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A SIMILARITY SOLUTION FOR LAMINAR THERMAL BOUNDARY LAYER OVER A FLAT PLATE WITH INTERNAL HEAT GENERATION AND A CONVECTIVE SURFACE BOUNDARY CONDITION

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Abstract: This paper aims to examine the problem of hydrodynamics and thermal boundary layers over a stationary flat plate in a uniform stream of fluid under a convective surface boundary condition when the cold fluid on the surface of the plate generates heat internally. The governing non – linear partial differential equations are transformed to ordinary differential equations by using a similarity variable and then solved numerically using shooting iteration method along with Runge Kutte integration technique. It is found that a similarity solution is possible in this case if the convective heat transfer associated with the hot fluid on the lower side of the plate is proportional to $x^{-1/2}$ and internal heat generated decays exponentially with the classical similarity variable. Further analysis is done to study the effects of the physical parameters on the dimensionless velocity and temperature profiles by presenting the results graphically. Finally local skin – friction coefficient and Nusselt number have been calculated and tabulated and are found to show excellent agreement with the previously published results for special case.

Keywords: Similarity solution, boundary layer, heat generation, flat plate, convective boundary conditions.

1. Introduction

Boundary layer flows with internal heat generation continues to be of considerable interest, mainly because of its numerous applications in the field of engineering systems like geothermal reservoirs, cooling of nuclear reactors, thermal insulation, combustion chamber, rocket engines etc. The problems arising in natural convection flows have been largely studied and widely quoted in various research papers and text books such as [3], [7]. The well known Blasius similarity solution [6] provides velocity distribution for the laminar boundary layer. The similarity solutions for the thermal boundary layer for the case of constant surface temperature at the plate and for the boundary condition of constant heat flux at the plate are also well known through numerous studies [5]. Aziz [2] in his paper attempted to obtain a solution for the thermal boundary layer problem considering a convective boundary condition at the plate.

In the present paper, the problem of Aziz[2] is extended to include the effect of internal heat generation. The numerical solutions of the resulting momentum and thermal boundary layer equations are established and shown graphically and also presented in the tabular form. These findings are also compared with the earlier published results by Aziz.[2]

2. Mathematical Analysis

A two dimensional steady laminar natural convectional flow of a viscous incompressible fluid over a semi- infinite flat plate in a stream of cold fluid at uniform ambient temperature T_∞ moving over the top surface of the plate with a uniform velocity U_∞ is considered. The flow is in the x-direction which is taken along the plate and y-axis is taken normal to the plate. The cold fluid on the surface of the plate generates heat internally at the volumetric rate $\dot{q}$. Further it is assumed that the bottom surface of the plate is heated by convection from a hot fluid at constant temperature $T_f > T_\infty$ which provides a heat transfer coefficient h_f. The governing equations of continuity, momentum and energy describing the flow can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad ...(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} \qquad ...(2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\dot{q}}{\rho C_p} \qquad ...(3)$$

where u and v are the x and the y components of the velocities respectively, T is the temperature, υ- the kinematic viscosity of the fluid, ρ- the fluid density, α- the thermal diffusivity of the fluid and C_p is the specific heat at constant pressure.

For the flow there is no-slip at the plate. The boundary conditions at the plate surface and far into the cold fluid can be expressed as :

$$u = 0, \quad v = 0, \quad -k\frac{\partial T}{\partial y} = h_f(T_f - T_w) \quad \text{at} \quad y = 0 \qquad ...(4)$$

$$u \rightarrow U_\infty, \quad T \rightarrow T_\infty, \quad \text{as} \quad y \rightarrow \infty \qquad ...(5)$$

where $T_w = T(x, 0)$ and k is the thermal conductivity.

In order to obtain similarity solution of the problem, we define an independent variable η and dependent functions $f(\eta)$ and $\theta(\eta)$ as follows:

$$\eta = y\left(\frac{U_\infty}{\upsilon x}\right)^{1/2}, \quad f(\eta) = \frac{\psi}{\sqrt{U_\infty \upsilon x}}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}$$

where ψ is the stream function.

