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C-FIELD COSMOLOGICAL MODEL WITH VARIABLE G AND BULK VISCOSITY IN FLAT FRW SPACE-TIME

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Abstract: Cosmological models with variable bulk viscosity and gravitational constant in C-field cosmology for flat FRW space-time are investigated. To find the deterministic model of the universe, we assume that $G = R^n$ where R is scale factor and n is a constant. We find that the creation field (C) increases with time, G and ρ (matter density) decreases with time and $\left|\frac{\dot{G}}{G}\right| = H(t)$ where H is the Hubble parameter. These results match with the astronomical observations.

Key words: C-field cosmology; Variable G and bulk viscosity, FRW space-time.

2010 Mathematics Subject Classification: 83CXX, 83FXX

1. Introduction

The importance of gravitation on the large scale is due to the short range of strong and weak forces and electromagnetic forces become weak due to global neutrality of matter as pointed by Dicke and Peebles [10]. Dirac [11] proposed a theory with gravitational constant motivated by the occurrence of large number hypothesis. Demarque et al.[9] considered an ansatz in which $G \, \alpha \, t^{-n}$ and obtained

$$\left|\frac{\dot{G}}{G}\right| < 2 \, x \, 10^{-11} \, yr^{-1} \text{ with } |n| < 0.1$$

Barrow[5] assumed that $G \, \alpha \, t^{-n}$ and obtained from helium abundances for $-5.9 \text{ x } 10^{-3} < n < 7 \text{ x } 10^{-3}$,

$$\left|\frac{\dot{G}}{G}\right| < (2 \pm 9.3) h \, x \, 10^{-12} \, yr^{-1}$$

by assuming a flat universe. Subsequently, alternative theories of gravity were developed to generalize Einstein's general theory of relativity by including variable G and satisfying conservation equation (Brans and Dicke [7]).

The big-bang model based on Einstein's field equations successfully explains the three important observations in Astronomy: (i) the phenomena of expanding universe, (ii) primordial nucleosynthesis, (iii) the observed isotropy of the cosmic back ground radiation. However, the big-bang model is known to have the short coming in the following aspects: (i) the model has singularity in the past and possible one in future, (ii) the conservation of energy is violated, (iii) it leads to a very small particle horizon, (iv) no consistent scenario exists that explains the origin, evolution and characteristic of structures in the universe at small scale, (v) flatness problem.

Therefore, it is natural to replace the simple 'big-bang' model and consideration of relevant improvement which may take care of the difficulties mentioned above. The attempts to improve upon the big-bang model may be classified into two models (i) quantum cosmological models (ii) inflationary models.

It is unfortunate fact that we do not have a complete quantum theory of gravity. The inflationary models (Linde[14]) are also not without serious drawbacks of their own. Therefore, it is interesting to consider the fact that if a model successfully explains creation of positive energy then it is necessary to have some degree of freedom which acts as a negative energy mode. All quantum gravitational models which describe the creation consistently (Atkatz and Pagels [1], Padmanabhan [17]) use such a energy mode arising from the scale degree of freedom of gravity. Thus introducing a negative-energy field may provide a natural way for creating the matter. Hoyle and Narlikar [13] adopted a field theoretic approach introducing a massless and chargeless scalar field in the Einstein-Hilbert action to account for creation of matter. In C-field (creation field), there is no big-bang type singularity as in the steady state theory of Bondi and Gold [6]. Narlikar [15] has pointed out that the proper consideration of matter creation can resolve the problem of singularity. Narlikar and Padmanabhan [16] have discussed the importance of creation field cosmology and have explained that it is the possible solution to singularity, horizon and flatness problems. Vishwakarma and Narilikar [20] emphasized that creation of matter plays a very crucial role in cosmology and provides a natural explanation to the various explosive phenomena occurring in local and extra galactic universe. Bali [3] investigated barotropic field model in creation field cosmology using FRW space-time.

The viscosity of a fluid is a measure of its resistance due to gradual deformation by shear stress. A realistic treatment of the problem requires the consideration of material distribution other than perfect fluid. The dissipative processes play a significant role for the high degree of isotropy we observe today. The effect of bulk viscosity on the cosmological evolution has been investigated by Bali [2], Gron [12], Chimento et al.[8], Ren and Meng [18], Verma and Ram [19], Bali and Singh [4].

In this paper, we have investigated C-field cosmological model with variable bulk viscosity and gravitational constant in the frame work of flat FRW space-time. To get the deterministic solution, we have G = R^n where R is the scale factor and n is a constant. The physical aspects of the model have been discussed and results match with the astronomical observations.

