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STRESS INTENSITY FACTORS OF A GRIFFITH CRACK OPENED BY ASYMMETRICAL FORCES AT CRACK FACES IN THE INFINITE ISOTROPIC STRIP

RAHUL BAJAJ
Deptt. of Mathematics, St.John's College, Agra, U.P.-28002, INDIA
E-mail: rahulbajajmaths2012@gmail.com
SONU SIDHWANI
Deptt. of Mathematics, ITM University, Turari, Gwalior, M.P.-474001 , INDIA
E-mail: sonu_sidhwani@rediffmail.com
D.V.S. KUSHWAH
Deptt. of Mathematics, IITM Universe, Sithauli, Gwalior, M.P.-474001, INDIA
E-mail: dvs.kushwaha@yahoo.com

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Abstract : The exact expressions of stress-intensity factors and of crack shape of a Griffith crack are obtained by using finite Fourier sine and cosine transforms. The crack is opened by asymmetrical system of force at crack surfaces. The crack is in isotropic infinite strip of width 2a.

Key Words : Finite Fourier transforms, Stress-Intensity factors, Crack opening displacement.

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1. Introduction

Rectangular strips are very commonly used in structures like wing or fuselage of an aircraft or in a machine. This strip strengthens the strength of the structures. After continuous use of structures these strips may develop cracks under hydro-static pressure or air pressure. Cracks are in opening mode or mode-I . Structure undergo twisting ,then strip may fracture or develop fracture under tearing mode or mode-II. Mode-II is called crack under shearing force.

Using global theory Zhou Shan et al. [13] discussed the behavior of a Griffith-crack in functionally graded piezoelectric material under anti- plane shear.The problem is reduced to dual integral equation. Then they used Schmidt method to solve the dual integral equations. Ruiz et al. [9] discussed a new theoretical model based on critical shear crack theory. They investigated the strength and ductility of shear reinforced slabs.

There are few more interesting problems dealing with shear crack theory [4],[5],[6],[8]and[12].

If the crack occupies the region y = 0, $0 \le |x| < b <$ a and the edges of the strip of width 2a are rigidly lubricated, then the physical problem is translated to mixed- boundary value problem as below :

$$\sigma_{xy}(\pm a, y) = 0 \tag{1}$$

$$u_x(\pm a, y) = 0\,,\ 0 \le |y| < \infty \tag{2}$$

and 2a is width of the strip.

Y-axis is bisecting the crack $y = 0\,, 0 \le |x| < b$.

The continuity conditions are

$$\sigma_{xy}(x,0^+) = \sigma_{xy}(x,0^-), b < |x| \le a \qquad 3)$$

$$\sigma_{yy}(x,0^+) = \sigma_{yy}(x,0^-), b < |x| \le a \tag{4}$$

$$u_x(x,0^+) = u_x(x,0^-), b \le |x| \le a \tag{5}$$

$$u_y(x,0^+) = u_y(x,0^-), b \le |x| \le a \tag{6}$$

The boundary conditions over crack faces are

$$\sigma_{yy}(x,0^\pm) = \begin{cases} p_1^\pm & , \quad 0 \le x < b \\ p_2^\pm & , \quad -b < x \le 0 \end{cases} \tag{7}$$

$$\sigma_{xy}(x,0^\pm) = \begin{cases} q_1^\pm & , \quad 0 \le x < b \\ q_2^\pm & , \quad -b < x \le 0 \end{cases} \tag{8}$$

The symbol $(\pm)$ over quantities refer to values of functions for $y > 0$ and $y < 0$ respectively. We found throughout that (see [1])

$$u_y(x,0^\pm) > 0 \quad , \qquad 0 \le |x| < b\,.$$

which means that the crack really opens, and the crack faces do not meet each other except at crack tips $(\pm b, 0)$.

