

## **INFLUENCES OF Soret AND DUFOUR EFFECTS ON DOUBLE-DIFFUSIVE CONVECTION IN A HORIZONTAL FLUID LAYER**

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**Abstract:** The onset of double diffusive convection in a horizontal fluid layer heated and salted from below in the presence of Soret and Dufour effects examined analytically and graphically. Normal mode technique has been used for the linear stability analysis of the problem. The Eigen value problem is obtained for stability investigations with emphasis on the role played by both Soret and Dufour effect on the onset of convection. The expressions for stationary and oscillatory Rayleigh number are obtained as a function of the governing parameters. The analysis reveals that the onset of convection in double diffusive flow strongly depends upon the cross-diffusion effect. The effects of Soret and Dufour parameters along with other physical parameters viz., Solutal Rayleigh numbers and that of Lewis number on stationary and oscillatory convections are studied and shown graphically. It is found that the Soret parameter has both stabilizing effect and destabilizing effect on the onset of stationary modes according as  $\tau < D_f$  or  $\tau > D_f$  while the Dufour coefficient has stabilizing effect for  $\tau \neq D_f$ . The solute gradient has stabilizing effect on the onset of stationary convection, whereas the effect of Lewis number is found to have destabilizing effect on the system. In the limiting cases some previous published results have been recovered.

**Keywords:** Double diffusive convection, Soret effect, Dufour effect, stationary and oscillatory convection, cross diffusion.

### **1. Introduction**

The study of the onset of thermal convection in a layer of fluid (or Rayleigh–Bernard convection) has its origin in the experimental observations of Bénard [3] in(1900). Lord Rayleigh [19] in1916 developed the mathematical theory of this problem and showed that the onset of convection of a fluid layer heated from below depends upon the numerical

value of a non-dimensional parameter, known as Rayleigh number. The main objective of the studies related to thermal instability, in particular, is to determine the value of the Rayleigh number which characterizes the stability or instability of the system or to derive certain criteria for the onset of instability through convection. A detailed account and significant contributions to the understanding of the instability mechanisms inherent in the Rayleigh Bernard convection were made by Chandrasekhar [5].

Double-diffusive convection (also known as thermohaline convection or thermosolutal convection) is an important fluid dynamic phenomenon that involves motions driven by two different density gradients diffusing at different rates (Mojtabi and Charrier-Mojtabi[17]). The problem of Double-diffusive convection in a fluid layer heated from above has been studied first by Melvin Stern [23] when the solute is salt and the solute concentration is increasing upwards. Veronis [28] studied the problem when the fluid layer is heated and soluted from below. Nield [18] discussed the problem when the fluid layer is heated from below and soluted from above. Many situations exist in oceanography, astrophysics and chemical engineering, where there are two components of different molecular diffusivities for example heat and salt, as in the oceanographic situation, heat and helium, as in the astrophysical situation, or two different solutes, as in chemical engineering situations. The study of double-diffusive convection of a fluid layer in different contexts has received considerable attention during the last several decades because of its occurrence in a wide range of applications in various fields; such as high-quality crystal production, food processing, hydrology, petrology, geosciences, liquid gas storage, solidification of molten alloys etc.

For double diffusive convection, the buoyancy forces can arise not only from density differences due to variations in temperature, but also from those due to variations in solute concentrations. The effect of double diffusive convection arises from the fact that heat diffuses more rapidly than a dissolved substance creating temperature and concentration difference under gravity. This presence of comparable magnitude of temperature and concentration gradients may play a vital role in the onset of double diffusive convection. Furthermore, when two transport processes occur simultaneously in a moving fluid, they interfere with each other, producing cross-diffusion effects (Soret and Dufour effects). The flux of heat caused by concentration gradient is termed as Dufour (or diffusion-thermo) effect [8,9]. The analogous effect, the flux of mass can also be created by temperature gradients and this embodies the Soret (thermal-diffusion) effect [21,22]. The coupling of the fluxes of the stratifying agents is a prevalent feature in double diffusive convection systems. In general, the cross diffusion effects have a large influence on the hydrodynamic stability relative to their contributions to the buoyancy of the fluid.

The growing importance of double diffusive convection system in modern technology and industries has led various researchers to attempt the problems with or without cross diffusion effect both theoretically and experimentally. There are only few studies available on the effect of cross diffusion on double diffusion convection largely because of the complexity in determining these coefficients. Veronis [28] was the first to examine

Dufour driven double-diffusive convection. Eckert and Drake [10] studies the various cases of Dufour effect of considerable magnitude. Tewfik et al. [27] were the first to study Soret-Dufour driven thermosolutal convection, followed by Sparrow et al. [24]. Hurlle and Jakeman [12] discuss Soret-driven thermosolutal convection and concluded that magnitude & sign of the Soret coefficient were changed by varying the composition of the mixture. Schechter et al [20] studies the effect of Soret coefficient on thermosolutal convection problem and neglect the Dufour coefficient with the argument that its influence is negligible ( $10^{-3}^{\circ}\text{C}$ ) in liquid mixtures, however, there are liquid mixtures for which cross diffusions are of the same order of magnitude as the diffusivities. In gas mixtures, however, the Dufour effects are very important. Antorang and Velarde [1] have analyzed the Soret driven convective instability with rotation. Legros et al.[14] had studied the effect of Soret coefficient on two-component Bénard convection in the benzene-methanol system. Thermal convection in a binary fluid driven by the Soret and Dufour effects has been investigated by Knobloch [13]. He has shown that equations are identical to the thermosolutal problem except for a relation between the thermal and solute Rayleigh numbers. McDougall [16] had made an in depth study of double-diffusive convection where both Soret and Dufour effects are important and observed that the spatiotemporal properties of convection in binary mixture show quite different trends from those of the double-diffusive systems without these cross diffusion effects. Malashetty and Gaikwad [15] investigated the effect of both Soret and Dufour coefficients on the onset of double-diffusive convection and established the dependence of growth rate of instability, the slope of wave front and wave number on both Soret and Dufour parameters. Dhiman and Goyal [7] recently studied the stability of Soret driven double-diffusive convection problem for the case of rigid, impervious and thermally perfectly conducting boundary conditions using variational principle. For a broader view of the subject one may refer to Brandt and Fernando [2], Schechter et al [20] Legro et al [14], Turner [26] Malashetty & Gaikwad [15] and Takashima [25].

