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BIANCHI TYPE V INFLATIONARY UNIVERSE WITH FLAT POTENTIAL AND STIFF FLUID DISTRIBUTION IN GENERAL RELATIVITY

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Abstract: Inflationary scenario for massless scalar field, flat potential and stiff fluid distribution in Bianchi Type V space-time is investigated. We find that the rate of Higgs field decreases with time. The spatial volume increases with time representing inflationary scenario. The model represents decelerating and accelerating phases of universe and also represents anisotropic space-time but isotropizes at late time. The Hubble parameter decreases with time. The model has Point Type singularity at T = 0. The particular case is also discussed. The results match with astronomical observations.

Key Words: Bianchi V, Inflationary, Stiff fluid, flat potential.

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1. Introduction

Stiff fluid models create more interest in the study because for these models the speed of sound is equal to speed of light and governing equations have the same characteristics as those of gravitational field (Zel'dovich [18]). Keeping in view the importance of stiff fluid models, Bali et al. [2,3], Mak and Harko [12] have investigated cosmological models for stiff fluid distribution in different contexts. Bianchi Type V cosmological models are the natural generalization of FRW (Friedmann-Robertson-Walker) models with negative curvature. These open models are favoured by the available evidences for low density universes (Gott et al.[10]). Bianchi Type V models are also studied by Roy and Singh [15], Banerjee and Sanyal [7], Coley [9], Bali and Meena [4].

Inflation is the extremely rapid exponential expansion of the early universe by a factor of at least 10^{78} in volume driven by a negative pressure vacuum energy density. Following the inflationary period, the universe continued to expand but at a slower rate. The inflationary hypothesis was originally proposed by Guth [11] who named it inflation. Later on, it was also proposed by Sato [16]. Inflationary universes play a significant role in solving number of outstanding problems in cosmology like homogeneity, the isotropy, horizon and primordial monopole problem in grand unified field theory. Anninos et al. [1] discussed the significance of inflation for isotropization of universe. Panchapakeshan and Sethi [13] discussed inflationary scenario in reference of large scale structure of universe. Burd [8] discussed inflationary scenario in FRW model. Rothman and Ellis [14] have pointed out that we can have solution of the isotropy problem if we work with anisotropic metric and these metrics can be isotropized in a very special circumstances. Stein-Schabes [17] has explained that inflation will take place if effective potential has flat region where Higgs field evolves slowly but the universe expands in an exponential way. Keeping in view of these studies, Bali and Jain [5], Bali [6] investigated inflationary cosmological models in anisotropic Bianchi Type I space-time with flat potential in different contexts. These models isotropize in special case.

The model represents decelerating and accelerating phases of universe and also represents anisotropic space-time but isotropizes at late time. The Hubble parameter decreases with time. The model has Point Type singularity at T = 0. The rate of Higgs field decreases with time.

2. Metric and Field Equations

We consider Bianchi Type V line-element in orthogonal form as

$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}(B^2dy^2 + C^2dz^2) \quad \ldots(1)$$

where A, B, C are metric potentials and are function of t-alone.

We assume the coordinates to be comoving so that $v^1 = 0 = v^2 = v^3, v^4 = 1.$

The action of gravitational field minimally coupled to a scalar field with potential V(ϕ) is given by Stein-Schabes [17]:

$$L = \int \sqrt{-g}\left[R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right]d^4x \quad \ldots(2)$$

The Einstein's field equations (in gravitational units 8πG = c = 1) in the case of massless scalar field ϕ with potential V(ϕ) are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad \ldots(3)$$

with

$$T_{ij} = (\rho + p)v_iv_j + pg_{ij} - \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right]g_{ij} \quad \ldots(4)$$

and

$$\frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}\partial^{\mu}\phi) = -\frac{dV}{d\phi} \qquad \dots(5)$$

The field equations (3) for the line-element (1) lead to non-linear differential equations

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = -\frac{1}{2}\dot{\phi}^2 + K - p \qquad \dots(6)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -\frac{1}{2}\dot{\phi}^2 + K - p \qquad \dots(7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = -\frac{1}{2}\dot{\phi}^2 + K - p \qquad \dots(8)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{3}{A^2} = \frac{1}{2}\dot{\phi}^2 + K + \rho \qquad \dots(9)$$

$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \qquad \dots(10)$$

Equation (10) leads to

$$A^2 = BC \qquad \dots(11)$$

The equation (5) for scalar field (ϕ) leads to

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\phi_4 = \frac{dV}{d\phi} \qquad \dots(12)$$

We are interested in inflationary solution so flat region is considered. Thus V(ϕ) is constant.

