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RELIABILITY ANALYSIS OF MULTI-PROCESSOR SYSTEM WITH COMMON CAUSE FAILURE

ASHISH MISHRA
Department of Mathematics, I.B.S., Khandari Campus, Dr. B.R. Ambedkar University, Agra-282002 (India)
E-mail : ashishddive@gmail.com

MADHU JAIN
Department of Mathematics, I.I.T. Roorkee-247667 (India)
E-mail : madhufma@iitr.erner.in

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Abstract : The processor's ability to share the entire memory space provides a convenient means of sharing information and provides flexibility in memory allocation. The objective of our study is to maintain the pre-defined performance level under degradation due to different failures in a multiprocessor system. We consider the gracefully degradable homogeneous and heterogeneous multi-processors system with common cause failure. The transient analysis has been provided in order to facilitate the computational work to predict the performance of the system. The expressions for reliability, mission time (MT) and availability have been derived. The sensitivity analysis of degradable multi-processor system is performed so as to compare the efficiency and effectiveness of the system under different environments.

Keyword: Reliability analysis, Multi-Processor, Mission time, Common Cause Failure

2010 Mathematics Subject Classification : 90B25 and 60K10

1. Introduction

Reliability is the measure of the operational success of the system. In a multi-processors system to reduce contention, the memory is usually splitted up into modules, which can be accessed independently and concurrently with other modules. When more than one processor attempts to access the same module, only one processor can access it successfully, while other processors must await their turn in a queue. The effect of such contention, or interference, is to increase the average memory access time. Multi processors are assumed to be fast enough to generate a new request as soon as their current request is satisfied. A processor generates a new request while waiting for the current request to be met. The operation of the multi-processors system can be visualized as a discrete parameter queueing network.

As the number of processors in such systems increases, the computational power increases with aggregated rate less than the sum of the rate of processors because of the added parallelization overhead. At the same time, the rate of failure of the system also increases due to increase of complexity in architecture of the system. In this direction, the performance analysis was done by several researchers in different frameworks by considering these constraints in isolation. In this investigation, we study the effect of failures over performance of degradable homogeneous and heterogeneous multiprocessors system. As multi-processor systems become more complex, their reliability will need to increase as well. System-level fault tolerance involves reliability redundant techniques incorporated within the system hardware and software whereas application-level fault tolerance involves reliability techniques incorporated within the application software. We assert that, for high reliability, a combination of system-level fault tolerance and application-level fault tolerance works best. In many systems, application-level fault tolerance can be used to bridge the gap when system-level fault tolerance alone does not provide the required reliability.

Some works have appeared on the fault tolerant multi-processor systems to maintain higher reliability. Hughes and Doone [9] analysed the reliability of multiprocessor systems. Marsland and Sutphen [19] considered the heterogeneous dual processor. Marsland *et al.* [20] and Fortes and Raghvendra [7] investigated the gracefully degradable multiprocessor tree search experiments. Agrawal and Agrawal [1] and Lyer and Rosett [18] gave the measure based model for workload dependence of CPU errors on fault tolerance algorithm by considering a network of computers in large multi-processor system. The coverage modeling for dependability analysis of fault tolerant systems was done by Dugan and Trivedi [6]. Islam and Ammar [10] gave analysis of the distributed real time system of multiprocessor. A recursive algorithm for fault tolerance to compute performance indices for parallel multiprocessing system of work load on the reliability of real time processor triads was considered by Krishna [13]. Haghighi [8] investigated the number of tasks in a parallel multiprocessor system with task splitting and feedback. A computer network and distribution system was studied by Sherif and Matt [24].