2. Metric and Field Equations

We consider the flat FRW space-time as

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, f\phi^2] \qquad \dots(1)$$

Einstein's modified field equations by the introduction of C-field are given by

$$R_i^j - \frac{1}{2} R \, g_i^j = -8\pi G \, [{}^m T_i^j + {}^C T_i^j] \qquad \dots(2)$$

where

$${}^m T_j^i = (\rho + p) v_i v^j - p \, g_i^j - \xi\theta \, (v_i v^j - g_i^j) \qquad \dots(3)$$

$${}^C T_j^i = -f \left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha \right) \qquad \dots(4)$$

where $f > 0$ and $C_i = \frac{dC}{dx^i}$. We assume that the coefficient of bulk viscosity (ξ) is inversely proportional to expansion (θ) i.e. ξθ = β (constant). Now the field equations (2) for metric (1) lead to

$$\frac{3\dot{R}^2}{R^2} = 8\pi G \left[\rho - \frac{1}{2} f \, \dot{C}^2 \right] \qquad \dots(5)$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} = 8\pi G \left[\frac{1}{2} f \dot{C}^2 + \beta - p \right] \qquad \dots(6)$$

3. Solution of Field Equations

The conservation equation

$$[8\pi G \, T_i^j]_{;j} = 0 \qquad \dots(7)$$

leads to

$$8\pi\dot{G}\left(\rho-\frac{1}{2}f\,\dot{C}^2\right)+8\pi G\left[\dot{\rho}-f\dot{C}\ddot{C}+3\rho\frac{\dot{R}}{R}-3f\dot{C}^2\frac{\dot{R}}{R}-3\beta\frac{\dot{R}}{R}+3p\frac{\dot{R}}{R}\right]=0$$ …(8)

which yields $\dot{C}=1$ when used in source equation.

Using $\dot{C}=1$ in equation (5), we get

$$8\pi\rho G=\frac{3\dot{R}^2}{R^2}+4\pi fG$$ …(9)

Using barotropic fluid condition $p=\gamma\rho\,\&\,\dot{C}=1$ in equation (6), we have

$$\frac{2\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}=4\pi Gf+8\pi G\beta-8\pi G\gamma\rho$$ …(10)

From equations (9) and (10), we have

$$\frac{2\ddot{R}}{R}+(1-3\gamma)\frac{\dot{R}^2}{R^2}=4\pi G\,(1-\gamma)f+2\beta]$$ …(11)

To get the deterministic solution in terms of cosmic time t, we assume

$$G=R^n$$ …(12)

where n is a constant and R the scale factor.
Equations (11) and (12) lead to

$$2\ddot{R}+(1+3\gamma)\frac{\dot{R}^2}{R}=4\pi[(1-\gamma)f+2\beta]R^{n+1}$$ …(13)

To get the solution of equation (13), we assume $\dot{R}=F(R)$. This leads to $\ddot{R}=FF'$ with $F'=\frac{dF}{dR}$. Thus equation (13) leads to

$$\frac{dF^2}{dR}+\frac{(1+3\gamma)}{R}F^2=4\pi[(1-\gamma)F+2\beta]R^{n+1}$$ …(14)

which leads to

$$F^2 = \frac{A\,R^{n+2}}{n+3\gamma+3} + \frac{L}{R^{3\gamma+1}} \qquad \ldots(15)$$

where $A = 4\pi[(1-\gamma)f + 2\beta)]$ and L is constant of integration.

From equation (15), we have

$$\frac{R^{\frac{3\gamma+1}{2}}\,dR}{\sqrt{R^{n+3\gamma+3} + \frac{L(n+3\gamma+3)}{A}}} = \sqrt{\frac{A}{n+3\gamma+3}}\,dt \qquad \ldots(16)$$

To obtain the deterministic value of R in terms of cosmic time t, we assume that

$$n = -\left(\frac{3\gamma+3}{2}\right) \qquad \ldots(17)$$

Using condition (17) in (16), we have

$$\frac{R^{\frac{3\gamma+1}{2}}\,dR}{\sqrt{R^{\frac{3\gamma+3}{2}} + \frac{L(3\gamma+3)}{2A}}} = \sqrt{\frac{2A}{3\gamma+3}}\,dt \qquad \ldots(18)$$