The finite Fourier transforms are defined as (see [10])

$$F_C(\alpha_n) = \int_0^a f(x)\cos\alpha_n x dx$$

$$F_S(\alpha_n) = \int_0^a f(x)\sin\alpha_n x dx$$

$$\alpha_n = nq \ or \qquad \alpha_n = (n - \frac{1}{2})q, \quad q = \frac{\pi}{a}$$

The plan of the paper is as follows: in §2 the problem is formulated and it will be reduced to dual series equations. The solution of dual series equations is given in §3. The physical quantities are given in§4. §5 will present the special type of loading. The Discussion and conclusion is given in §6.

2. Formation of problem

The physical quantities at a general point (x, y) are divided into two groups namely

$$\left[\sigma_{xy}, \sigma_{yy}\right] = \left[\sigma_{xy}^{(s)} + \sigma_{xy}^{(a)}, \sigma_{yy}^{(s)} + \sigma_{yy}^{(a)}\right]$$

$$\left[u_x, u_y\right] = \left[u_x^{(s)} + u_x^{(a)}, u_y^{(s)} + u_y^{(a)}\right]$$

where superscript (s) and (a), refers to symmetrical and anti symmetrical respectively.

The physical problem is divided into two parts namely (a) symmetrical problem (b) anti- symmetrical problem. These problems satisfy separately the boundary conditions over crack faces given by (7) and (8) respectively. We take the solutions of equations of equilibrium through stress- strain relations as (see [2] or [10-11]) :

$$u_x^{sk}(x,y) = \alpha_0 \sum_{n=1}^{\infty} \frac{\sin(\alpha_n x)}{\alpha_n} \left[(1-\eta)\frac{\partial^2 G^{sk}}{\partial y^2} + \eta\alpha_n^2 G^{sk} \right] \tag{9}$$

$$u_x^{ak}(x,y) = \alpha_0 \sum_{n=1}^{\infty} \frac{\cos(\beta_n x)}{\beta_n} \left[(1-\eta)\frac{\partial^2 G^{ak}}{\partial y^2} + \eta\alpha_n^2 G^{ak} \right] \tag{10}$$

$$u_y^{sk}(x,y) = \frac{u_0}{2} + \alpha_0 \sum_{n=1}^{\infty} \frac{\cos(\alpha_n x)}{\alpha^2{}_n} \left[(1+\eta)\frac{\partial^3 G^{sk}}{\partial y^3} + (2+\eta)\alpha_n^2 \frac{\partial G^{sk}}{\partial y} \right] \tag{11}$$

$$u_y^{ak}(x,y) = \alpha_0 \sum_{n=1}^{\infty} \frac{\sin(\beta_n x)}{\beta^2{}_n} \left[(1+\eta)\frac{\partial^3 G^{ak}}{\partial y^3} + (2+\eta)\beta_n^2 \frac{\partial G^{ak}}{\partial y} \right] \tag{12}$$

$$\alpha_0 = \frac{2}{a}\frac{1+\eta}{E} \quad ,\alpha_n = nq, \qquad \beta_n = (n-\frac{1}{2})q$$

and

$$\left.\begin{array}{l} G^{sk}(\alpha_n, y) = (A_{k-2} + yB_{k-2})e^{\mp\alpha_n y} \\ G^{ak}(\beta_n, y) = (C_{k-2} + yD_{k-2})e^{\mp\beta_n y} \end{array}\right\} \tag{13}$$

with k = 3, 4 referring to y > 0 and y < 0, respectively, u_0 is constant, η is Poisson ratio of the medium and E is Young's modulus of the medium. The displacement components above, along with stress- strain relations for plane strain conditions, satisfy equations of equilibrium and compatibility relations.