The concept of coverage and its effects on the reliability model of a repairable system with common cause reduction rules for fault tolerant system via simulation using a stress strength failure model was considered by Bukoswski and Gobie [2]. Performance analysis of the simultaneous optical multiprocessor exchange bus was made by Kalsinis [12]. Shekhar [23] gave the multiprocessor system which is gracefully degradable due to different failures. The optimization survivability of multi state systems with multi level protection by multi-processor genetic algorithm was investigated by Levintin *et al.* [15]. Jain *et al.* [11] gave the reliability of redundant repairable system with degraded failure. Li *et al.* [17] suggested the reliability estimation and performance prediction of multi state components and coherent system for two parallel machines with an availability constraint. Xing *et al.* [27] and Levitin *et al.* [16] studied the reliability of fault-tolerant systems with parallel task processing and hierarchical computer based systems subject to common cause failures. Lazaro *et al.* [14] analysis the long term availability prediction for groups of volunteer resources. Distefano *et al.* [5] investigating the dynamic reliability and availability through state space models. The reliability of k-out-of-n systems with phased mission requirements and imperfect fault coverage was made by Xing *et al.* [26]. Dashti and Yousefi [4] studied the reliability based asset assessment in electrical distribution systems. Damghani *et al.* [3] considered a new multi objective particle swarm optimization method for solving reliability redundancy allocation problems. Schneider *et al.* [22] gave the social network analysis via multi state reliability and conditional influence models. Xing and Levitin [25] studied the BDD based reliability evaluation of phased mission systems with internal/external common cause failures. The application of fault tree for customer reliability assessment of a distribution power system was considered by Rahman et al. [21].

In this paper, we investigate the homogenous and heterogeneous multi processor system with state dependent and common cause failure. The purpose of our investigation is to develop a comprehensive approach to compute and analyze performance indices for a multi processor system. A discrete state and continuous time Markov chain is used to construct a set of differential difference equations for transient probabilities governing the model. The rest of the paper is organized as follows. Section 2 deals with model description by stating the requisite assumptions and notations. Section 3 contains governing equations and mathematical analysis. The performances indices are derived in section 4. In section 5, numerical results are given. Finally section 6 is devoted to conclusion and further research directions related to our study.

2. Model Description

Consider a degradable multiprocessor system with a main unit which works as supervisor in the system. We formulate the Markov model to analyze the effect of failures over performance of gracefully degradable homogenous and heterogeneous multiprocessor systems. To maintain the pre-defined performance level, the degradation due to different failure rates is studied. For formulating the mathematical model, the following assumptions are made:

- The system comprises of N adiabatically homogenous and heterogeneous multiprocessors, where degradation states are D (where D = N–1).
- The failure density of each unit in the system is load dependent. The inter-failure time of individual processor is exponentially distributed with parameter Λ_i (i = 0,1,…,D)
- The system fails completely (F) due to common cause with failure rate λ_c.

  λ_p is the failure rate of main unit, which supervises the system and causes the total system failure and is constant.
- The probability of coverage 'c' for successful recovery from an individual failure in any state i (i = n, n+1,…,D) is constant. After a threshold level (n) of degration, i.e when i e" n; the system may completely fail with state dependent failure rates μ_i (n d" i d" D) which includes the failure rate of main unit and common cause failure rate.
- $P_i(t)$ is the occupation probability of system being in state i where (i = 0,1,…,D) at any time t.
- The state dependent failure rate are given as follows:

$$\Lambda_i = \begin{cases} N\lambda_i\,, & 0 < i < n \\ c\,(N+n-i)\,\lambda_i, & n \le i \le D \end{cases}$$

and $\mu_i = \{\lambda_i(1-c)(N+n-i) + \lambda_p + \lambda_c\},\ n \le i \le D$

3. Governing Equations and Analysis

The set of differential difference equations governing the model based on transition diagram shown in fig. 1, is given by

$$P_0'(t) = -(N\lambda_0)P_0(t) \tag{1}$$

$$P_i'(t) = -(N\lambda_i)P_i(t) + (N\lambda_{i-1})P_{i-1}(t), \quad 1 \le i < n-1 \tag{2}$$

$$P_n'(t) = -(N\lambda_n + \lambda_p + \lambda_c)P_n(t) + (N\lambda_{n-1})P_{n-1}(t) \tag{3}$$

$$P_i'(t) = -[(N-i+n)\lambda_i + \lambda_p + \lambda_c]P_i(t) + [c(N-i+n+1)\lambda_{i-1})P_{i-1}(t), \quad n+1 \le i < D \tag{4}$$

$$P_F'(t) = \sum_{j=n}^{D}[\lambda_j(1-c)(N+n-j) + \lambda_c + \lambda_p] \tag{5}$$

