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SUBCLASSES OF ANALYTIC FUNCTIONS WITH RESPECT TO SYMMETRIC AND CONJUGATE POINTS

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Abstract: In this paper, we introduce new subclasses of Convex and Starlike functions with respect to other points. Further we obtain co-efficient inequalities for these classes.
Key words and phrases. Analytic function, univalent function, Starlike with respect to symmetric points, Coefficient estimates.

1. Introduction

Let A be the class function which are analytic and univalent in the open unit disc $U = \{z : z \in \mathbb{C}\ |z| < 1\}$ given by

$$\omega(z) = z + \sum_{n=1}^{\infty} b_n z^n,$$

and satisfying the conditions $\omega(0) = 0,$ and $|\omega(z)| \le 1, \qquad z \in \mathcal{U}.$

Let S denote the class of functions f which are analytic and univalent in $\mathcal{U}$ of the from

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad z \in \mathcal{U}. \qquad ...(1)$$

Also, let S_s^{λ} be the subclass of S of consisting of functions given by (1) satisfying the condition. [1]

$$\Re\left\{\frac{(1-t)zf^{|}(z)}{f(z)-f(tz)}\right\} > 0, \qquad z \in \mathcal{U}, \quad |t| \le 1.$$

The class S_s^λ be the subclass of S consisting of functions given by (1) satisfying the condition

$$\Re\left\{\frac{(1-t)\left(zf^{|}(z)\right)^{|}}{\left(f(z)-f(tz)\right)^{|}}\right\}>0, \qquad z\in U, \quad |t|\le 1.$$

In 1962, Goel and Mehrok [2], in terms of subordination introduced a subclasses of S_s^λ denoted by $S_s^\lambda(A,B)$. So in the same manner, the author here wish to give the analogue definitions by estimation as follows.

Let $S_s^\lambda(A,B,t)$ be the subclass of S consisting of functions given by (1) satisfying the condition

$$\frac{(1-t)zf^{|}(z)}{\left(f(z)-f(tz)\right)}\prec\frac{1+Az}{1+Bz}, \qquad -1\le B<A\le 1, Z\in U$$

and t is a fixed point in U.

Let $S_s^\lambda(A,B)$ be the subclass of S consisting of functions given by (1) satisfying the condition

$$\frac{(1-t)zf^{|}(z)}{\left(f(z)-\overline{f(tz)}\right)}\prec\frac{1+Az}{1+Bz}, \qquad -1\le B<A\le 1, Z\in U$$

and t is a fixed point in U.

The class S_s^c be the subclass of S consisting of functions given by (1) satisfying the condition

$$\frac{(1-t)\left(zf^{|}(z)\right)^{|}}{\left(f(z)-f(tz)\right)^{|}}\prec\frac{1+Az}{1+Bz}, \qquad -1\le B<A\le 1, Z\in U$$

The class S_c^c be the subclass of S consisting of functions given by (1) satisfying the condition

$$\frac{(1-t)\left(zf^{|}(z)\right)^{|}}{\left(f(z)-\overline{f(tz)}\right)^{|}}\prec\frac{1+Az}{1+Bz}, \qquad -1\le B<A\le 1, Z\in U$$

and t is a fixed point in $\mathcal{U}$.

In the present paper, we introduce the class $M_c(\alpha,t,A,B)$ consisting of analytic functions f of the form (1) and satisfying [3,4]:

$$\frac{(1-t)zf^{\prime}(z)+\alpha(1-t)z^{2}f^{\prime\prime}(z)}{(1-\alpha)\big(f(z)-f(tz)\big)+\alpha z\big(f(z)-f(tz)\big)^{\prime}}\prec\frac{1+Az}{1+Bz}$$

We note that $M_s(0,0,A,B)=S_s^{*}(A,B)$ *and* $M_s(1,1,A,B)=C_s(A,B)$.

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$$\frac{(1-t)zf^{\prime}(z)+\alpha(1-t)z^{2}f^{\prime\prime}(z)}{(1-\alpha)\Big(f(z)+\overline{f(tz)}\Big)+\alpha z\Big(f(z)-\overline{f(tz)}\Big)^{\prime}}\prec\frac{1+Az}{1+Bz}\qquad -1<A\le 1,\quad \le\alpha\le 1,\qquad \in\mathcal{U},t\in\mathcal{C}$$

We note that $M_s(0,0,A,B)=S_s^{*}(A,B)$ *and* $M_s(1,1,A,B)=C_s(A,B)$.