In most of the studies that related to double diffusive flow in the presence of cross diffusion effect, it has been noticed that either the influence of Dufour or both Soret and Dufour effect are neglected on the basis that they are of smaller order of magnitude in liquid mixtures (Mojtabi and Charrier-Mojtabi [17], Schechter et al [20]). The problem of onset of convective motion in a double diffusive flow in the presence of Soret and Dufour effects has received very scant attention in the literature. The cross diffusion effects, however small they may be, are present in double diffusive convections and are equally important and they have a large influence on hydrodynamic stability relative to their contributions to the buoyancy of the fluid, hence cannot be easily discarded. Keeping in mind the importance of cross diffusion effect, convection in double diffusive system, the present paper attempts to studied the effect of Soret and Dufour on Double diffusive convection in horizontal fluid layer by means of a linear stability analysis. The effects of Soret coefficient and that of Dufour along with the other parameters of the flow on the onset of double diffusive convection are investigated both analytically and numerically. In the limiting cases some previous published results have been recovered.

## 2. Physical Configuration and Basic Equations

Consider an infinite horizontal layer of two component viscous quasi-incompressible (Boussinesq) fluid heated and salted from below, confined between two horizontal boundaries  $z = 0$  and  $z = d$  which are respectively maintained at uniform temperature  $T_0$  and  $T_1$  ( $T_0 > T_1$ ) and at uniform concentrations  $C_0$  and  $C_1$  ( $C_0 > C_1$ ) in the field of gravity. Both the boundaries are assumed to be dynamically free, pervious and perfectly heat conducting while the adjoining medium is assumed to be electrically non-conducting.

The phenomenological equations relating the heat flux  $J_Q$  and the solute flux  $J_c$  to the thermal and solute gradients present in two component fluid mixture are given by (see for instance, De Groot and Mazur [6]) as;

$$J_Q = -\kappa \frac{\partial T}{\partial x_j} - \rho T C \frac{\partial \mu}{\partial c} D' \frac{\partial C}{\partial x_j} \quad \dots(1)$$

$$J_c = -\rho \kappa' \left[ \frac{\partial C}{\partial x_j} + S_T C (1-C) \frac{\partial T}{\partial x_j} \right] \quad \dots(2)$$

where,  $T$  is the temperature,  $C$  is the concentration,  $\rho$  is the density,  $\kappa$  is the thermal conductivity,  $D' (= S_T \kappa')$  is the *Dufour* coefficient and  $\mu$  is the chemical potential of the solute,  $\kappa'$  is the solutal diffusivity,  $\alpha$  and  $\alpha'$  are respectively thermal and concentration expansion coefficient.

Following Mcdougall, [16] and De Groot [6], the basic equations (*i.e.* the equations of continuity, motion, heat conduction, mass diffusion and the equation of state) under Boussinesq [4] approximation that govern the present physical configuration are given by (*cf.* Chandrasekhar [5]):

$$\frac{\partial u_j}{\partial x_j} = 0 \quad \dots(3)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial}{\partial x_i} \frac{p}{\rho_0} + \left[ 1 + \frac{\delta \rho_1}{\rho_0} + \frac{\delta \rho_2}{\rho_0} \right] X_i + \nu \nabla^2 u_i \quad \dots(4)$$

$$\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \kappa \nabla^2 T + D_{12} \nabla^2 C \quad \dots(5)$$

$$\frac{\partial C}{\partial t} + u_j \frac{\partial C}{\partial x_j} = \kappa' \nabla^2 C + D_{21} \nabla^2 T \quad \dots(6)$$

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \alpha'(C - C_0)] \quad \dots(7)$$

In the above equations  $D_{12}$  is the Dufour coefficient;  $D_{21}$  is the Soret coefficient;  $X_i = (0,0,-g)$  is the external force;  $g$  is gravity;  $u_i = (u, v, w)$  are the components of velocity;  $p$  is the pressure,  $\mu$  is the coefficient of viscosity and  $\nu = \frac{\mu}{\rho_0}$  is the coefficient of kinematic viscosity. Since the temperature and concentration are constant at the boundaries, therefore the boundary conditions are

$$\begin{aligned} w=0, \quad T=T_0, \quad C=C_0 \quad \text{at} \quad z=0 \\ w=0, \quad T=T_1, \quad C=C_1 \quad \text{at} \quad z=d \end{aligned}$$

### 2.1 Basic State and its Solutions

The initial stationary state of the fluid whose stability we want to examine is characterized by,

$$(u,v,w)=(0,0,0), \quad T=T(z), \quad C=C(z), \quad p=p(z), \quad \rho=\rho(z) \quad \dots(8)$$

Using equation (8), equations (3) to (7) yield the following stationary solution

$$(u,v,w)=(0,0,0),$$

$$T = T_0 - \beta z,$$

$$C = C_0 - \beta' z,$$

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \alpha'(C - C_0)] = \rho_0 [1 + \alpha\beta z - \alpha'\beta' z]$$

$$p = p_0 - \rho_0 g \left[ z + \frac{1}{2} (\alpha\beta - \alpha'\beta') z^2 \right]$$

where,  $\beta = \frac{T_0 - T_1}{d} > 0$  and  $\beta' = \frac{C_0 - C_1}{d} > 0$  are maintained uniform temperature and concentration gradients respectively and  $p_0$  the is the value of  $p$  at  $z = 0$ .