Now equation (12) leads to

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\phi_4 = 0 \qquad \dots(13)$$

where suffix ‘4’ indicates ordinary partial derivative with respect to t.

From equation (13), we have

$$\phi_4 = \frac{\ell}{ABC} \qquad \text{...(14)}$$

where ℓ is constant of integration.

The scale factor (R) for line-element (1) is given by

$$R^3 = ABC = A^3 \qquad \text{...(15)}$$

as

$$BC = A^2 \qquad \text{(from equation (11))} \qquad \text{...(16)}$$

From equation (7) and (8), we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = 0 \qquad \text{...(17)}$$

which leads to

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = -\frac{1}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)$$

which leads to

$$C^2\left(\frac{B}{C}\right)_4 = \frac{L}{(BC)^{1/2}} \qquad \text{...(18)}$$

where L is constant of integration.

To find the solution of equation (18), we assume that BC = μ and $\frac{B}{C} = \nu$.

Thus equation (18) leads to

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^{3/2}} \qquad \text{...(19)}$$

From equations (6) and (9), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{2B_4 C_4}{BC} - \frac{4}{A^2} = 2K + \rho - p \qquad \text{...(20)}$$

Applying stiff fluid condition ρ = p, we have

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{1}{2}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)^2+\frac{2B_4C_4}{BC}-\frac{4}{A^2}=2K \qquad \ldots(21)$$

Equation (21) leads to

$$2\mu_{44}+\frac{1}{\mu}\mu_4^2=4K\mu+8 \qquad \ldots(22)$$

which leads to

$$f^2=\frac{4K}{3}\mu^2+4\mu+\frac{\gamma}{\mu} \qquad \ldots(23)$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2=-\frac{T}{\frac{4K}{3}T^3+4T^2+\gamma}dT^2+TdX^2+e^{2X}T$$
$$.\left\{e^{\int\frac{L}{T\sqrt{\frac{4K}{3}T^3+4T^2+\gamma}}dT+N}dY^2+e^{-\int\frac{L}{T\sqrt{\frac{4K}{3}T^3+4T^2+\gamma}}dT+N}dZ^2\right\} \qquad \ldots(24)$$

where

$$\mu=T \qquad \ldots(25)$$

$$\nu=e^{\int\frac{L}{T\sqrt{\frac{4K}{3}T^3+4T^2+\gamma}}dT+N} \qquad \ldots(26)$$

where N is constant of integration

$$A^2=BC=\mu=T \qquad \ldots(27)$$

$$B^2=\mu\nu=T\,e^{\int\frac{L}{T\sqrt{\frac{4K}{3}T^3+4T^2+\gamma}}dT+N} \qquad \ldots(28)$$

$$C^2 = \mu/\nu = T\, e^{-\int \frac{L}{T\sqrt{\frac{4K}{3}T^3+4T^2+\gamma}}dT+N} \qquad \ldots(29)$$

Particular Case

Put $\gamma = 0$ in equation (23), we have

$$f^2 = \frac{4K}{3}\mu^2 + 4\mu \qquad \ldots(30)$$

$$\Rightarrow \frac{d\mu}{\sqrt{\left(\mu+\frac{2}{\beta^2}\right)^2 - \left(\frac{2}{\beta^2}\right)^2}} = \beta\, dt \qquad \ldots(31)$$

where

$$\beta^2 = \frac{4K}{3}$$

Equation (31) leads to

$$\mu = \frac{4}{\beta^2}\sinh^2\left(\frac{\beta t+\alpha}{2}\right) \qquad \ldots(32)$$

From equation (19), we have

$$\frac{\nu_4}{\nu} = \frac{L\beta^3}{8\sinh^3\left(\frac{\beta t+\alpha}{2}\right)} \qquad \ldots(33)$$

which leads to

$$\nu = a\exp b\left\{-\operatorname{cosech}\left(\frac{\beta t+\alpha}{2}\right)\coth\left(\frac{\beta t+\alpha}{2}\right)\right.$$

$$\left.+\log\left[\operatorname{cosech}\left(\frac{\beta t+\alpha}{2}\right)+\coth\left(\frac{\beta t+\alpha}{2}\right)\right]\right\} \qquad \ldots(34)$$

where $b = \frac{L\beta^2}{8}$ and a is constant of integration.