The boundary conditions (1) and (2) are satisfied identically for symmetric as well as for anti- symmetric cases. After separating the stress components into symmetric and anti-symmetric parts, and using continuity conditions (3)-(4) and conditions over crack faces (7)-(8), we get

$$\left.\begin{array}{l} A_1 - A_2 = \alpha_n^{-2}\int_{-b}^{b} p_1^e(x)\cos(\alpha_n x)dx \\ C_1 - C_2 = \beta_n^{-2}\int_{-b}^{b} p_1^0(x)\sin(\beta_n x)dx \end{array}\right\} \tag{14}$$

$$\left.\begin{array}{l} \alpha_n(A_1 + A_2) - (B_1 - B_2) = \alpha_n^{-1}\int_{-b}^{b} Q_1^e(x)\sin(\alpha_n x)dx \\ \beta_n(C_1 + C_2) - (D_1 - D_2) = \beta_n^{-1}\int_{-b}^{b} Q_1^0(x)\cos(\beta_n x)dx \end{array}\right\} \tag{15}$$

with

$$\left.\begin{array}{l} p_1^{(e)}(x) = \frac{1}{2}\left[p_1^+(x) + p_2^+(x) - (p_1^-(x) + p_2^-(x))\right] \\ p_1^{(0)}(x) = \frac{1}{2}\left[p_1^+(x) - p_2^+(x) - (p_1^-(x) - p_2^-(x))\right] \end{array}\right\} \tag{16}$$

$$\left.\begin{array}{l} Q_1^{(e)}(x) = \frac{1}{2}\left[q_1^+(x) - q_2^+(x) - (q_1^-(x) - q_2^-(x))\right] \\ Q_1^{(0)}(x) = \frac{1}{2}\left[q_1^+(x) + q_2^+(x) - (q_1^-(x) + q_2^-(x))\right] \end{array}\right\} \tag{17}$$

Now we use continuity conditions (5) and (6) for displacement and using (14)-(15), we get

$$\frac{B_0}{2}+\sum_{n=1}^{\infty}(B_1-B_2)\cos(\alpha_n x)=t_1F_{12}(x), b\le |x|\le a, t_1=\frac{E}{2}(1-\eta^2)^{-1} \quad (18)$$

$$\sum_{n=1}^{\infty}(D_1-D_2)\sin(\beta_n x)=t_1F_{22}(x), b\le |x|\le a \quad (19)$$

$$\sum_{n=1}^{\infty}(B_1+B_2)\sin(\alpha_n x)=t_1F_{32}(x), b\le |x|\le a \quad (20)$$

$$\sum_{n=1}^{\infty}(D_1+D_2)\cos(\beta_n x)=t_1F_{42}(x), b\le |x|\le a \quad (21)$$

$$F_{12}(x)=\frac{q}{2}\int_{-b}^{b}Q_1^{(e)}(x)dx \;, \qquad F_{22}(x)=\frac{q}{2}\int_{-b}^{b}Q_1^{(0)}(x)dx$$

$$F_{32}(x)=\frac{q}{2}\int_{-b}^{b}p_1^{(e)}(x)dx \;, \qquad F_{42}(x)=\frac{q}{2}\int_{-b}^{b}p_1^{(0)}(x)dx$$

We evaluate for symmetric and anti –symmetric cases separately after using stress-strain relations and (9)-(12) for displacement components and (14)-(15) for removal of constants, we get

$$\sum_{n=1}^{\infty}\alpha_n(B_1-B_2)\cos(\alpha_n x)=T_{11}(x) \quad (22)$$

$$\sum_{n=1}^{\infty}\beta_n(D_1-D_2)\sin(\beta_n x)=T_{21}(x) \quad (23)$$

$$\sum_{n=1}^{\infty}\alpha_n(B_1+B_2)\sin(\alpha_n x)=T_{31}(x) \quad (24)$$

$$\sum_{n=1}^{\infty}\beta_n(D_1+D_2)\cos(\beta_n x)=T_{41}(x) \quad (25)$$

where

$$\left.\begin{aligned} T_{11}(x) &= p_{11}(x) + q\int_{-b}^{b} \frac{\sin(qy)Q_n^{(e)}(y)}{G_1(y,x)}dy \\ T_{21}(x) &= p_{21}(x) + q\int_{-b}^{b} \frac{\sin(qy)Q_n^{(0)}(y)}{G_1(y,x)}dy \\ T_{31}(x) &= Q_{11}(x) + q\int_{-b}^{b} \frac{p_n^{(e)}(y)}{G_1(y,x)}dy \\ T_{41}(x) &= Q_{21}(x) + q\int_{-b}^{b} \frac{p_n^{(0)}(y)}{G_1(y,x)}dy \\ G_1(y,x) &= \frac{G(y,x)}{\sin(\frac{qx}{2})\cos(\frac{qy}{2})} \end{aligned}\right\} \tag{26}$$