### 2.2 The Perturbation Equations

Let the initial state described by (8) be slightly perturbed so that perturbed state is given by

$$\left. \begin{aligned} (u, v, w) &= (0 + u', 0 + v', 0 + w'), \quad T = T_b + \theta' \\ C &= C_b + \phi', \quad \rho = \rho_0 [1 - \alpha(T_b - T_0) + \alpha'(C_b - C_0) - \alpha\theta' + \alpha'\phi'] \\ p &= p_b + \delta p' \end{aligned} \right\} \dots(9)$$

$$\delta p = -\rho_0 (\alpha\theta' - \alpha'\phi')$$

Where,  $(u', v', w')$   $\delta p', \theta', \phi'$  denote respectively, the perturbations in three components of velocity, pressure, temperature and concentration.. Then the perturbed state defined by (9) together with the governing equations (3) – (7) determine the linearized perturbation equations given by

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \quad \dots(10)$$

$$\rho_0 \frac{\partial u'}{\partial t} = -\frac{\partial}{\partial x} \delta p' + \mu \nabla^2 u' \quad \dots(11)$$

$$\rho_0 \frac{\partial v'}{\partial t} = -\frac{\partial}{\partial y} \delta p' + \mu \nabla^2 v' \quad \dots(12)$$

$$\rho_0 \frac{\partial w'}{\partial t} = -\frac{\partial}{\partial z} \delta p' + \mu \nabla^2 w' + g \rho_0 \alpha \theta' - g \rho_0 \alpha' \phi' \quad \dots(13)$$

$$\frac{\partial \theta'}{\partial t} - \beta w' = \kappa \nabla^2 \theta' + D_{12} \nabla^2 \phi' \quad \dots(14)$$

$$\frac{\partial \phi'}{\partial t} - \beta' w' = \kappa' \nabla^2 \phi' + D_{21} \nabla^2 \theta' \quad \dots(15)$$

### 3. Linear Stability Analysis

In this section, using linear stability analysis, we predict the thresholds of both stationary and oscillatory convections. First we analyze an arbitrary perturbation into two-dimensional periodic waves and assume that the all quantities describing the perturbation, a dependence on x, y, and t are of the form

$$[w', \theta', \phi'] = [W(z), \Theta(z), \Gamma(z)] \exp [i(k_x x + k_y y) + nt] \quad \dots(16)$$

Where  $k = \sqrt{k_x^2 + k_y^2}$  is the resultant wave member of the perturbation,  $k_x$  and  $k_y$  being real constants and n is a constant which can be complex in general. Substituting equation (16) in the linearized perturbation equations (10) – (16) and then using the following non-dimensional quantities

$$z_* = \frac{z}{d}; \quad \tau_* = \frac{\kappa'}{\kappa}; \quad a_* = kd; \quad \sigma_* = \frac{\nu}{\kappa}; \quad \frac{D_*}{d} = \frac{d}{dz}; \quad p_* = \frac{nd^2}{\kappa};$$

$$w_* = \frac{d}{\kappa} W; \quad \theta_* = \frac{Ra^2 \Theta}{\beta d}; \quad \phi_* = \frac{R'a^2 \Gamma}{\beta' d}; \quad R^* = \frac{g \alpha \beta d^4}{\nu \kappa}; \quad R'^* = \frac{g \alpha' \beta' d^4}{\nu \kappa'} \quad \dots(17)$$

in the resulting equations and omitting the asterisks for simplicity, we obtain the following system of non-dimensional linearized perturbation equations;

$$(D^2 - a^2) \left( D^2 - a^2 - \frac{p}{\sigma} \right) w - Ra^2\theta + R'\tau a^2\phi = 0 \quad \dots(18)$$

$$w + (D^2 - a^2 - p)\theta + D_f(D^2 - a^2)\phi = 0 \quad \dots(19)$$

$$w + S_T(D^2 - a^2)\theta + (\tau(D^2 - a^2) - p)\phi = 0 \quad \dots(20)$$

together with the boundary conditions,

$$w = 0 = \theta = \phi = D^2w \quad \text{at } z = 0, \text{ and } z = 1 \quad \dots(21)$$

(Both boundaries are dynamically free)

In the above equations;  $D \equiv d/dz$  represents the derivative with respect to the vertical co-ordinate  $z$  ( $0 \leq z \leq 1$ ); and  $R = \frac{g\alpha\beta d^4}{\kappa\nu}$  is the thermal Rayleigh number;  $R' = \frac{g\alpha'\beta'd^4}{\kappa\nu}$  is the solutal Rayleigh number.  $D_f = \frac{D_{12}\beta'}{\kappa\beta}$  is the Dufour parameter,  $S_T = \frac{D_{21}\beta}{\kappa\beta'}$  is the Soret parameter.

The system of equations (18)-(20) together with boundary conditions (21) constitutes an eigenvalue problem for  $R$  for the prescribed values of other parameters namely;  $p, R', a^2, \sigma, \tau, S_T$  and  $D_f$ . We assume the solutions to  $w, \theta, \phi$  that satisfy the boundary condition (21) are in the form

$$(w, \theta, \phi) = (W_0, \Theta_0, \Gamma_0) \sin \pi z \quad \text{at } z = 0, \text{ and } z = 1 \quad \dots(22)$$

Substituting equation (22) into equations (18) – (20), integrating each equation within the range of  $z$  by parts, we get the following matrix equation

$$\begin{bmatrix} Q \left( Q + \frac{p}{\sigma} \right) & -a^2 R & a^2 \tau R' \\ -1 & Q + p & D_f Q \\ -1 & S_T Q & Q\tau + p \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{where } Q = \pi^2 + a^2 \quad \dots(23)$$

