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SOME METHODS OF CONSTRUCTION OF RECTANGULAR DESIGNS

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Abstract : Some new methods of construction of rectangular designs are described. Consequently, some series of rectangular design are obtained. A table of new designs in the range of parameters r, k < 10 is presented.

Key Words: Rectangular association scheme, Rectangular design.

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1. Introduction

Rectangular designs introduced by Vartak [11], are three associate class PBIB designs based on a rectangular association scheme having v = mn treatments arranged in a rectangular array of m rows and n columns. For the definition of rectangular designs along with their combinatorial properties, refer, Raghavarao [5]. Following the standard notations, the parameters of a rectangular design are denoted by v = mn, b, r, k, λ_1, λ_2, λ_3. These designs have recently been studied among others, by Suen [10], Sinha [6], Sinha et al. ([7], [8], and [9]), Kageyama and Miao [3], Parihar et al. [4]. The rectangular designs are useful as factorial experiments, having factorial balance as well as orthogonality (cf. Dey, [1], Section 6.5.3). In addition, if λ_3 is greater than λ_1 and λ_2 the loss of information on the main effects becomes small (Suen. [10]) when these designs are used as $m \times n$ complete confounded factorial experiment. In this paper, some new methods for the construction of rectangular designs have been developed. Some new series of rectangular designs are obtained. A table of certain rectangular designs obtained herein but not included in the tables of Suen [10] and Sinha et al.([7], [8], and [9]) in the parametric range r, k $\leq$ 10 is given.

2. Construction

In this section few methods of construction of rectangular designs will be presented.

Theorem 2.1: Let $A_1, A_2, \ldots, A_m$ be sets of equal size for $m \geq 2$. Then on composition of these sets a rectangular design with parameters

$$v = ms,\ b = \frac{s(s-1)}{2}, r = s-1, k = 2m, \lambda_1 = 1, \lambda_2 = s-1,\ \lambda_3 = 1;$$

$$n_1 = s-1,\ n_2 = m-1,\ \ n_3 = (m-1)(s-1) \qquad \ldots.(2.1)$$

can always be constructed.

Proof: Let sets $A_1, A_2, \ldots, A_m$ be defined as follows:

$A_1 = \{1, 2, \ldots, s\}$
$A_2 = \{s+1, s+2, \ldots, 2s\}$
$A_3 = \{2s+1, 2s+2, \ldots, 3s\}$
.
.
$A_m = \{(m-1)s+1, (m-1)s+2, \ldots, ms\}$.

For the construction of rectangular design, identify these m sets each having s treatments as a rectangular association scheme of v = ms treatments arranged in a rectangular array of m rows and s columns. The (s-1) treatments occurring in a row will be the first associate of a particular treatment θ contained in that row. There are m rows in the association scheme; hence (m-1) treatments occurring in a column will be second associate of θ contained in that column. Remaining (m-1) (s-1) treatments are third associate of θ.

In the composition process pickup one pair of distinct treatments (θ , ϕ) from each of m sets to form a block of the design in the following manner:

$\{1, 2, s+1, s+2, 2s+1, 2s+2, \ldots, (m-1)s+1, (m-1)s+2\} \rightarrow$: 1st block.
$\{1, 3, s+1, s+3, 2s+1, 2s+3, \ldots, (m-1)s+1, (m-1)s+3\} \rightarrow$: 2nd block.
. .
. .
. .
$\{s-1, s, 2s-1, 2s, 3s-1, 3s, \ldots, ms-1, ms\} \rightarrow: \binom{s}{2}$ th block.

The parameters given in (2.1) of the rectangular design are obvious from construction itself.

Example: Let $A_1 = \{1, 2, 3, 4\}, A_2 = \{5, 6, 7, 8\}, A_3 = \{9, 10, 11, 12\}, A_4 = \{13, 14, 15, 16\}$. Then $A_1 \otimes A_2 \otimes A_3 \otimes A_4$ is a rectangular design with parameters v = 16, b = 6, r = 3, k = 8, $\lambda_1 = 1$, $\lambda_2 = 3$, $\lambda_3 = 1$; $n_1 = 3$, $n_2 = 3$, $n_3 = 9$, having the following blocks written in the form of columns:

1	1	1	2	2	3
2	3	4	3	4	4
5	5	5	6	6	7
6	7	8	7	8	8
9	9	9	10	10	11
10	11	12	11	12	12
13	13	13	14	14	15
14	15	16	15	16	16

The rectangular association scheme is written as

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

This design seems to be new because it is not listed in Suen [10], Sinha et al. ([7], [8], and [9]), Kageyama and Sinha [3], Parihar et al. [4].