$$\left.\begin{aligned} 2P_{11}(x) &= p_1^{(+)} + p_1^{(-)} + p_2^{(+)} + p_2^{(-)} \\ 2P_{21}(x) &= p_1^{(+)} - p_1^{(-)} + p_2^{(+)} - p_2^{(-)} \\ 2Q_{11}(x) &= q_1^{(+)} + q_1^{(+)} + q_2^{(-)} + q_2^{(-)} \\ 2Q_{21}(x) &= q_1^{(-)} - q_1^{(-)} + q_2^{(+)} - q_2^{(+)} \end{aligned}\right\} \tag{27}$$

Thus we get dual series relations (18) –(21) and (22) – (25). In next section we will solve these series relations.

3. Solution of dual series equations

Before solving the series relations we shall use the geometrical symmetry, for symmetrically & anti-symmetrically problem separately and the solution domain [-a, a] U $(-\infty,\infty)$ is reduced to [0, a] U $(0,\infty)$.

The trial solutions for dual series (18) and (22), (19) and (23),(20) and (24), (21) and (25) as (sec[7])

$$\alpha_n(B_1 - B_2) = 2\left[\int_0^b g_1(t)\sin(\alpha_n t)dt - a^{-1}\int_b^a F_{12}(t)\sin(\alpha_n t)dt\right] \tag{28}$$

$$B_0 = 2\left[\int_0^b tg_1(t)\sin(\alpha_n t)dt - ta^{-1}\int_b^a tF_{12}(t)\sin(\alpha_n t)dt\right] + tF_{12}(a) \tag{29}$$

$$\beta_n(D_1 - D_2) = 2\left[\int_0^b g_2(t)\cos(\beta_n t)dt - a^{-1}t_1\int_b^a F_{22}(t)\cos(\beta_n t)dt\right] \tag{30}$$

$$\alpha_n(B_1+B_2)=2\left[\int_0^b g_3(t)<1-\cos(\alpha_n t)>dt-a^{-1}\int_b^a F_{32}(t)<1-\cos(\alpha_n t)>dt\right] \tag{31}$$

$$\beta_n(D_1+D_2)=2\left[\int_0^b g_4(t)\sin(\beta_n t)dt-a^{-1}\int_b^a F_{42}(t)\sin(\beta_n t)dt\right] \tag{32}$$

The equations (18), (20) and (21) are satisfied by (28),(29),(31),(32) and(19), if

$$\int_0^b g_2(t)dt=t_1F_{22}(b) \tag{33}$$

The substitution of (28),(30),(31) and (32) into (22), (23),(24)and (25) respectively and then using Parihar method [7] . With no loss of generality, we take

$g_1(0)=g_2(0)=g_4(0)=0$, then solution of dual series are given as below :

$$g_1(t)=2\frac{\sin(qt/2)}{a^2\sqrt{G(t,b)}}\Delta_1(t),\quad 0\le t\le b,$$

$$\Delta_1(t)=\int_0^b\frac{\cos(qx/2)\sqrt{G(x,b)}L_{11}(x)}{G(x,t)}dx$$

$$g_2(t)=2\frac{\cos(qt/2)}{a^2\cos qt\sqrt{G(t,b)}}\Delta_2(t),0\le t\le b,$$

$$\Delta_2(t)=\int_0^b\frac{\sin(qx/2)\sqrt{G(x,b)}L_{21}(x)}{G(x,t)}dx$$

$$g_3(t)=2\frac{\cos(qt/2)G(0,t)}{a^2\sqrt{G(t,b)}}\Delta_3(t),\qquad 0\le t\le b,$$