This matrix equation has a non zero solution implies that determinant of the coefficient matrix is equal to zero which requires that

$$R = \frac{Q(Q+p)\left(Q + \frac{p}{\sigma}\right)(\tau Q + p) - D_f S_T Q^3 \left(Q + \frac{p}{\sigma}\right)}{a^2((\tau - D_f)Q + p)} + \frac{\tau R' \{(1 - S_T)Q + p\}}{((\tau - D_f)Q + p)} \quad \dots(24)$$

$$Q(Q+p)\left(Q + \frac{p}{\sigma}\right)(\tau Q + p) - D_f S_T Q^3 \left(Q + \frac{p}{\sigma}\right) - Ra^2((\tau - D_f)Q + p) + \tau R' a^2 \{(1 - S_T)Q + p\} = 0 \quad \dots(25)$$

Using the dispersion relation (25) we now discuss stationary and oscillatory modes of instability. The growth rate  $p = p_r + ip_i$  is, in general, a complex quantity such that the given state of the system is *stable*, *neutral* or *unstable* according as,  $p_r < 0$ ,  $p_r = 0$  or  $p_r > 0$ . We are interested in the marginal stability analysis and in that case  $p_r = 0$  and it is apparent that the marginal state ( $p_r = 0$ ) occurs with two cases  $p_i = 0$  and  $p_i \neq 0$ . When  $p_i = 0$  then the marginal state is characterized by the stationary (steady) convection and when  $p_i \neq 0$  then the instabilities are characterized by a marginally oscillatory mode and the instability sets in as oscillatory convection of growing amplitude, known as 'overstability'.

### 3.1 Stationary convection

For stationary convection putting  $p = 0$  in the equation (25), we get

$$R^{st} = \frac{1}{\tau - D_f} \left[ \frac{(\pi^2 + a^2)^3}{a^2} (\tau - D_f S_T) + \tau(1 - S_T)R' \right] \quad \dots(26)$$

It is clear from equation (27) that the stationary Rayleigh number  $R$  is a function of dimensionless wave number  $a$ , Lewis number  $\tau$ , Soret parameter  $S_T$ , Dufour parameter  $D_f$  and solutal Rayleigh number  $R'$  but independent of Prandtl number  $\sigma$

The critical stationary Rayleigh number  $R_c^{st}$  occurs when

$$\frac{\partial R^{st}}{\partial a^2} = 0 \Rightarrow a^2 = \frac{\pi^2}{2} \quad \text{or} \quad a_c = \frac{\pi}{\sqrt{2}}$$

and hence the critical Rayleigh number for the stationary convection is given by

$$R_c^{st} = \frac{1}{\tau - D_f} \left[ \frac{27\pi^4}{4} (\tau - D_f S_T) + \tau(1 - S_T)R' \right] \quad \dots(27)$$

(i) when  $D_f = 0$  (In the absence of Soret effect) then the equation (27) reduces to

$$R_c^{st} = \left[ \frac{27\pi^4}{4} + (1 - S_T)R' \right], \text{ which occurs at critical wave number } a = a_c = \frac{\pi}{\sqrt{2}} \quad \dots(28)$$

(ii) when  $S_T = 0$  (In the absence of Soret effect) then the equation (27) gives

$$R_c^{st} = \frac{\tau}{\tau - D_f} \left[ \frac{27\pi^4}{4} + R' \right] \text{ which occurs at critical wave number } a = a_c = \frac{\pi}{\sqrt{2}} \quad \dots(29)$$

(iii) when  $D_f = S_T = 0$  (in the absence of Soret and Dufour effect) then the equation (27) reduces to

$$R_c^{st} = \frac{27\pi^4}{4} + R' \quad \dots(30)$$

is the critical value of stationary Rayleigh number in the absence of Soret and Dufour effect, which occurs at  $a_c = \frac{\pi}{\sqrt{2}}$

This result is same as reported by Gupta et-al [8]

(iv) when  $D_f = S_T = R' = 0$ , (For single component fluid) the stationary Rayleigh numbers is given by equation (27) reduces to

$$R_c^{st} = \frac{27\pi^4}{4} \quad \text{at} \quad a_c = \frac{\pi}{\sqrt{2}} \quad \dots(31)$$

which coincides with the results obtained by Nield [18] and Chandrasekher [5]

### 3.2 Oscillatory Convection

For oscillatory convection at the marginal state ( $p_r = 0$ ), putting  $p = ip_i$  (where  $p_i \neq 0$ ) real and is the frequency of oscillation) in equation (25) and clear the complex quantities from the denominator, we get

$$R^{osc} = A + ip_i B \quad \text{where}$$

$$A = \frac{1}{a^2 \sigma (\tau - D_f)^2 Q^2 + a^2 \sigma p_i^2}$$

$$\left[ \left\{ (\tau - D_f)(1 - S_T)Q^2 + p_i^2 \right\} \tau a^2 \sigma R' + \begin{Bmatrix} \sigma(\tau - D_f)(\tau - D_f S_T)Q^4 \\ -(\tau - D_f)(1 + \sigma + \tau)Q^2 p_i^2 \\ -p_i^4 + (\tau + \sigma + \tau\sigma - D_f S_T)Q^2 p_i^2 \end{Bmatrix} Q \right] \quad \dots(32)$$

$$B = \frac{Q}{a^2 \sigma (\tau - D_f)^2 Q^2 + a^2 \sigma p_i^2} \left[ \begin{Bmatrix} (1 + \sigma + D_f)Q p_i^2 \\ -\tau a^2 \sigma \{ (1 - S_T) - (\tau - D_f) \} R' + Q^3 \begin{Bmatrix} (\tau - D_f)(\tau + \tau\sigma - D_f S_T) \\ -\sigma(1 - S_T)D_f \end{Bmatrix} \end{Bmatrix} \right] \quad \dots(33)$$