Using Laplace transformation, the set of equations (1)-(4) with initial conditions, $P_0(0)=1$, $P_i(0) = 0$, $i \neq 0$ can be solved. The Laplace transforms of equations (1)-(4) yield

$$1 = (s + N\lambda_0)\widetilde{P}_0(t) \tag{6}$$

$$0 = (s + N\lambda_i)\widetilde{P}_i(t) - (N\lambda_{i-1})\widetilde{P}_{i-1}(t), \quad 1 \le i < n-1 \tag{7}$$

$$0 = (s + N\lambda_n + \lambda_p + \lambda_c)\widetilde{P}_n(t) - (N\lambda_{n-1})\widetilde{P}_{n-1}(t) \tag{8}$$

$$0 = (s + \{N-i+n\}\lambda_i + \lambda_p + \lambda_c)\widetilde{P}_i(t) - (c\{N-i+n+1\}\lambda_{i-1})\widetilde{P}_{i-1}(t), \quad n+1 \le i < D \quad \ldots(9)$$

The inverse Laplace transforms of equations (6)-(9) yield

$$P_i(t)=\begin{cases} e^{-(N\lambda_0)t} \\ N^i\prod_{i=1}^{n-1}\lambda_{i-1}\left\{\dfrac{e^{-(N\lambda_0)t}}{\prod_{i=1}^{n-1}(N\lambda_i-N\lambda_0)}+\sum_{i=1}^{n-1}\dfrac{e^{-(N\lambda_i)t}}{(N\lambda_0-N\lambda_i)\prod_{\substack{i=1\\k\neq i}}^{n-1}(N\lambda_k-N\lambda_i)}\right\}, & i=1,2,...,n-1 \\ N^i\prod_{j=1}^{n}\lambda_{j-1}\left\{\dfrac{e^{-(N\lambda_i+\lambda_c+\lambda_p)t}}{\prod_{j=0}^{n-1}(N\lambda_j-N\lambda_i+\lambda_c+\lambda_p)}+\sum_{j=0}^{n-1}\dfrac{e^{-(N\lambda_j)t}}{(N\lambda_i+\lambda_c+\lambda_p-N\lambda_j)\prod_{\substack{j=0\\k\neq j}}^{n-1}(N\lambda_j-N\lambda_k)}\right\}, & i=n \\ cN^{i-1}(N+n+1-i)\prod_{j=0}^{D}\lambda_j \\ \times\left\{e^{-(N\lambda_0)t}A_0+\sum_{j=1}^{n-1}e^{-(N\lambda_j)t}A_r+\sum_{j=n}^{D}e^{-\{(N-j+n)\lambda_j+\lambda_c+\lambda_p\}t}A_q\right\}, & n+1\leq i\leq D \end{cases} \tag{10}$$

where

$$A_0=\frac{1}{\prod_{j=1}^{n-1}(N\lambda_j-N\lambda_0)\prod_{j=n}^{D}\{(N-j+n)\lambda_j+\lambda_c+\lambda_p-N\lambda_0\}}$$

$$A_r=\frac{1}{(N\lambda_0-N\lambda_r)\prod_{\substack{j=1\\j\neq r}}^{n-1}(N\lambda_j-N\lambda_r)\prod_{j=n}^{D}\{(N-j+n)\lambda_j+\lambda_c+\lambda_p-N\lambda_j\}},r=1,2,...,n-1$$

$$A_q=\frac{1}{\{N\lambda_0-\{(N-q+n)\lambda_q+\lambda_c+\lambda_p\}\}\prod_{j=1}^{n-1}\{N\lambda_j-(N-q+n)\lambda_q+\lambda_c+\lambda_p\}}$$

$$\times\prod_{\substack{j=n\\j\neq q}}^{D}\{(N-j+n)\lambda_j-(N-q+n)\lambda_q\};\quad q=n,n+1,...,D$$