Theorem 2.2: Suppose $A_1, A_2, \ldots, A_m$ be the sets of equal size for $m \geq 3$. Then on composition of these sets a rectangular design with parameters

$$v = ms, b = \binom{s}{2}, r = \binom{s-1}{2}, k = 2m, \lambda_1 = s-2, \lambda_2 = \binom{s-1}{2}, \lambda_3 = s-2$$

$$n_1 = s-1,\ n_2 = m-1, n_3 = (m-1)(s-1) \qquad \ldots(2.2)$$

can always be constructed.

Proof: Let $A_1, A_2, \ldots, A_m$ be defined as in proof of Theorem 2.1.

In the composition processes pick up one triplet of distinct treatments (q,f,d) from each of m sets to form a block of the design in the following manner:

{1, 2, 3; s+1, s+2, s+3; 2s+1, 2s+2, 2s+3; ... , (m-1) s+1, (m-1) s+2, (m-1) s+3} $\rightarrow$: 1st block.

{1, 2, 4; s+1, s+2, s+4; 2s+1, 2s+2, 2s+4; ... , (m-1) s+1, (m-1) s+2, (m-1) s+4} $\rightarrow$: 2nd block.

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{s-2, s-1, s; 2s-2, 2s-1, 2s; 3s -1, 3s-2, 3s; ... , (ms-2, ms-1, ms)} –: $\binom{s}{3}$th block.

The parameters given in (2.2) of the rectangular design are obvious from the construction itself.

Example: Let A_1 = {1, 2, 3, 4}, A_2 = {5, 6, 7, 8}, A_3 = {9, 10, 11, 12}, Then $A_1 \otimes A_2 \otimes A_3$ is a rectangular design with parameters v = 12, b = 4, r = 3, k = 9, $\lambda_1 = 2$, $\lambda_2 = 3$, $\lambda_3 = 2$; $n_1 = 3$, $n_2 = 2$, $n_3 = 6$, having the following blocks written in the form of columns:

1	1	1	2
2	2	3	3
3	4	4	4
5	5	5	6
6	6	7	7
7	8	8	8
9	9	9	10
10	10	11	11
11	12	12	12

The rectangular association scheme is written as

1	2	3	4
5	6	7	8
9	10	11	12

This design seems to be new because it is not available in the existing lists of rectangular designs.

The results given in this paper produce the following new rectangular designs within the parametric range of r, k $\leq$ 10, that are not found in table by Suen [10] and Sinha et al. ([7], [8], and [9]).

Table Rectangular designs with r, k $\leq$ 10

No.	v	b	r	k	λ_1	λ_2	λ_3	m	n	Source
1	10	10	4	4	1	4	1	2	5	Th. 2.1
2	10	10	6	6	3	6	3	2	5	Th. 2.2
3	12	4	3	9	2	3	2	3	4	Th. 2.2
4	12	15	5	4	1	5	1	2	6	Th. 2.1
5	12	20	10	6	4	10	1	2	6	Th. 2.2
6	14	21	6	4	1	6	1	2	7	Th. 2.1
7	15	10	4	6	1	4	1	3	5	Th. 2.1
8	15	10	6	9	3	6	3	3	5	Th. 2.2
9	16	6	3	8	1	3	1	4	4	Th. 2.1
10	16	28	7	4	1	7	1	2	8	Th. 2.1
11	18	15	5	6	1	5	1	3	6	Th. 2.1
12	18	36	8	4	1	8	1	2	9	Th. 2.1
13	18	20	10	9	4	10	4	3	6	Th. 2.2
14	20	10	4	8	1	4	1	4	5	Th. 2.1
15	20	45	9	4	1	9	1	2	10	Th. 2.1
16	21	21	6	6	1	6	1	3	7	Th. 2.1
17	22	55	10	4	1	10	1	2	11	Th. 2.1
18	24	15	5	8	1	5	1	4	6	Th. 2.1
19	24	28	7	6	1	7	1	3	8	Th. 2.1
20	25	10	4	10	1	4	1	5	5	Th. 2.1
21	27	36	8	6	1	8	1	3	9	Th. 2.1
22	28	21	6	8	1	6	1	4	7	Th. 2.1
23	30	15	5	10	1	5	1	5	6	Th. 2.1
24	30	45	9	6	1	9	1	3	10	Th. 2.1
25	32	28	7	8	1	7	1	4	8	Th. 2.1
26	35	21	6	10	1	6	1	5	7	Th. 2.1
27	36	36	8	8	1	8	1	4	9	Th. 2.1
28	40	28	7	10	1	7	1	5	8	Th. 2.1
29	40	45	9	8	1	9	1	4	10	Th. 2.1

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