

STRING DUST COSMOLOGICAL MODEL FOR BAROTROPIC FLUID DISTRIBUTION WITH VACUUM ENERGY DENSITY IN FRW SPACE-TIME

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Abstract. String dust cosmological model for barotropic fluid distribution with vacuum energy density in the frame work of FRW space-time is investigated. The model starts with a big-bang at $\tau = 0$ and the expansion in the model decreases with time. The spatial volume increases with time representing inflationary scenario. The vacuum energy density $\Lambda \sim \tau^{-2}$ which matches with the result as obtained by Bertolami [10]. The energy density and string tension density are initially large but decreases with time. The model also represents decelerating phase of universe. The special cases for dust model ($p = 0$), radiation dominated model and stiff fluid model are also discussed.

Key Words. String dust, Cosmological, barotropic fluid, vacuum energy density
2010 Mathematics Subject Classification. 83 C XX, 83 F XX

1. Introduction

The object of string theory is to promote better understanding of the evolution in the early stages of universe after its beginning as a singular event. The universe passes through a string era and evolves in to its present state after formation of matter. This gives us the possibility that after the formation of matter, the universe might have passed through a state with matter and remnant string cloud as a transition state with energy-momentum tensor. After the formation of mater, the string cloud in the lemnant form may consist of purely geometric strings with energy density same as the string tension. The presence of strings in early universe can be explained by grand unified field theories (GUT) as discussed by Kibble [12]. These strings have stress energy and classified as geometric and massive string. Letelier [14] formulated the form of energy-momentum tensor for massive strings and used it to obtain cosmological models for massive string using Bianchi Type I and Kantowski space-time. Stachel [18] considered a massless (geometric) strings ($\rho = \lambda$) to examine the relevance of cosmic evolution. Later on several studies viz. Banerjee et al. [8], Krori et al. [13], Tikekar and Patel [19], Barrow

and Kunj [9], Singh and Singh [17], Bali et al. [2,3,4,5] aimed at exploring the relevance of cosmological models based on string hypothesis.

Einstein introduced cosmological constant (Λ) (Vacuum energy density) to find the solution of static cosmological models using his field equations because at that time universe was supposed to be static, homogeneous and isotropic. The relevance of cosmological constant related with the observations is given by Zel'dovich [20]. Riess et al. [16] and Perlmutter et al. [15] investigated that the universe is not only expanding but it is also accelerating and the accelerating behaviour of universe is due to the dominance of vacuum energy density. Many authors viz. Bertolami [10], Chen and Wu [11], Bali et al. [6,7] investigated cosmological model with vacuum energy density in different contexts.

In this paper, we have investigated string dust cosmological model for barotropic fluid distribution with vacuum energy density in the frame work of FRW space-time. To get the deterministic model of the universe, we use the condition $\Lambda \sim \frac{1}{R^2}$ as used by Chen and Wu [11] where Λ is vacuum energy density and R is the scale factor. The physical aspects of the models are also discussed.

2. Metric and Field Equations

We consider FRW (Friedmann-Robertson-Walker) space-time as

$$ds^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad \dots(1)$$

where $k = 0, -1, 1$ represents curvature of space section, R the scale factor.

Einstein's field equation with vacuum energy density (Λ) is given by

$$R_i^j - \frac{1}{2} \bar{R} g_i^j = -T_i^j - \Lambda g_i^j \quad \dots(2)$$

\bar{R} being Ricci scalar.

The energy-momentum tensor (T_i^j) for matter with string used by Letelier [14] is taken as

$$T_i^j = (\rho + p) v_i v^j - p g_i^j - \lambda x_i x^j \quad \dots(3)$$

with

$$v_i v^j = -x_i x^j = 1; v_i x^i = 0 \quad \dots(4)$$

where ρ is the matter density, p the isotropic pressure, λ the string tension density and x^i the direction of string.