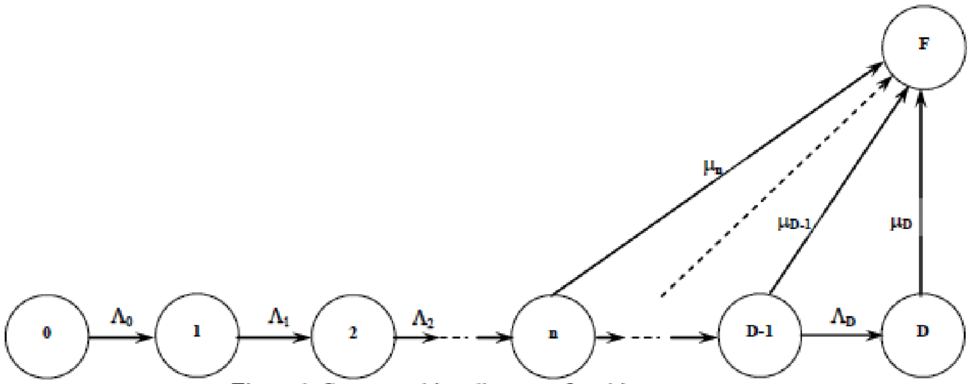

Figure 1: State transition diagram of multi-processor system

4. Performance Indices

Now we derive various reliability indices by using transient probabilities obtained in previous section as follows:

- Reliability of the system is given by

$$R(t) = \sum_{i=0}^{D} P_i(t) \tag{11}$$

The mission time (MT) is the time interval during which the R(t) is greater than pre-defined minimum reliability $R(t)_{min}$. Thus MT is determined by using following inequality

$$R(t) \geq R(t)_{min} \quad \forall\, t < MT \tag{12}$$

- Point availability of the system is obtained as

$$A_p(t) = P_0\,(t) \tag{13}$$

Interval availability of the system for the interval [0,T] is

$$\overline{A_V}\,(T) = \frac{1}{T}\int_0^T A_p(t)dt = \frac{1}{N\lambda_0 T}\left[1 - e^{-(N\lambda_0)T}\right] \tag{14}$$

5. Numerical Results

Numerical illustrations have been made to calculate the reliability, mission time and availability. The point availability and interval availability profiles for different values of failure rate λ are displayed in figures 2-3. From figure 2, we see that the point availability initially decreases sharply with respect to time (t), but after some time, the increasing trends slow down. In figure 3, interval availability decreases very sharply in the beginning with respect to the interval duration T but the rate of decrement becomes slow as T increases. In both figures 2 and 3, the availability decreases as failure rate λ increases, which coincides with the physical situations.

The reliability profiles for different failures rates λ, λ_c and λ_p and coverage for successfully recovery (c) are displayed in figures 4-7 and figures 8-11. For heterogeneous and homogeneous cases, in figures 4-6 and 8-10, we show the effect of failure rates λ, λ_c and λ_p on the reliability. It is observed that the reliability decreases sharply for lower values of 't' but further decrease becomes slow as t increases. So far as the variations with respect to λ, λ_c and λ_p are concerned, the reliability is less for larger values of failures rates.

The effects of successfully coverage recovery (c) on the reliability by varying t are depicted in figures 7 and 11 for heterogeneous and homogeneous rates, respectively. For both cases, we see that the reliability decreases remarkably for lower values of 't' but later on it tends asymptotically to zero. The effect of c on the reliability is positive i.e. by increasing c, the reliability also increases.

In tables 1-3, we provide the results of mission time by varying λ, λ_c and λ_p, respectively for different values of R_{min}. It is noticed that the mission time (MT) is higher if the predefined acceptable value of minimum reliability (R_{min}) is low. Also failure rates λ, λ_c and λ_p have reverse effect on MT, which decreases as these parameters increase; however the effect is more prominent for lower values of failure rates.

Overall, based on numerical experiment, we conclude that

- As we expect, by increasing the failure rates λ, λ_c and λ_p, the system availability, reliability and mission time decrease but effect diminishes as time t increases.
- The reliability increases as the recovery rate c increases.