$$\Delta_3(t)=\int_0^b\frac{\cos(qx/2)\sqrt{G(x,b)}L_{31}(x)}{G(x,t)}dx$$

$$g_4(t)=2\frac{\sin(qt/2)}{a^2\sqrt{G(t,b)}}\Delta_4(t),\quad 0\le t\le b,$$

$$\Delta_4(t) = \int_0^b \frac{\cos(qx/2)\sqrt{G(x,b)}L_{41}(x)}{\cos(qx)G(x,t)}dx$$

$$L_{11} = T_{11} + t_1 \int_b^a \frac{F'_{12}(t)\sin(qt)}{G(x,t)}dt \tag{34}$$

$$L_{31} = T_{31} + t_1 \int_b^a F'_{32}(t)\{\tan(qt/2) - \frac{\sin qt}{G(x,t)}\}dt \tag{35}$$

$$L_{21} = T_{21} + t_1 \int_b^a F'_{22}(t)\{\pi - qt\}dt \tag{36}$$

$$L_{41} = T_{41} + t_1 \int_b^a F'_{42}(t)\{\pi - qx\}dt \tag{37}$$

4. Physical quantities

The quantities important in fracture designing are stress-intensity factors and the crack opening displacement. We shall evaluate symmetric and anti-symmetric quantities separately

(a) SYMMETRIC (CRACK SHAPE)

We use k =3 and k =4 in (11) and put y = 0, we get

$$u_y^{(s_3)}(x,0) - u_y^{(s_4)}(x,0) = a\int_x^b g_1(t) + N_1^{(e)}(b) + (5+4\eta)\sum_{n=1}^{\infty}\frac{T_0\cos(\alpha_n x)}{\alpha_n},$$

$$0 \le |x| < b$$

$$N_1^{(e)}(x) = F_{12}(x), \qquad T_0(\alpha_n) = \int_{-a}^{a}(q_1^+ + q_2^+ - q_1^- - q_2^2)\sin\alpha_n x dx$$

$$u_y^{(s_3)}(x,0) + u_y^{(s_4)}(x,0) = \frac{2(5+4n)}{q}\Big[-\int_0^b g_3(t)\{\log 2(1-\cos qx)^2\}G(x,t)$$

$$+a^{-1}\int_a^b s_1^{(e)}(t)\{\log 2(1-\cos qx)^2\}G(x,t)\Big] - (3+2n)\sum\frac{F^{(e)}(\alpha_n)\cos\alpha_n x}{\alpha_n}, 0 \le |x| < b$$

$$S_1(x) = F_{32}(x)$$

Again, we use k =3 and k =4 in (9) and put y = 0, we get

$$u_x^{(s_3)}(x,0)-u_x^{(s_4)}(x,0)=\alpha_0\pi(1+n)\left[\int_x^b g_2(t)-q<s_1^{(e)}(a)-s_1^{(e)}(b)>\right]+\alpha_n t_1(\pi/2-qx/2)$$

$$u_x^{(s_3)}(x,0)+u_x^{(s_4)}(x,0)=\alpha_0\sum\frac{Q_1^{(e)}(\alpha_n)\sin\alpha_n x}{\alpha_n},\qquad S_1^{(e)}(x)=F_{32}(x)$$

$$2Q_1^{(e)}(\alpha_n)=\left[q_1^- - q_2^- -(q_1^+ - q_2^+)\right]$$

(b) SYMMETRIC (STRESS COMPONENTS)

