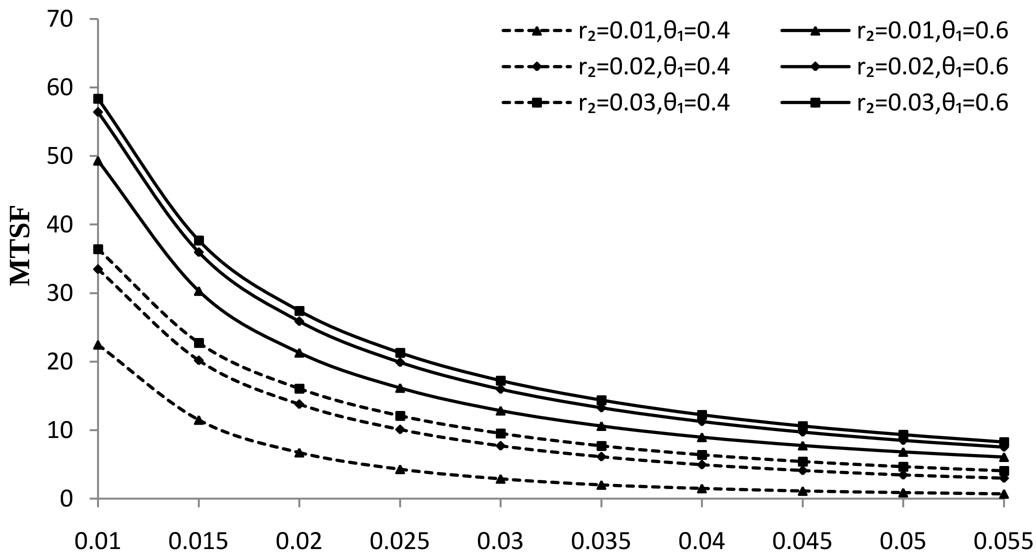

Fig.2

p

Behavior of Profit (P) with respect to p, r_2 and θ_1

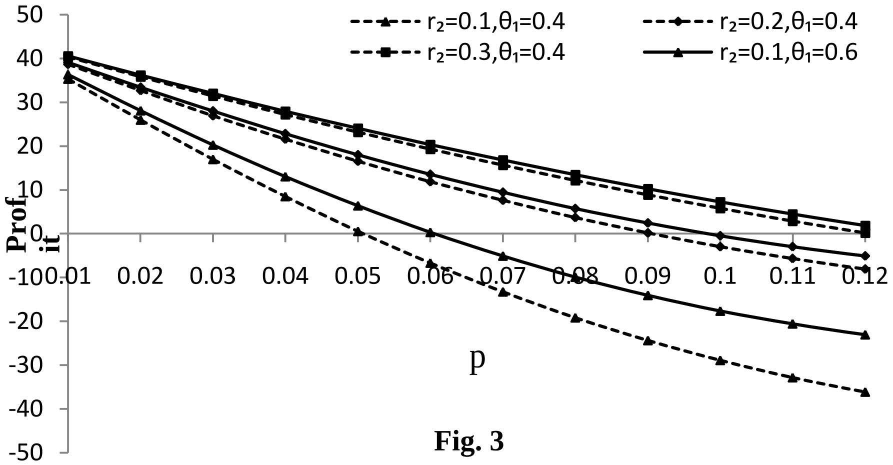

Fig. 3

Acknowledgement: The authors are thankful to the Referee for valuable comments and suggestions.

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