Thus, we have

$$A^2 = BC = \mu = \frac{4}{\beta^2}\sinh^2 T \qquad \ldots(35)$$

$$B^2 = \mu\nu = \frac{4a}{\beta^2}\sinh^2 T \exp b\{-\mathrm{cosech}T\coth T + \log[\mathrm{cosech}T + \coth T]\} \qquad \ldots(36)$$

$$C^2 = \mu/\nu = \frac{4\sinh^2 T}{\beta^2 a \exp b\{-\mathrm{cosech}T\coth T + \log[\mathrm{cosech}T + \coth T]\}} \qquad \ldots(37)$$

where $\frac{\beta t + \alpha}{2} = T$.

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = -\frac{4}{\beta^2}dT^2 + \sinh^2 T\, dX^2 + e^{\beta X}\sinh^2 T$$

$$\{e^{b[-\mathrm{cosech}T\mathrm{coth} + \log(\mathrm{cosech}T + \coth T)]}dY^2 + e^{-b[-\mathrm{cosech}T\coth T + \log(\mathrm{cosech}T + \coth T)]}dZ^2\} \qquad \ldots(38)$$

where $\frac{2}{\beta}x = X$

$$\frac{2\sqrt{a}}{\beta}y = Y$$

$$\frac{2}{\beta\sqrt{a}}z = Z$$

3. Physical and Geometrical Aspects

The Higgs field (ϕ), the spatial volume (R^3), the expansion (θ), shear (σ), Hubble parameter (H), the deceleration parameter (q) for the model (24) are given by

The rate of Higgs field (ϕ) is given by equation (14) as

$$\phi_4 = \frac{\ell}{A^3} = \frac{\ell}{T^{3/2}} \qquad \ldots(39)$$

which leads to

$$\phi = \int \frac{\ell}{T\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}} dT + M \qquad \ldots(40)$$

where M is constant of integration.

The spatial volume (R^3) for the model (24) is given by

$$R^3 = T^{3/2} \qquad \ldots(41)$$

The expansion $\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{3}{2}\frac{\mu_4}{\mu}$

$$= \frac{3\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}{2T^{3/2}} \qquad \ldots(42)$$

Shear $(\sigma) = \frac{1}{2}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{1}{2}\frac{\nu_4}{\nu}$

$$= \frac{L}{2T^{3/2}} \qquad \ldots(43)$$

Deceleration parameter $(q) = -\frac{R_{44}/R}{R_4^2/R^2}$

$$= -\frac{\frac{4K}{3} - \frac{2\gamma}{T^3}}{\frac{4K}{3} + \frac{4}{T} + \frac{\gamma}{T^3}}; \quad \begin{array}{l} q < 0 \text{ if } \frac{4K}{3} > \frac{2\gamma}{T^3} \\ q > 0 \text{ if } \frac{4K}{3} < \frac{2\gamma}{T^3} \end{array} \qquad \ldots(44)$$

$$\frac{\sigma}{\theta} = \frac{L}{3\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}} \qquad \ldots(45)$$

$\frac{\sigma}{\theta} \to 0$ for large value of T.

H = Hubble parameter = $\frac{R_4}{R}$

$$= \frac{\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}{2T^{3/2}} \qquad \text{…(46)}$$

Calculation of anisotropy parameter

If $\widehat{A}$ is anisotropy parameter and H_1, H_2, H_3 are Hubble constants in x, y, z direction then anisotropy parameter $\widehat{A}$ is defined as

$$\widehat{A} = \frac{1}{3}\left[\left(\frac{H_1}{H} - 1\right)^2 + \left(\frac{H_2}{H} - 1\right)^2 + \left(\frac{H_3}{H} - 1\right)^2\right] \qquad \text{…(47)}$$

where $H_1 = \frac{A_4}{A}, H_2 = \frac{B_4}{B}, H_3 = \frac{C_4}{C}$

which leads to

$$\frac{A_4}{A} = H_1 = \frac{f}{2T} = \frac{\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}{2T^{3/2}}$$

$$\frac{B_4}{B} = H_2 = \frac{\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}{2T^{3/2}} + \frac{L\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}{2T^{3/2}\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}$$

$$\frac{C_4}{C} = H_3 = \frac{\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}{2T^{3/2}} - \frac{L\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}{2T^{3/2}\sqrt{\frac{4K}{3}T^3 + 4T^2 + \gamma}}$$