Normal stress components are evaluated as below:

$$\sigma_{yy}^{(s_3)}(x,0)-\sigma_{yy}^{(s_4)}(x,0)=0\qquad b<x\le a \tag{38}$$

$$\sigma_{yy}^{(s_3)}(x,0)+\sigma_{yy}^{(s_4)}(x,0)=\frac{2}{a}\frac{\sin(qx/2)}{\sqrt{G(b,x)}}\Delta_1(x)+N_2^{(e)}(x)-a^{-1}\int_b^a N_1^{(e)}(x)\frac{\sin qt}{G(x,t)}dt \tag{39}$$

$$N_2^{(e)}(x)=a^{-1}t_1\int_b^a F'_{12}(t)\frac{\sin qt}{G(x,t)}dt \tag{40}$$

Shear stress components are evaluated below:

$$\sigma_{xy}^{(s_3)}(x,0)-\sigma_{xy}^{(s_4)}(x,0)=0\qquad b<x\le a \tag{41}$$

$$\sigma_{xy}^{(s_3)}(x,0)+\sigma_{xy}^{(s_4)}(x,0)=\frac{2\sin(qx/2)}{a\sqrt{G(b,x)}}\Delta_2(x)+\cot(qx/2)\left[\int_0^b g_2(x)a^{-1}(F_{32}(a)-F_{32}(b))\right]$$
$$+a^{-1}\int_b^a F'_{32}(t)\frac{\sin qt}{G(x,t)}dt \tag{42}$$

(c) ANTI- SYMMETRIC (CRACK SHAPE)

$$u_y^{(a_3)}(x,0)-u_y^{(a_4)}(x,0)=\sum_{n=1}^{\infty}\beta_n^{-1}\sin(\beta_n)x\int_{-b}^{b}p_1^{(0)}\cos(\beta_n x)dx$$
$$+(5+4n)\sum T_0(\alpha_n)\sin(\beta_n x)$$

$$u_y^{(a_3)}(x,0)+u_y^{(a_4)}(x,0)=-(1-n)\left[\int_0^b g_2(t)\frac{\sin qx\cos qt}{G(x,t)}dt\right]$$

$$+2(1-n)\left[\int_x^b g_2(t)\frac{\sin qx\cos qt}{G(x,t)}dt\right]+a^{-1}\left[\int_b^a F_{22}(t)\frac{\sin qx\cos qt}{G(x,t)}dt\right]$$

$$+\sum_{n=1}^{\infty}\left[\int_{-b}^{b} Q_1^{(0)}(x)\cos(\beta_n x)dt\right] \tag{43}$$

$$u_x^{(a_3)}(x,0)-u_x^{(a_4)}(x,0)=-2\alpha_0(1-n)\cos(qx)\left[(\int_x^b g_4(t)-t_1a^{-1}\int_b^a F_4(t))\frac{\sin qt}{G(x,t)}dt\right]$$
$$+\alpha_0\sum_{n=1}^{\infty}\frac{\cos\beta_n x}{\beta_n}\int_{-b}^{b} p_1^{(0)}(x)\sin(\beta_n x)dx$$

$$u_x^{(a_3)}(x,0)-u_x^{(a_4)}(x,0)=(1+2n)a\alpha_0\left[t_1F_{22}(a)\right]$$
$$+\alpha_0\sum_{n=1}^{\infty}\frac{\sin\beta_n x}{\beta_n}\int_{-b}^{b} Q_1^{(0)}(x)\cos(\beta_n x)dx$$

(d) ANTI-SYMMETRIC (STRESS COMPONENTS)

$$\sigma_{yy}^{(a_3)}(x,0)-\sigma_{yy}^{(a_4)}(x,0)=0 \qquad b<x\le a \tag{44}$$

$$\sigma_{yy}^{(a_3)}(x,0)+\sigma_{yy}^{(a_4)}(x,0)=\frac{2}{q}\frac{\cos(qx/2)\sin(qx)}{\cos(qx)\sqrt{G(b,x)}}\Delta_3(x)-t_1a^{-1}\int_b^a F'_{22}(t)\frac{\sin qx\cos qt}{G(x,t)}dt$$

$$+\sum_{n=1}^{\infty}\sin\beta_n x\int_{-b}^{b} Q_1^{(0)}(x)\cos(\beta_n x)dx \tag{45}$$