We assume the coordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3; v^4 = 1 \quad \dots(5)$$

So we get

$$T_1^1 = -p + \lambda = T_2^2 = T_3^3; T_4^4 = \rho \quad \dots(6)$$

Thus equation (2) leads as

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = \rho + \Lambda \quad \dots(7)$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -p + \Lambda + \lambda \quad \dots(8)$$

Using geometric string ($\rho = \lambda$) condition as used by Stachel [18] in equation (7) and (8), we have

$$-\frac{2\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{2k}{R^2} = p \quad \dots(9)$$

Applying barotropic fluid condition $p = \gamma\rho$ and $\Lambda = \frac{\alpha}{R^2}$ as used by Chen and Wu [11],

we get

$$\frac{2\ddot{R}}{R} + (3\gamma - 2)\frac{\dot{R}^2}{R^2} = \frac{(2k - 3k\gamma + \alpha\gamma)}{R^2}$$

Thus, we have

$$2\ddot{R} + (3\gamma - 2)\frac{\dot{R}^2}{R} = \frac{\beta}{R} \quad \dots(10)$$

where $\beta = 2k - 3k\gamma + \alpha\gamma$.

To get the deterministic solution, we suppose

$$\dot{R} = f(R)$$

Thus, we have

$$\ddot{R} = \frac{df}{dt} = \frac{df}{dR} \frac{dR}{dt} = f' f$$

where

$$f' = \frac{df}{dR}$$

Now equation (10) leads to

$$2ff' + \frac{(3\gamma - 2)}{R} f^2 = \frac{\beta}{R} \quad \dots(11)$$

Thus, we have

$$\frac{d}{dR}(f^2) + \frac{(3\gamma - 2)}{R} f^2 = \frac{\beta}{R} \quad \dots(12)$$

Equation (12) leads to

$$f^2 = \frac{\beta}{3\gamma - 2} + \delta R^{-(3\gamma - 2)} \quad \dots(13)$$

where δ is constant of integration.

From equation (13), we have

$$\frac{dR}{\sqrt{a + \delta R^{-(3\gamma - 2)}}} = dt \quad \dots(14)$$

which leads to

$$\frac{dR}{dt} = \sqrt{a + \delta R^{-(3\gamma - 2)}} \quad \dots(15)$$

where

$$a = \frac{\beta}{3\gamma - 2}$$

By suitable transformation of coordinates, the metric (1) leads to

$$ds^2 = \frac{d\tau^2}{a + \delta\tau^{-(3\gamma-2)}} - \tau^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad \dots(16)$$

where $R = \tau$ and cosmic time t is defined as

$$t = \int \frac{d\tau}{\sqrt{a + \delta R^{-(3\gamma-2)}}} \quad \dots(17)$$

3. Special Models

(i) Dust Fluid Model ($\gamma = 0$)

For $\gamma = 0$, the equation (14) leads to

$$R = \sqrt{\frac{a}{\delta}} \sinh T \quad \dots(18)$$

where $\sqrt{\delta}t + b = T$, b being constant of integration.

Thus the metric (1) leads to

$$ds^2 = dt^2 - \frac{a}{\delta} \sinh^2 T \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad \dots(19)$$

(ii) Radiation Dominated Model ($\gamma = 1/3$)

For $\gamma = 1/3$, the equation (14) leads to

$$R = \frac{1}{\delta} (T^2 - a) \quad \dots(20)$$

where

$$T = \frac{\delta t + b}{2} \quad \dots(21)$$

Thus the metric (1) leads to

$$ds^2 = dt^2 - \frac{(T^2 - a)^2}{\delta^2} \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \quad \dots(22)$$

(iii) Stiff Fluid Model ($\gamma = 1$)

For $\gamma = 1$, the equation (14) leads to

$$\frac{dR}{dt} = \frac{\sqrt{aR + \delta}}{R^{1/2}} \quad \dots(23)$$

Thus the metric (1) leads to

$$\begin{aligned} ds^2 &= \left(\frac{dt}{dR}\right)^2 dR^2 - R^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \\ &= \frac{R}{aR + \delta} dR^2 - R^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \end{aligned} \quad \dots(24)$$

where R is defined by

$$\sqrt{b+R} \sqrt{R} - b \log \left\{ \frac{\sqrt{b+R} + \sqrt{R}}{\sqrt{b}} \right\} = \sqrt{a} t + L \quad \dots(25)$$

L being constant of integration.