Equation (47) leads to

$$\Rightarrow \widehat{A} = \frac{2L^2}{3\left(\frac{4K}{3}T^3 + 4T^2 + \gamma\right)}$$

= 0, for large value of T.

The Higgs field (ϕ), the spatial volume (R^3), the expansion (θ), shear (σ), Hubble parameter (H), the deceleration parameter (q) for the model (38) are given by

The rate of Higgs field (ϕ) is given by equation (14) as

$$\phi_4 = \frac{\ell}{A^3}$$

$$= \frac{\ell \beta^3}{8\sinh^3 T} \quad \text{…(48)}$$

which leads to

$$\phi = \frac{\ell \beta^2}{8}\{-\text{cosech}T \coth T + \log(\text{cosech}T + \coth T)\} + M_1 \quad \text{…(49)}$$

where M_1 is constant of integration.

The spatial volume

$$(R^3) = \frac{8\sinh^3 T}{\beta^3} \quad \text{…(50)}$$

The expansion

$$(\theta) = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{3}{2}\frac{\mu_4}{\mu}$$

$$= \frac{3\beta \coth T}{2} \quad \text{…(51)}$$

Shear $(\sigma) = \frac{1}{2}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{1}{2}\frac{\nu_4}{\nu}$

$$=\frac{L\beta^3}{16\sinh^3 T} \qquad \ldots(52)$$

The deceleration parameter $(q) = -\frac{R_{44}/R}{R_4^2/R^2}$

$$= -\tanh^2 T \qquad \ldots(53)$$

Hubble parameter $(H) = \frac{R_4}{R}$

$$= \frac{\beta \coth T}{2} \qquad \ldots(54)$$

$$\frac{\sigma}{\theta} = \frac{L\beta^2}{24\sinh^2 t \cosh T}$$

$\frac{\sigma}{\theta} \to 0,$ for large value of T ...(55)

Calculation of anisotropy parameter

If $\widehat{A}$ is anisotropy parameter and H_1, H_2, H_3 are Hubble constants in x, y, z directions then anisotropy parameter $\widehat{A}$ is defined as

$$\widehat{A} = \frac{1}{3}\left[\left(\frac{H_1}{H} - 1\right)^2 + \left(\frac{H_2}{H} - 1\right)^2 + \left(\frac{H_3}{H} - 1\right)^2\right] \qquad \ldots(56)$$

where $H_1 = \frac{A_4}{A}, H_2 = \frac{B_4}{B}, H_3 = \frac{C_4}{C}$

which leads to

$$\frac{A_4}{A} = H_1 = \frac{\beta \coth T}{2}$$

$$\frac{B_4}{B}=H_2=\frac{\beta\coth T}{2}+b\left\{\frac{\beta}{4\sinh T}\left(\frac{1+\cosh^2 T}{\sinh^2 T}\right)-\frac{\beta}{4\sinh T}\right\}$$

$$\frac{C_4}{C}=H_3=\frac{\beta\coth T}{2}-b\left\{\frac{\beta}{4\sinh T}\left(\frac{1+\cosh^2 T}{\sinh^2 T}\right)-\frac{\beta}{4\sinh T}\right\}$$

Equation (56) leads to

$$\widehat{A}=\frac{2b^2}{3\cosh^2 T\sinh^4 T}$$

= 0, for large value of T.

Discussion and Conclusion

The spatial volume increases with time representing inflationary scenario. The first model represents decelerating and accelerating phases of universe which matches with recent Astronomical observations. The model in general represents anisotropic space-time but isotropizes at late time. The Hubble parameter decreases with time. The second model represents accelerating universe as deceleration parameter $q < 0$. The rate of Higgs field decreases with time. The model has Point Type singularity at $T = 0$.

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