$$\sigma_{xy}^{(a_3)}(x,0)-\sigma_{xy}^{(a_4)}(x,0)=0 \qquad b<x\le a \tag{46}$$

$$\sigma_{xy}^{(a_3)}(x,0)+\sigma_{xy}^{(a_4)}(x,0)=\frac{2}{q}\frac{\cos(qx)}{\sqrt{G(b,x)}}\Delta_4(x)-t_1\frac{a^{-1}}{\pi}\cos(qx)\int_b^a F'_{42}(t)\frac{\cos qt}{G(x,t)}dt$$

$$+\sum_{n=1}^{\infty}\cos\beta_n x\int_{-b}^{b} Q_1^{(0)}(x)\sin(\beta_n x)dx \tag{47}$$

THE STRESS INTENSITY FACTORS

The stress intensity factors at crack tip (b,0) are defined as

$$[K_b,N_b]=\lim_{x\to b^-}\sqrt{x-b}[\sigma_{yy}(x,0),\sigma_{xy}(x,0)] \tag{48}$$

where

$$\sigma_{yy}(x,0)=\left[\sigma_{yy}^{(s_3)}+\sigma_{yy}^{(a_3)},\sigma^{(s_4)}{}_{yy}+\sigma_{yy}^{(a_4)}\right]$$

$$\sigma_{xy}(x,0)=\left[\sigma_{xy}^{(s_3)}+\sigma_{xy}^{(a_3)},\sigma_{xy}^{(s_4)}+\sigma_{xy}^{(a_4)}\right]$$

FOR SYMMETRIC PART

$\sigma_{yy}^{(s_3)}(x,0),\sigma^{(s_4)}{}_{yy}(x,0);\ \ \sigma_{xy}^{(s_3)}(x,0),\sigma_{xy}^{(s_4)}(x,0)$ are evaluated from (38), (39); (41), (42) respectively. Then we substitute these expressions in (47) and evaluate the limit we get

$$K_b^{(s_3)}=K_b^{(s_4)}=\frac{1}{a}\sqrt{\frac{\tan(qb/2)}{q}}\Delta_1(b) \tag{49}$$

$$N_b^{(s_3)}=N_b^{(s_4)}=-\frac{\sin(qb/2)}{a}\sqrt{\frac{\sin(qb/2)}{q}}\Delta_3(b) \tag{50}$$

FOR ANTI-SYMMETRIC PART

The stress components are $\sigma_{yy}^{(a_3)}(x,0),\sigma_{yy}^{(a_4)}(x,0)\ \ ;\sigma_{xy}^{(a_3)}(x,0),\sigma_{xy}^{(a_4)}(x,0)$ evaluated from (44),(45);(46),(47) respectively. Then use the definitions for stress-intensity factor (after evaluating the limits), we get

$$\begin{aligned}K_b^{(a_3)}&=K_b^{(a_4)}=\frac{1}{a}\sqrt{\frac{\cot(qb/2)}{2q}}\frac{\Delta_2(b)}{\cos(qb)}\\N_b^{(a_3)}&=N_b^{(a_4)}=-\frac{1}{a}\sqrt{\frac{\tan(qb/2)}{q}}\Delta_4(b)\end{aligned} \tag{51}$$

5. Special case of loading

Now we assume that the system of forces is self equilibrating in the sense that the resultant of the forces and moments over crack faces vanish identically. We take

$$\left.\begin{aligned}p_1^+=p_2^+=p_1^-=p_2^-=p_0=const\\q_1^+=q_2^+=q_1^-=q_2^-=q_0=const\end{aligned}\right\} \tag{52}$$

Using (52) in (13)-(16) we get

$$p_1^{(e)}(x) = p_1^{(0)}(x) = Q_1^{(e)}(x) = Q_1^{(0)}(x) = 0$$

and $F_{i2} = 0 \qquad i = 1,2,3.....$ (53)

We use (52) in (27),we get

$$\left.\begin{array}{ll} P_{11}(x) = 2p_0, & Q_{11}(x) = 2q_0 \\ p_{21}(x) = 0 = Q_{21}(x) & \end{array}\right\}$$