Since the Rayleigh number  $R$  is a physical quantity so it must be real hence the imaginary part of  $R$  must be zero, it follows from equation (33) that  $B=0$  which yield the square of frequency of oscillation given by

$$\begin{aligned} & \left\{ (1 + \sigma + D_f) \right\} Q p_i^2 \\ & - \tau a^2 \sigma \left\{ (1 - S_T) - (\tau - D_f) \right\} R' + Q^3 \begin{Bmatrix} (\tau - D_f)(\tau + \tau\sigma - D_f S_T) \\ -\sigma(1 - S_T)D_f \end{Bmatrix} = 0 \end{aligned} \quad \dots(34)$$

Therefore,

$$p_i^2 = \frac{a^2 \tau \sigma \left\{ (1 - S_T) - (\tau - D_f) \right\} R' - \left\{ (\tau - D_f)(\tau + \tau\sigma - D_f S_T) - \sigma(1 - S_T)D_f \right\} Q^3}{(1 + \sigma + D_f)Q} \quad \dots(35)$$

Now the equation (28) on putting  $B=0$  reduces to

$$R^{osc} = \frac{1}{a^2 \sigma (\tau - D_f)^2 Q^2 + a^2 \sigma p_i^2} \left[ \left\{ (\tau - D_f)(1 - S_T)Q^2 + p_i^2 \right\} \tau a^2 \sigma R' + \begin{Bmatrix} \sigma(\tau - D_f)(\tau - D_f S_T)Q^4 \\ -(\tau - D_f)(1 + \sigma + \tau)Q^2 p_i^2 \\ -p_i^4 + (\tau + \sigma + \tau\sigma - D_f S_T)Q^2 p_i^2 \end{Bmatrix} Q \right] \quad \dots(36)$$

Since for oscillatory convection  $p_i \neq 0, p_i^2 > 0$

Therefore, the condition for oscillatory convection derived from equation (34) is given by

$$a^2\tau\sigma\{(1-S_T)-(\tau-D_f)\}R' > \{(\tau-D_f)(\tau+\tau\sigma-D_fS_T)-\sigma(1-S_T)D_f\}Q^3 \quad \dots(37)$$

Thus the oscillatory mode of instability is possible only if the condition (36) is satisfied for some fixed values of parameters involved otherwise such type of modes do not exist. In the absence of Soret and Dufour parameters  $p_i^2$  is negative if  $\tau \geq 1$ . Therefore, in the absence of cross diffusion effect the oscillatory modes do not exist if  $\tau \geq 1$ . The analytical expression for oscillatory Rayleigh number given by equation (35) is minimized with respect to the wave number numerically, after substituting for  $p_i^2$  from equation (34), for various values of physical parameters in order to know their effects on the onset of oscillatory convection. Further, the equation (27) and the condition (36) imply that the onset of stationary or oscillatory modes of instability strongly depends upon Dufour and Soret parameters (cross diffusion effect).

#### 4. Results and Discussion

The onset of double diffusive convection in a horizontal fluid layer heated and salted from below in the presence of Soret and Dufour effect is examined analytically and graphically. The expressions for stationary and oscillatory Rayleigh number are obtained as function of governing parameters. We choose the values of these parameters as suggested in literature in the range of  $10^2 \leq R \leq 10^5$ ,  $10^2 \leq R' \leq 10^3$  and  $0 \leq D_f \leq S_T \leq 1$  and  $\tau = 10^{-2}$ . (where  $\tau \neq D_f$ ). First, we shall first investigate analytically the effects of Dufour parameter  $D_f$  and Soret parameter  $S_T$ , solute gradient  $R'$  (in absence of Dufour and Soret effect) and that of Lewis number  $\tau$  on the onset of stationary convection in the double diffusive system.

(a) In the presence of both Soret and Dufour effects ( $D_f \neq 0, S_T \neq 0$ ). From equation (26), we have

$$R^{st} = \frac{1}{\tau - D_f} \left[ \frac{(\pi^2 + a^2)^3}{a^2} (\tau - D_f S_T) + \tau(1 - S_T)R' \right]$$

$$(i) \quad \frac{\partial R^{st}}{\partial D_f} = \frac{\tau(1 - S_T)}{(\tau - D_f)^2} \left[ \frac{(\pi^2 + a^2)^3}{a^2} + R' \right] > 0$$

which implies that for all wave number  $a^2$  ( $a^2 \neq 0$ ) and  $\tau \neq D_f$ , the value of stationary Rayleigh number increases with increasing values of Dufour parameter for a given value of Soret parameter ( $0 \leq S_T \leq 1$ ) and fixed Lewis and solutal Rayleigh number. Thus for the stationary convection Dufour parameter has stabilizing effect on the double diffusive system. Further, we can have from equation (26) that

$$(ii) \quad \frac{\partial R^{st}}{\partial S_T} = \frac{-1}{(\tau - D_f)} \left[ D_f \frac{(\pi^2 + a^2)^3}{a^2} + \tau R' \right]$$

$$\frac{\partial R^{st}}{\partial S_T} > 0 \text{ if } \tau - D_f < 0 \text{ or if } \tau < D_f$$

$$\frac{\partial R^{st}}{\partial S_T} < 0 \text{ if } \tau - D_f > 0 \text{ or if } \tau > D_f$$

This yields that for all wave number  $a^2 (a^2 \neq 0)$  and for given positive values of  $\tau, D_f$  and  $R'$ , the value of the stationary Rayleigh number increases for increasing values of  $S_T$  if  $\tau < D_f$  and decreases with increasing values of  $S_T$  if  $\tau > D_f$ . Thus the Soret parameter has both stabilizing effect and destabilizing effect on the onset of stationary modes according as  $\tau < D_f$  or  $\tau > D_f$

$$(iii) \quad \frac{\partial R^{st}}{\partial R'} = \frac{\tau(1 - S_T)}{(\tau - D_f)}$$

$$\frac{\partial R^{st}}{\partial R'} > 0 \text{ if } \tau > D_f \qquad \frac{\partial R^{st}}{\partial R'} < 0 \text{ if } \tau < D_f$$

which means that for all wave number  $a^2 (a^2 \neq 0)$  and for given positive values of Soret coefficient ( $0 \leq S_T \leq 1$ ), Dufour coefficient  $D_f$  and fixed Lewis number  $\tau$ , the value of stationary Rayleigh number increases with increasing values of solutal Rayleigh number if  $\tau > D_f$  and decreases with increasing values of solutal Rayleigh number if  $\tau < D_f$ . Thus for the stationary convection solutal Rayleigh number has dual stabilizing and destabilizing effect according as  $\tau > D_f$  or  $\tau < D_f$  on the double diffusive system.