By definition of subordination it follows that $f\in M_s(\alpha,t,A,B)$ if and only if

$$\frac{(1-t)zf^{\prime}(z)+\alpha(1-t)z^{2}f^{\prime\prime}(z)}{(1-\alpha)\big(f(z)-f(tz)\big)+\alpha z\Big(f(z)-f(tz)^{\prime}\Big)}=\frac{1+Aw(z)}{1+Bw(z)}=\wp(z)\ w\in\mathcal{U}\qquad \text{...(2)}$$

$$-1<A\le 1,\qquad \le\alpha\le 1,\qquad \in\mathcal{U},t\in\mathcal{C}.$$

and that $f\in M_s(\alpha,t,A,B)$ if and only if

$$\frac{(1-t)zf^{\prime}(z)+\alpha(1-t)z^{2}f^{\prime\prime}(z)}{(1-\alpha)\Big(f(z)+\overline{f(tz)}\Big)+\alpha z\Big(f(z)-\overline{f(tz)}\Big)^{\prime}}=\frac{1+Aw(z)}{1+Bw(z)}=\wp(z),\ w\in\mathcal{U}\qquad \text{...(3)}$$

$$-1\le B<A\le 1,\in\mathcal{U},\qquad t\in\mathcal{C}.$$

Where

$$\wp(z)=1+\sum_{n=1}^{\infty}\wp_n z^n.\qquad \text{...(4)}$$

In this work, we obtain the coefficient estimate of the classes $M_s(\alpha,t,A,B)$ and $M_c(\alpha,t,A,B)$.

2. Preliminary Result

To prove our results, we need the following lemma:

Lemma 2.1. If $\wp(z)$ is given by (4) then

$$|\wp_n| \le A-B, \quad =1,2,3,4,...$$

3. Main Result

In this section, we give the coefficient inequalities for classes $M_s(\alpha,t,A,B)$ *and* $M_c(\alpha,t,A,B)$.

Theorem 3.1. Let $f \in M_s(\alpha,t,A,B)$. Then for $n \ge 1, 0 \le \alpha \le 1$,

$$|a_2| \le \frac{A-B}{(1+\alpha)(2-u_2)}$$

$$|a_3| \le \frac{(A-B)\left[(A-B-1)u_2+2\right]}{(1+2\alpha)(2-u_2)(3-u_3)}$$

$$|a_4| \le \frac{(A-B)\left[(A-B-1)u_2+2\right]\left[(A-B-1)u_3+3\right]}{(1+3\alpha)(2-u_2)(3-u_3)(4-u_4)}$$

$$|a_5| \le \frac{(A-B)\left[(A-B-1)u_2+2\right]\left[(A-B-1)u_3+3\right]\left[(A-B-2)u_4+4\right]}{(1+3\alpha)(2-u_2)(3-u_3)(4-u_4)(5-u_5)}$$

Proof. From (2) and (4), we have

$$(1-t)\left[\left(z+2a_2z^2+3a_3z^3+4a_4z^4+5a_5z^5+...\right)+\alpha\left(2a_2z^2+6a_3z^3+12a_4z^4+20a_5z^5+...\right)\right]=$$
$$(1-t)\left[(1-t)\left(z+a_2u_2z^2+a_3u_3z^3+a_4u_4z^4+...\right)+\alpha\left(z+2a_2u_2z^2+3a_3u_3z^3+4a_4u_4z^4+...\right)\right].$$
$$\left(1+\wp_1z+\wp_2z^2+\wp_3z^3+\wp_4z^4+\wp_5z^5+...\right).$$

Equating the coefficients of like powers of z, we have:

$$\wp_1 = a_2(1+\alpha)(2-u_2), \qquad \wp_2 = a_3(1+2\alpha)(3-u_3)-\wp_1a_2u_2(1+\alpha)$$
$$\wp_3 = a_4(1+3\alpha)(4-u_4)-\wp_1a_2u_3(1+2\alpha)-a_2\wp_2(1+\alpha)$$
$$\wp_4 = a_5(1+4\alpha)(5-u_5)-\wp_1a_4u_4(1+3\alpha)-a_3\wp_2(1+2\alpha)-a_2p_3u_2(1+\alpha)$$
$$\wp_5 = a_6(1+5\alpha)(6-u_6)-\wp_1a_5u_5(1+4\alpha)-a_4\wp_2u_4(1+3\alpha)-a_3p_3u_3(1+2\alpha)-a_2\wp_2u_2(1+\alpha)$$

Using lemma 2.1 on the above we have

$$|a_2| \le \frac{A-B}{(1+\alpha)(2-u_2)}$$

$$|a_3| \le \frac{(A-B)\left[(A-B-1)u_2+2\right]}{(1+2\alpha)(2-u_2)(3-u_3)}$$

$$|a_4| \le \frac{(A-B)\left[(A-B-1)u_2+2\right]\left[(A-B-1)u_3+3\right]}{(1+3\alpha)(2-u_2)(3-u_3)(4-u_4)}$$

$$|a_5| \le \frac{(A-B)\left[(A-B-1)u_2+2\right]\left[(A-B-1)u_3+3\right]\left[(A-B-2)u_4+4\right]}{(1+3\alpha)(2-u_2)(3-u_3)(4-u_4)(5-u_5)}$$

and this complete the proof of Theorem 3.1.