6. Conclusion

The performance of large multi-processor systems raises new issues to research in the design and development of highly fault-tolerant architectures because of it's applicability in large distributed systems, communication and computer networks, etc.. We have studied the effect of failure and recovery of homogeneous and heterogeneous multi-processors system due to individual as well as common cause failure. The explicit expressions for the reliability, availability and mission time are provided which may be employed by the system designers to evaluate optimal inherent parameters to maintain the minimum prescribed reliability level at reasonable cost.

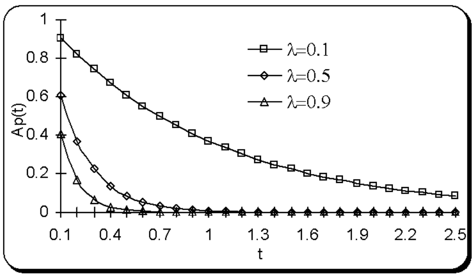

Fig. 2: Point availability vs t for different values of λ

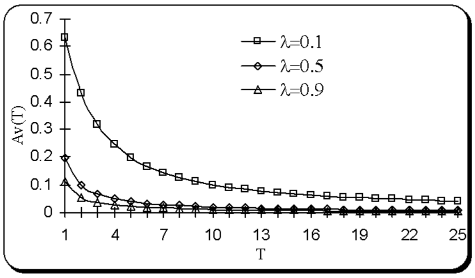

Fig. 3: Interval availability vs T for different values of λ

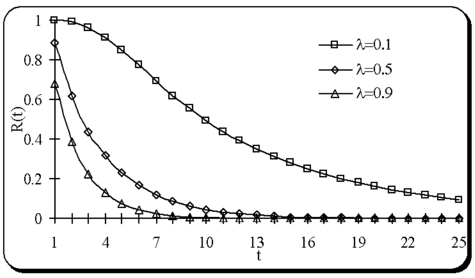

Fig. 4: R(t) vs t for different value of λ and heterogeneous failure rate ($c = 0.9$, $\lambda_c = 0.09$, $\lambda_p = 0.05$)

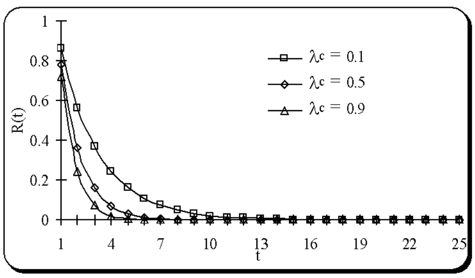

Fig. 5: R(t) vs t for different value of λ_c and heterogeneous failure rate ($\lambda = 0.1$, $c = 0.9$, $\lambda_p = 0.009$)

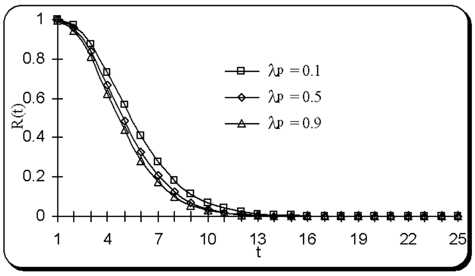

Fig. 6: R(t) vs t for different value of λ_p and heterogeneous failure rate ($\lambda = 0.1$, $c = 0.9$, $\lambda_c = 0.5$)

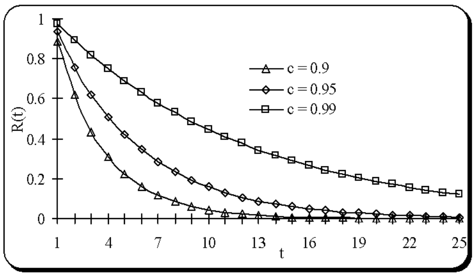

Fig. 7: R(t) vs t for different value of c and heterogeneous failure rate ($\lambda = 0.1$, $\lambda_c = 0.0001$, $\lambda_p = 0.005$)

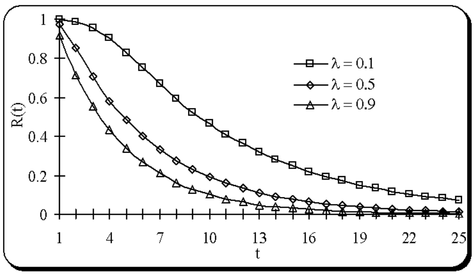

Fig. 8: R(t) vs t for different value of λ and homogeneous failure rate ($\lambda_c = 0.09$, $\lambda_p = 0.005$, $c = 0.95$,)