**(b)** In the presence of Dufour effect only ( $D_f \neq 0, S_T = 0$ ). From equation (26), we have

$$R^{st} = \frac{\tau}{\tau - D_f} \left[ \frac{(\pi^2 + a^2)^3}{a^2} + R' \right]$$

$$\frac{\partial R^{st}}{\partial D_f} = \frac{\tau}{(\tau - D_f)^2} \left[ \frac{(\pi^2 + a^2)^3}{a^2} + R' \right] > 0$$

which yield that for all wave numbers  $a^2 (a^2 \neq 0)$  and in the absence of Soret effect, the value of stationary Rayleigh number increases with increasing values of Dufour number for fixed values of Lewis and solutal Rayleigh number. Thus the system stabilizes with increasing values of Dufour parameter in the absence of Soret effect.

(c) In the presence of Soret effect only ( $D_f = 0, S_T \neq 0$ ). From equation (26), we have

$$R^{st} = \frac{(\pi^2 + a^2)^3}{a^2} + (1 - S_T)R' \quad \frac{\partial R^{st}}{\partial S_T} = -R' < 0$$

which implies that for all wave numbers  $a^2 (a^2 \neq 0)$  and in the absence of Dufour effect, the value of stationary Rayleigh number decreases with increasing values of Soret number for fixed values of Lewis and solutal Rayleigh number. Thus the system destabilizes with increasing values of Soret parameter in the absence of Dufour effect.

(d) In the absence of both Soret and Dufour effects ( $D_f = 0, S_T = 0$ ). From equation (26), we have

$$(i) \quad \frac{\partial R^{st}}{\partial R'} = 1$$

which is positive for all wave number  $a^2 (a^2 \neq 0)$  in the absence of both Soret and Dufour effects implies that the value of stationary Rayleigh number increases with increasing values of solutal Rayleigh number. Thus the solute gradient has stabilizing effect in the double diffusive system in the absence of Soret and Dufour effects.

$$(ii) \quad \frac{\partial R^{st}}{\partial \tau} = -\frac{(1 - S_T)D_f}{(\tau - D_f)^2} \left[ \frac{(\pi^2 + a^2)^3}{a^2} + R' \right]$$

which is negative for all wave number  $a^2 (a^2 \neq 0)$  and  $\tau \neq D_f$  and for given positive values of  $S_T$ ,  $D_f$  and  $R'$ . This implies that the value of the stationary Rayleigh number decreases with increasing values of  $\tau$ . Thus the Lewis number  $\tau$  has destabilizing effect on the onset of stationary convection in the double diffusive system.

Now we shall discuss the effect of various governing parameters in the double diffusive system for stationary convection graphically. The figures 1-6 displays the curves of Rayleigh number versus the square of wave numbers; representing the effects of Dufour parameter, Soret parameter, solute gradient and Lewis number on the stationary convection in the double diffusive system. We fixed the values for the parameters except the varying parameter.

- Variation of stationary Rayleigh number  $R$  with respect to wave number 'a' for fixed values of  $R' = 50$ ,  $\tau = \frac{1}{100}$  and  $S_T = 0.5$  for different values of Dufour number.

$D_f=0.002$		$D_f=0.0035$		$D_f=0.005$	
A	R	a	R	A	R
1	1475.75	1	1658.21	1	1976.00
2	781.25	2	879.82	2	1050.38
3	871.00	3	980.16	3	1169.67
4	1247.37	4	1403.44	4	1672.95
5	1939.33	5	2177.84	5	2593.71
6	3047.08	6	3420.40	6	4071.11
7	4715.24	7	5291.07	7	6295.32
8	7116.55	8	7983.89	8	9497.07
9	10452.37	9	11724.6	9	13944.8
10	14951.56	10	16770.0	10	19943.9

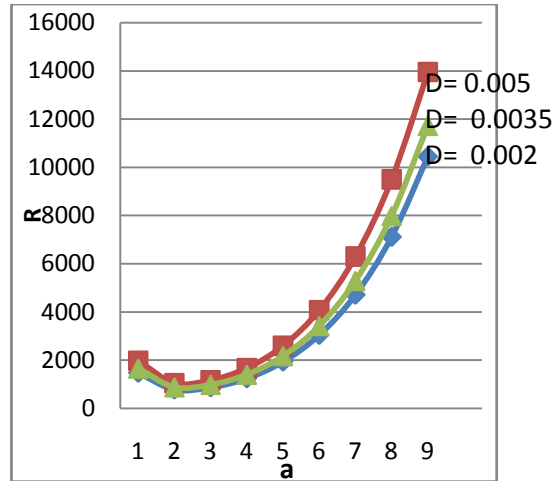


Table1: Values of  $R$  for different values of

Fig. 1: Plot of  $R$  verses wave number 'a'

- Variation of stationary Rayleigh number  $R$  with respect to wave number 'a' when  $\tau > D_f$  and for fixed values of  $R' = 50$ ,  $\tau = \frac{1}{100}$  and  $D_f = 0.002$  for different values of Soret number.