Equation (18) leads to

$$R^{\frac{3\gamma+3}{2}} = [(at+b)^2 - k] \qquad \ldots(19)$$

where

$$a = \frac{1}{2}\sqrt{\frac{A(3\gamma+3)}{2}} \qquad \ldots(20)$$

$$b = \frac{N(3\gamma+3)}{4} \qquad \ldots(21)$$

$$k = \frac{L(3\gamma+3)}{2A} \qquad \ldots(22)$$

where N is a constant of integration. Thus, we have

$$G = R^{n} = R^{-\left(\frac{3\gamma+3}{2}\right)} = [(at+b)^2 - k]^{-1} \qquad \ldots(23)$$

From equations (9), (19) and (23), we have

$$8\pi\rho = \frac{48a^2(at+b)^2}{(3\gamma+3)^2[(at+b)^2-k]^2} + 4\pi f \qquad \ldots(24)$$

After using the value of R given by (19), the metric (1) led to

$$ds^2 = dt^2 - [(at+b)^{4/3\gamma+3} - k]^2[dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2] \qquad \ldots(25)$$

Now equation (8) leads to

$$8\pi\rho\dot{G} + 8\pi\dot{\rho}F - 4\pi\dot{G}f\dot{C}^2 - 8\pi fG\dot{C}\ddot{C} + 24\pi G\rho\frac{\dot{R}}{R} - 24\pi Gf\dot{C}^2\frac{\dot{R}}{R}$$

$$-24\pi\beta G\frac{\dot{R}}{R} + 24\pi Gp\frac{\dot{R}}{R} = 0 \qquad \ldots(26)$$

Using equations (19), (23) and (24) into equation (26), we get

$$\dot{C}^2 t^{\frac{6(3-\gamma)}{(3\gamma+3)}} = \frac{1}{4\pi f}\int\left[\frac{96 + 8\pi f(3\gamma+3)^2 - 96\pi\beta(3\gamma+3)}{(3\gamma+3)^2}\right]t^{\frac{15-9\gamma}{3\gamma+3}} \qquad \ldots(27)$$

where we have set the constants a = 1, b = 0, k = 0 to get deterministic solution. Now (27) leads to

$$\dot{C}^2 = \frac{1}{4\pi f}\left[\frac{96 + 8\pi f(3\gamma+3)^2 - 96\pi\beta(3\gamma+3)}{(3\gamma+3)6(3-\gamma)}\right] \qquad \ldots(28)$$

using $a = 1, a = \frac{1}{2}\sqrt{\frac{A(3\gamma+3)}{2}}$ and $A = 4\pi[(1-\gamma)f + 2\beta]$ in equation (28), we have

$$\dot{C}^2 = 1 \qquad \ldots(29)$$

which leads to

$$C = t \qquad \ldots(30)$$

The metric (1) for a = 1, b = 0, k = 0 leads to

$$ds^2 = dt^2 - t^2[dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2] \qquad \ldots(31)$$

4. Physical Aspects

The homogeneous mass density ρ, gravitational constant G, scale factor $\bar{a}(t)$ and deceleration parameter (q) for the model (25) are given by

$$8\pi\rho = \frac{48a^2(at+b)^2}{(3\gamma+3)^2[(at+b)^2-k]^2} + 4\pi f \quad \ldots(32)$$

$$G = [(at+b)^2-k]^{-1} \quad \ldots(33)$$

$$\bar{a}(t) = [(at+b)^2-k] \quad \ldots(34)$$

$$q = -\left[\frac{4a^2(at+b)^2(1-3\gamma)-4a^2k(3\gamma+3)}{16a^2(at+b)^2}\right] \quad \ldots(35)$$

Setting a = 1, b = 0, k = 0, we have

$$8\pi\rho = \frac{48}{(3\gamma+3)^2} + 4\pi f \quad \ldots(36)$$

$$G = \frac{1}{t^2} \quad \ldots(37)$$

$$\bar{a}(t) = t^2 \quad \ldots(38)$$

$$q = -\left[\frac{1-3\gamma}{4}\right] \quad \ldots(39)$$

5. Conclusion

The matter density is initially large but decreases with time. The deceleration parameter q < 0 indicating accelerating universe. The spatial volume increases with time indicating inflationary scenario. The creation field (C) increases with time. The gravitational constant decreases with time. Setting a = 1, b = 0, k = 0, we find that matter density (ρ) is constant, creation field increases with time and gravitation constant decreases with time. The spatial volume increases with time. The deceleration parameter q < 0 for dust filled universe (γ = 0) indicating accelerating universe while for γ = 1, the model decelerates as q > 0. There is no singularity in the model (31).

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