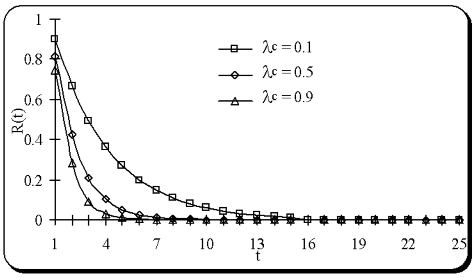

Fig. 9: R(t) vs t for different value of λ_c and homogeneous failure rate ($\lambda = 0.1$, $c = 0.95$, $\lambda_p = 0.005$)

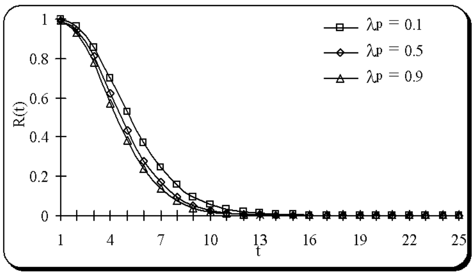

Fig. 10: R(t) vs t for different value of λ_p and homogeneous failure rate ($\lambda = 0.1$, $c = 0.95$, $\lambda_c = 0.005$)

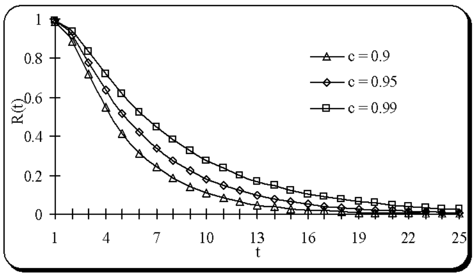

Fig. 11: R(t) vs t for different value of c and homogeneous failure rate ($\lambda = 0.1$, $\lambda_c = 0.005$, $\lambda_p = 0.005$)

R_{min} / λ	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	9.0	8.0	6.5	5.4	5.0
0.2	6.2	5.8	4.0	3.3	2.3
0.3	4.9	4.8	3.0	2.5	1.8
0.4	5.0	4.6	2.4	2.3	1.7
0.5	5.2	4.4	1.9	1.7	1.7
0.6	3.0	2.7	1.8	1.3	1.0
0.7	2.5	2.1	1.5	1.2	1.0
0.8	2.5	1.9	1.4	1.1	1.0
0.9	2.2	1.5	1.2	1.1	1.0
1.0	1.5	1.3	1.2	1.1	1.0

Table 1: Mission time (MT) for different failure rates λ

R_{min} / λ_c	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	9.0	7.0	6.3	5.0	4.0
0.2	7.4	6.0	5.3	4.2	3.4
0.3	6.2	5.5	4.7	4.0	3.0
0.4	5.5	5.0	4.3	3.5	2.7
0.5	5.3	4.8	4.0	3.4	2.5
0.6	5.1	4.4	4.2	3.2	2.4
0.7	5.0	4.0	3.9	3.1	2.3
0.8	4.9	4.2	3.3	2.3	2.2
0.9	4.9	4.0	2.4	2.3	2.2
1.0	4.3	3.2	2.4	2.2	2.0

Table 2: Mission time (MT) for different common cause failure rates λ_c

R_{min} / λp	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	5.2	4.5	4.0	3.3	1.9
0.2	4.9	4.4	3.9	3.2	1.8
0.3	4.8	4.2	3.8	3.0	1.8
0.4	4.7	4.1	3.6	3.5	1.6
0.5	4.5	4.0	3.5	3.4	1.5
0.6	4.4	3.9	3.4	3.2	1.5
0.7	4.2	3.8	3.2	3.1	1.4
0.8	4.1	3.7	3.0	1.4	1.3
0.9	4.0	3.5	2.8	1.3	1.0
1.0	3.8	2.2	1.5	1.0	1.0

Table 3: Mission time (MT) for different failure rates λ_p of main unit

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