$S_T$	$S_T=0.3$	$S_T=0.6$	$S_T=0.9$
A	R		
1	1552.45	1437.4	1322.35
2	827.38	758.61	689.84
3	920.82	846.09	771.36
4	1315.06	1215.16	1115.26
5	2036.32	1890.39	1744.45
6	3193.61	2973.81	2754.00
7	4935.92	4604.90	4273.88
8	7443.95	6952.85	6461.75
9	10928.03	10214.54	9501.05
10	15627.18	14613.75	13600.31

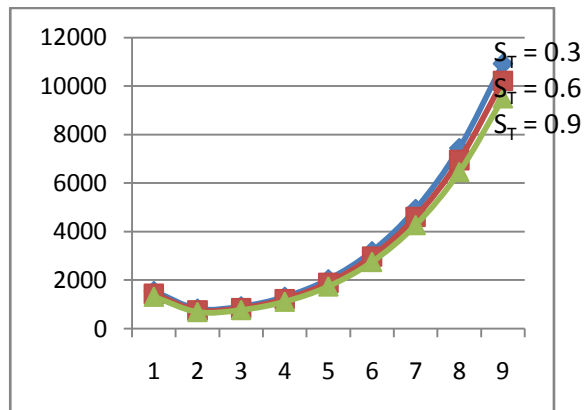


Table 2: Values of  $R$  for different values of  $a$

Fig.2: Plot of  $R$  verses wave number 'a'

3. Variation of stationary Rayleigh number  $R$  with respect to wave number 'a' for fixed values of  $R' = 50, D_f = 0.3$  and  $S_T = 0.5$  for different values of Lewis number.

$\tau = .01$		$\tau = .05$		$\tau = .09$	
A	R	a	R	A	R
1	1475.75	1	1336.57	1	1323.73
2	781.53	2	706.04	2	699.84
3	871	3	787.91	3	779.93
4	1248.46	4	1130.37	4	1119.47
5	1939.03	5	1756.89	5	1740.09
6	3047.08	6	2762.18	6	2735.89
7	4715.24	7	4275.64	7	4235.07
8	7116.55	8	6454.26	8	6393.14
9	10452.37	9	9480.72	9	9391.04
10	14951.56	10	13562.67	10	13434.47

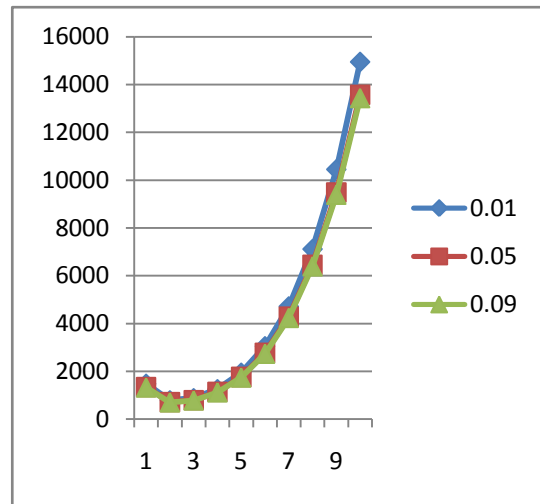


Table3: Values of R for different values of a

Fig. 3: Plot of R verses wave number 'a'

4. Variation of stationary Rayleigh number  $R$  with respect to wave number 'a' when  $\tau > D_f$  and for fixed values of  $\tau = \frac{1}{100}$  and  $S_T = 0.5, D_f = 0.002$  for different values of solutal Rayleigh number.

$R' = 100$		$R' = 200$		$R' = 300$	
A	R	a	R	a	R
1	1507	1	1569.5	1	1632
2	812.78	2	875.28	2	937.78
3	902.25	3	964.75	3	1027.25
4	1279.71	4	1342.21	4	1404.71
5	1970.28	5	2032.78	5	2095.28
6	3078.33	6	3140.83	6	3203.33
7	4746.49	7	4808.99	7	4871.49
8	7147.50	8	7210.30	8	7272.80
9	10483.62	9	10546.12	9	10608.62
10	14982.81	10	15045.31	10	15107.81

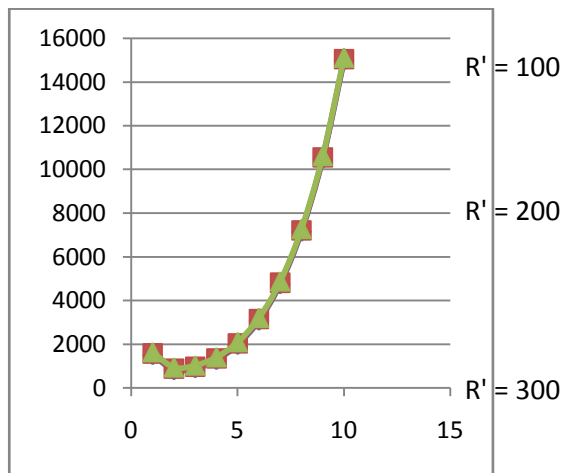


Table 4: Values of R for different values of a

Fig. 4:Plot of R verses wave number 'a'

## 5. Conclusion

In the present paper, the onset of double diffusive convection in a horizontal fluid layer heated and salted from below is examined analytically and graphically with the emphasis on the role played by the Soret and Dufour effect (cross diffusion effect) by means of linear stability analysis. The expressions for stationary and oscillatory Rayleigh number are obtained as a function of the governing parameters, which characterize the stability of the system. The analysis reveals that the onset of convection in double diffusive flow strongly depends upon the cross-diffusion effect. The effect of various parameters such as Soret parameter, Dufour parameter, solutal Rayleigh number and Lewis number on the onset of convection has been investigated analytically as well as numerically. Our investigation leads to the following conclusions.