With various choices of A, B, α many existing and new results in this dimension could be obtained. For example, if we set $t = 0$ Theorem 3.1 we have

Corollary 3.2. Let $f \in M_s(\alpha, t, A, B)$. Then for $n = 2,3,4,5, 0 \le \alpha \le 1$

$$|a_2| \le \frac{(A-B)}{(1+\alpha)2}, \qquad |a_3| \le \frac{(A-B)}{(1+2\alpha)3}$$

$$|a_4| \le \frac{(A-B)}{(1+3\alpha)4}, \qquad |a_5| \le \frac{(A-B)}{(1+4\alpha)5}$$

If we set $\alpha = 1$ in Corollary 3.2, we have

Corollary 3.3. Let $f \in M_s(\alpha, t, A, B)$. Then for $n = 2,3,4,5, 0 \le \alpha \le 1$

$$|a_2| \le \frac{(A-B)}{2^2}, \qquad |a_3| \le \frac{(A-B)}{3^2}$$

$$|a_4| \le \frac{(A-B)}{4^2}, \qquad |a_5| \le \frac{(A-B)}{5^2}$$

Theorem 3.4. Let $f \in M_c(\alpha, t, A, B)$. Then for $n \ge 1, 0 \le \alpha \le 1$,

$$|a_2| \le \frac{A-B}{2(1+\alpha)}$$

$$|a_3| \le \frac{(A-B)}{(1+2\alpha)(3-u_3)}$$

$$|a_4| \le \frac{(A-B)(A-B-1)u_3+3}{4(1+3\alpha)(3-u_3)}$$

$$|a_5| \le \frac{(A-B)\left[(A-B-1)u_3+3\right]}{(1+4\alpha)(3-u_3)(5-u_5)}$$

Proof. From (2) and (4), we have

$$(1-t)\left[\left(z+2a_2z^2+3a_3z^3+4a_4z^4+5a_5z^5+...\right)+\alpha\left(2a_2z^2+6a_3z^3+12a_4z^4+20a_5z^5+...\right)\right]=$$

$$(1-t)\left[\left(z+a_3u_3z^3(1+2\alpha)+a_5u_5z^5(1+4\alpha)+....\right)\right].\left(1+\wp_1z+\wp_2z^2+\wp_3z^3+\wp_4z^4+\wp_5z^5+...\right)$$

Equating the coefficients of like powers of z, we have:

$$\wp_1 = a_2(1+\alpha), \qquad \wp_2 = a_3(1+2\alpha)(u_3-3)$$

$$\wp_3 = 4a_4(1+3\alpha)(1+3\alpha)-\wp_1 a_3 u_3(1+2\alpha)$$

$$\wp_4 = a_5(1+4\alpha)(5-u_5)-\wp_3 a_2 u_3(1+2\alpha)$$

Using Lemma 2.1 on the above we have

$$|a_2| \le \frac{A-B}{2(1+\alpha)}$$

$$|a_3| \le \frac{(A-B)}{(1+2\alpha)(3-u_3)}$$

$$|a_4| \le \frac{(A-B)(A-B-1)u_3+3}{4(1+3\alpha)(3-u_3)}$$

$$|a_5| \le \frac{(A-B)\left[(A-B-1)u_3+3\right]}{(1+4\alpha)(3-u_3)(5-u_5)}$$

and this complete the proof of Theorem 3.2.

With various choices of A, B, α many existing and new results in this dimension could be obtained. For example, if we set $t=0$ in Theorem 3.1 we have

Corollary 3.5. Let $f \in M_c(\alpha, t, A, B)$. Then for $n = 2,3,4,5, 0 \le \alpha \le 1$

$$|a_2| \le \frac{(A-B)}{2(1+\alpha)}, \qquad |a_3| \le \frac{(A-B)}{(1+2\alpha)3}$$

$$|a_4| \le \frac{(A-B)}{4(1+3\alpha)}, \qquad |a_5| \le \frac{(A-B)}{(1+4\alpha)5}$$

If we set $\alpha = 1$ in Corollary 3.2, we have

Corollary 3.6. Let $f \in M_s(\alpha, t, A, B)$. Then for $n = 2,3,4,5, 0 \le \alpha \le 1$

$$|a_2| \le \frac{(A-B)}{2^2}, \qquad |a_3| \le \frac{(A-B)}{3^2}$$

$$|a_4| \le \frac{(A-B)}{4^2}, \qquad |a_5| \le \frac{(A-B)}{5^2}$$

References

[1] El-Ashwah, R. M. and Thomas, D. K. (1982). *Some subclass of close-to-convex functions*, J.Ramanujan Math. Soc., 2, 86-100.

[2] Goel, R. M. and Mehrok, B. C., (1982). *A subclasses of starlike functions with respect to other symmetric points*, Tamkang J. Math., 13(1), 11-24.

[3] Oladip, T. (2012). *New subclasses of analytic functions with respect to other point s*, J. Ineq.in Pure and Appl. Math., 1, art 75.

[4] Selvaraj, C. and Vasanthi, N. (2011). *subclasses of analytic functions with respect to symmetric an d conjugate points*, Tamkang J. Math., 1, art 42.