Equations (1) – (5) reduce to

$$2f''' + f f'' = 0 \qquad ...(6)$$

$$\theta'' + \frac{1}{2}\Pr f\theta' + Qe^{-\eta} = 0 \qquad ...(7)$$

$$f(0) = 0, \quad f'(0) = 0, \quad , \quad f'(\infty) = 1 \qquad ...(8)$$

$$\theta'(0) = -Bi[1 - \theta(0)], \quad \theta(\infty) = 0 \qquad ...(9)$$

where $Bi = \frac{h_f}{k}\left(\frac{\upsilon x}{U_\infty}\right)^{1/2}$ is the local Biot number,

$\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number,

$Q = \frac{\upsilon x \dot{q} e^{\eta}}{k U_\infty (T_f - T_\infty)}$ is the local heat generation parameter.

In order to have a similarity solution, the parameters Q and Bi must be constant. Thus we assume that

$$\dot{q} = a\, x^{-1} e^{-\eta}, \quad h_f = b x^{-1/2}$$

so that $Bi = \frac{b}{k}\left(\frac{\upsilon}{U_\infty}\right)^{1/2}$ and $Q = \frac{b\nu}{k U_\infty (T_f - T_\infty)}$

where a, b are constants with appropriate dimensions.

3. Numerical Solution

Equations (6) and (8) represent the classical Blasius problem. The numerical solutions to this problem has been largely studied and analysed by various authors, Cortell [4], Andrzejczak [1] to mention a few. Our focus, therefore will be on analysing the energy equation represented by equations (7) and (9). For the purpose we have used the computational software Mathematica to solve the equations. The boundary conditions at infinity have been replaced by η =10 which is the standard practice in the boundary layer analysis.

The local surface heat flux q_x'' and the local heat transfer rate q can be expressed in terms of θ'(0) as follows:

$$q_x'' = -kW(T_f - T_\infty)\left(\frac{U_\infty}{\upsilon x}\right)^{1/2} \theta'(0),$$

q = -2kW$(T_f - T_\infty)\left(\frac{U_\infty L}{\upsilon}\right)^{1/2}$ θ'(0), when Bi is constant,

q = -2kW$(T_f - T_\infty)\left(\frac{U_\infty}{\upsilon}\right)^{1/2} \int_0^L x^{-1/2}\ \theta'(0)$dx, when Bi is a function of x,

where L is the plate length and W is the plate width.

This is because, when Biot number is a function of x, the surface temperature θ'(0) depends on x and thus integration over the entire plate is necessary to obtain the total heat transfer rate.

4. Result and Discussions

In order to get a clear insight of the problem considered above, numerical calculations have been carried out for different values of the parameters. In this paper the problem attempted by Aziz[2] has been extended and numerical data for θ(0) and -θ'(0) has been presented in the tabular form. Table I gives these values for a fixed Prandtl number 0.72 and for a range of values of the parameter Bi and it shows a perfect agreement with the existing data in the paper by Aziz[2]. Also we observe that whereas for the problem where there is no heat generation, the values of both θ(0) and -θ'(0) increase as Bi increases for a fixed Prandtl number, the values of both decrease when the internal heat generation is taken into account. In Table II, the numerical data showing values of θ(0) and -θ'(0) for fixed Biot number is listed for a range of Prandtl number. It is observed that θ(0) decreases but -θ'(0) increases with the increase in Prandtl number. These observations are also shown in the graphs depicted in Figures-I(a), I(b), II(a) and II(b).

Computations showing variation of -θ'(0) and θ(0) at the plates for different parameter values:

Table I. Effect of Bi for Pr = 0.72

Bi	WITHOUT INTERNAL HEAT GENERATION (Aziz 2009)		WITH INTERNAL HEAT GENERATION (Present work)	
	θ(0)	-θ'(0)	θ(0)	-θ'(0)
0.05	0.144661	0.0427669	2.32195	-0.0660973
0.1	0.252758	0.0747242	2.15488	-0.115488
0.2	0.403523	0.119295	1.92187	-0.184374
0.4	0.575014	0.169994	1.65683	-0.26273
0.6	0.669916	0.198051	1.51015	-0.306092
0.8	0.73017	0.215864	1.41703	-0.333623
1.0	0.771822	0.228178	1.35265	-0.352654
5.0	0.944174	0.279131	1.08628	-0.431403
10.0	0.971285	0.287146	1.04438	-0.443791
20.0	0.985434	0.291329	1.02251	-0.450255