### For the case of stationary convection:

- (i) It has been found that in the presence of both Soret and Dufour effects ( $D_f \neq 0, S_T \neq 0$ ) The Dufour parameter has stabilizing effect on the double diffusive system whereas the Soret parameter has both stabilizing and destabilizing effect on the onset of stationary modes according as  $\tau < D_f$  or  $\tau > D_f$ . Figures (1)-(2) support the analytical results graphically.
- (ii) The Lewis number  $\tau$  has destabilizing effect on the onset of stationary convection in the double diffusive system whereas the solutal Rayleigh number has dual stabilizing and destabilizing effect according as  $\tau > D_f$  or  $\tau < D_f$  Figures (3) depict these effects graphically.
- (iii) In the absence of Soret effect ( $D_f \neq 0, S_T = 0$ ), the Dufour parameter has a stabilizing influence on the onset of stationary convection system.
- (iv) In the absence of Dufour effect ( $D_f = 0, S_T \neq 0$ ), the Soret parameter has a destabilizing influence on the system.
- (v) In the absence of Soret and Dufour effect ( $D_f = 0, S_T = 0$ ), the solutal gradient has stabilizing influence on the onset of stationary convection in the double diffusive system which is displayed graphically in Figure 4.

### For the case of oscillatory convection:

- (i) It has been found that the oscillatory mode of instability exists only if the condition (32) is satisfied for some fixed values of parameters involved and violation of which imply that such type of modes do not exists.
- (ii) In the absence of cross diffusion effect the oscillatory modes do not exists if  $\tau \geq 1$ .

Finally, it is observed that the numerical results are in close agreement with the analytical results.

## References

- [1] Antoranz JC and Velarde MG (1978): Soret driven convective instability with rotation. *Physics Letters*, 65(a), pp5-6.
- [2] Brandt A and H. J. S. Fernando, *Double Diffusive Convection*, Am. Geophys. Union, Washington, DC, 1996.
- [3] Bénard H (1900): Les tourbillions cellulaires dans un liquide, *Revue generale des sciences pures et appliquées*, 11, pp1261.
- [4] Boussinesq JE (1903): *Theorie analytique de la chaleur*, 2, Paris: Gauthier Villars, pp172.
- [5] Chandrasekhar S (1961): *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, Amen House, London.
- [6] De Groot SR and Mazur P (1962): *Non-equilibrium Thermodynamics* (North-Holland, Amsterdam) pp273
- [7] Dhiman JS and Goyal MR (2013) : Stability Analysis of Soret driven Double diffusive Steady Convection Problems for Rigid and Impervious Boundaries, *International Journal of Applied Mathematics and Mechanics*. 2(1) pp 11-20.
- [8] Dufour, L., *Archives of Physical and Natural Sciences Geneve*-45, (1872).
- [9] Dufour, L., "On the diffusion of gases through porous walls and the accompanying changes in temperature", *Annals of Physics*, Vol. 224, (1873), 490.
- [10] Eckert, E.R.G. and Drake, R.M., "Analysis of Heat and Mass Transfer", McGraw Hill, New York (1972).
- [11] Gupta JR, Dhiman JS and Thakur J (2001): Thermohaline Convection of Veronis and Stern types: Revisted, *Journal of Mathematical Analysis and Applications*. 287, pp398.
- [12] Hurle DTJ and Jakeman E (1971): Soret-driven thermosolutal convection. *Journal of Fluid Mechanics* 47(4) pp667-687.
- [13] Knobloch, E., "Convection in binary fluids", *The Physics of Fluids*, Vol. 23, No. 9, (1980), 1918-1920.
- [14] Legro JC, Goemare P and Platten JK (1985) : *Physics Review Letter*, 32(3), pp 1903.
- [15] Malashetty MS and Gaikwad SN (2002) : Effect of cross diffusion on double diffusive convection in the presence of horizontal gradients. *International Journal of Engineering Science*. 40 pp 773–787.
- [16] McDougall TJ (1983): Double-diffusive convection caused by coupled molecular diffusion. *Journal of Fluid Mechanics*, 126 pp379.

- [17] Mojtabi A, Charrier-Mojtabi, M.C. : Double diffusive convection in porous media, in: K. Vafai (Ed.), *Hand Book of Porous Media*, Marcel Dekkes, NewYork, 2000, pp. 559–603.
- [18] Nield, D A., Bejan, *Convection in Porous Media*, Springer-Verlag, Berlin, 2006.
- [19] Rayleigh L (1916): On the convective current in a horizontal layer of fluid when the higher temperature is on the upper side, *Phil. Mag.* **32**pp 529.
- [20] Schechter RS, Prigogine I and Hamm JR (1972): Thermal Diffusion and Convective Stability. *Physics of Fluids*, 15pp 379.
- [21] Soret, C.H., *Archives of Physical and Natural Sciences Geneve-2*, (1879), 48-61.
- [22] Soret, C.H., *Archives of Physical and Natural Sciences Geneve-4*, (1880), 209-213.
- [23] Stern M.E (1960), “The salt fountain and thermohaline convection”, *Tellus*, 12, 172-175.
- [24] Sparrow, E.M., Minkowycz, W.J. and Eckert, E.R.G., “Transpiration induced buoyancy and thermal diffusion -diffusion thermo in helium air free convection boundary layer”, *ASME Journal of Heat Transfer*, Vol. 86, No.4, (1964), 508-520
- [25] Takashima M (1976): The Soret effect on the thermal instability of two component fluid layer. *Journal of Physical Society of Japan*, 41pp4.
- [26] Turner JS (1974): Double diffusive Phenomena, *Ann. Rev. Fluid Mech.* 6 pp37.
- [27] Tewfik, O.E. and Yang, J.W., “The thermodynamics coupling between heat and mass transfer in free convection”, *International Journal of Heat Mass Transfer*, Vol. 6, No. 10, (1963), 915-923.
- [28] Veronis, G. (1965) “On finite amplitude instability in thermohaline convection”, *J. Mar. Res.*, 23 (1965).