4. Physical and Geometrical Aspects

The expansion (θ), vacuum energy density (Λ), spatial volume (R^3), deceleration parameter (q), the energy density (ρ), string tension density (λ) and the isotropic pressure (p) for the model (16) are given by

$$\begin{aligned} \theta = v_{;i}^i &= \frac{\dot{R}}{R} \\ &= \frac{1}{\tau} \sqrt{a + \frac{\delta}{\tau^{3\gamma-2}}} \end{aligned} \quad \dots(26)$$

$$\Lambda = \frac{\alpha}{R^2} = \frac{\alpha}{\tau^2} \quad \dots(27)$$

$$R^3 = \tau^3 \quad \dots(28)$$

$$q = \frac{(3\gamma - 2)\delta\tau^{-3\gamma+2}}{2(a + \delta\tau^{-3\gamma+2})} \quad \dots(29)$$

$$\rho = \lambda = \frac{1}{\tau^2} \{3a + 3\delta\tau^{-3\gamma+2} + 3k - \alpha\} \quad \dots(30)$$

$$p = \gamma\rho = \frac{\gamma}{\tau^2} \{3a + 3\delta\tau^{-3\gamma+2} + 3k - \alpha\} \quad \dots(31)$$

Now the expansion (θ), decelerating parameter (q), vacuum energy density (Λ), spatial volume (R^3), the energy density (ρ), string tension density (λ), and the isotropic pressure (p) for the model (19) are given by

$$\theta = \sqrt{a} \coth T \quad \dots(32)$$

$$q = -\tanh^2 T \quad \dots(33)$$

$$\Lambda = \frac{\alpha}{\frac{a}{\delta} \sinh^2 T} \quad \dots(34)$$

$$R^3 = \left(\frac{a}{\delta}\right)^{3/2} \sinh^3 T \quad \dots(35)$$

$$\rho = \lambda = \frac{3a \cosh^2 T + 3k - \alpha}{\frac{a}{\delta} \sinh^2 T} \quad \dots(36)$$

$$p = \frac{\gamma(3a \cosh^2 T + 3k - \alpha)}{\frac{a}{\delta} \sinh^2 T} \quad \dots(37)$$

and the expansion (θ), deceleration parameter (q), vacuum energy density (Λ), spatial volume (R^3), the energy density (ρ), string tension density (λ) and the isotropic pressure (p) for the model (22) are given by

$$\theta = \frac{T \delta}{T^2 - a} \quad \dots(38)$$

$$q = -\frac{(T^2 - a)}{2T^2} \quad \dots(39)$$

$$\Lambda = \frac{\alpha \delta^2}{(T^2 - a)^2} \quad \dots(40)$$

$$R^3 = \left(\frac{T^2 - a}{\delta} \right)^3 \quad \dots(41)$$

$$\rho = \lambda = \frac{(3T^2 + 3k - \alpha)\delta^2}{(T^2 - a)^2} \quad \dots(42)$$

$$p = \frac{\gamma(3T^2 + 3k - \alpha)\delta^2}{(T^2 - a)^2} \quad \dots(43)$$

and the expansion (θ), deceleration parameter (q), vacuum energy density (Λ), spatial volume (R^3), the energy density (ρ), string tension density (λ) and the isotropic pressure (p) for the model (24) are given by

$$\theta = \frac{\sqrt{aR + \delta}}{R^{3/2}} \quad \dots(44)$$

$$q = \frac{\delta}{2(aR + \delta)} \quad \dots(45)$$

$$\Lambda = \frac{\alpha}{R^2} \quad \dots(46)$$

$$\rho = \lambda = \frac{1}{R^2} \left[3 \left(\frac{aR + \delta}{R} \right) + 3k - \alpha \right] \quad \dots(47)$$

$$p = \frac{\gamma}{R^2} \left[3 \left(\frac{aR + \delta}{R} \right) + 3k - \alpha \right] \quad \dots(48)$$

where R is defined by equation (25).

5. Discussion and Conclusion

The model (16) starts with a big-bang at $\tau = 0$ and the expansion in the model decreases. The spatial volume increases with time representing inflationary scenario. The deceleration parameter $q < 0$ which shows that the model represents accelerating universe for barotropic fluid distribution. The energy density (ρ) and string tension density are initially large but decreases with time. The model has singularity at $\tau = 0$. The vacuum

energy density (Λ) $\sim \frac{1}{\tau^2}$ which matches with the result as obtained by Bertolami [10].

Similar is the case for dust distribution, radiation dominated model while for stiff fluid

case, the model represents decelerating universe as $q > 0$. We also find similar type of results for other parameters as for barotropic fluid distribution.

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