Table II. Effect of Pr for Bi = 1.0

Pr	WITHOUT INTERNAL HEAT GENERATION		WITH INTERNAL HEAT GENERATION	
	θ(0)	-θ'(0)	θ(0)	-θ'(0)
0.1	0.871701	0.128299	1.62367	-0.62367
0.5	0.794093	0.205907	1.41146	-0.411456
0.72	0.771822	0.228178	1.35265	-0.352654
7.1	0.606424	0.393576	0.95412	0.0458797
10.0	0.578656	0.421344	0.893789	0.106211

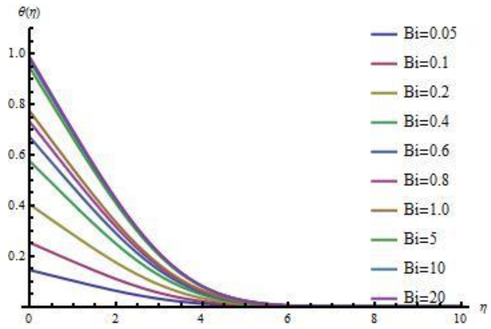

Fig.I(a): Temperature distribution for various values of Bi for Pr = 0.72 (without internal heat generation).

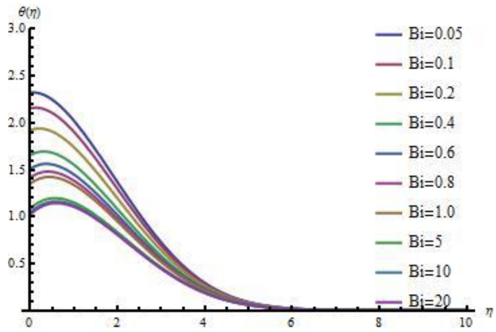

Fig.I(b): Temperature distribution for various values of Bi for Pr = 0.72 (with internal heat generation).

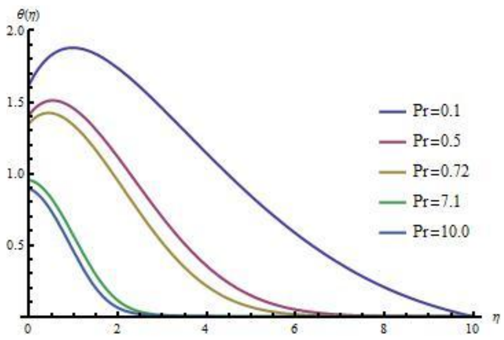

Fig.II(a):Temperature distribution for various values of Pr for Bi = 1 (without internal heat generation).

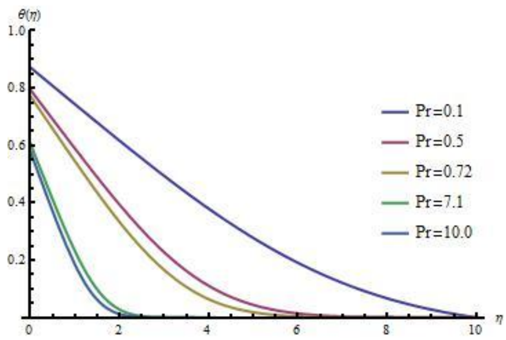

Fig.II(b): Temperature distribution for various values of Pr for Bi = 1 (with internal heat generation).

5. Conclusions

A similarity solution for the laminar thermal boundary layer over a horizontal plate with internal heat generation and a convective surface boundary condition in a uniform stream of fluid is possible if the convective heat transfer of the fluid heating the plate on its lower surface is proportional to $x^{-1/2}$ and the internal heat generated decays exponentially with the similarity variable η. As one would expect the temperature across the plate in the presence of internal heat generation is higher than when the heat generation is absent.

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