Using above in (26),we get

$$T_{11}(x) = 2p_0, T_{21}(x) = 0, T_{31}(x) = 2q_0, T_{41}(x) = 0 \tag{54}$$

Using (53) and (54) in (34)-(37) we get

$$L_{11}(x) = 2p_0, L_{21}(x) = 0, L_{31}(x) = 2q_0, L_{41}(x) = 0$$

$$N_1^{(e)}(x) = 0, \quad T_0(\alpha_n) = 0, F^{(e)}(\alpha_n) = 0, \quad S_1^{(e)}(x) = 0,$$
$$Q_1^{(e)} = 0, \qquad N_2^{(e)} = 0 = N_1^{(e)}$$

Thus we get for symmetric problem

$$g_1(t) = \frac{2}{q}\frac{\sin(qH2)}{\sqrt{G(t,b)}}p_0 \ , \ g_3(t) = \frac{2}{q}\frac{\cos(qH2)q_0G(0,t)}{\sqrt{G(t,b)}}$$

For this loading , the anti-symmetric problem is zero.

Thus the stress intensity factors for symmetric problem are given as

$$K_b^{(s_3)} = K_b^{(s_4)} = 2p_0\sqrt{\frac{\tan(qb/2)}{2q}} \tag{55}$$

$$N_b^{(s_3)} = N_b^{(s_4)} = -2q_0\frac{\sin(qb/2)}{a}\sqrt{\frac{\sin(qb)}{q}} \tag{56}$$

The displacement components for symmetric problem are evaluated as

$$u_y^{(s_3)}(x,0) - u_y^{(s_4)}(x,0) = \frac{p_0}{\sqrt{2}}\log\left|\frac{\cos(qx/2) + \sqrt{\cos^2(qx/2) - \cos^2(qb/2)}}{\cos(qb/2)}\right| \tag{57}$$

and $u_y^{(s_3)}(x,0)+u_y^{(s_4)}(x,0)$ is to be evaluated numerically by equation (43).

Also, $u_x^{(s_3)}(x,0)-u_x^{(s_4)}(x,0)$ is given as

$$u_x^{(s_3)}(x,0)-u_x^{(s_4)}(x,0)=-\frac{2q_0}{q}(5+4n)\left[1-\sin(qx/2)\sqrt{\frac{G(x,b)}{2}}\right.$$

$$\left.+\{\frac{a}{\sqrt{2}}G(0,b)\}\log\left|2G(0,x)\right|\right]$$

and $u_x^{(s_3)}(x,0)+u_x^{(s_4)}(x,0)=0$ (59)

6. Discussion and conclusion

In the present paper we deal with stress- intensity factors of a Griffith crack , opened by asymmetrical system of forces acting at crack faces in an isotropic infinite strip with rigidly lubricated edges.

The title problem can be obtained from the problem of isotropic infinite medium having equal and equally spaced Griffith- cracks loaded arbitrarily. It is assumed that the medium is under plane –strain conditions. It is also assumed that each crack has similar type of loading. Let each crack is divided into four quardrants with x-axis coinciding with common crack axes and Y- axis dividing the single crack at its middle. The crack surfaces lying in I , II ,III and IV quardrants have forces parallel to y-axis(positive and negative directions having (+) and (-) signs over quantites) as $p_1^+(x)$, $p_2^+(x)$ is positive and $p_1^-(x)$, $p_2^-(x)$ in negative directions respectively, which are acting normal to crack surfaces. The forces in positive x-axis direction as $q_1^+(x)$, $q_2^+(x)$ acting tangentially in I and II quadrants while $q_1^-(x)$, $q_2^-(x)$ acting tangentially in III and IV quadrants along negative of x-axis direction.

Then we divide the infinite medium by lines , parallel to Y- direction drawn from the middle points of line of separation of two Griffith cracks. These lines form the edges of the infinite strips.

One strip is considered in the paper. Thus, if cracks are of length 2b and spacing between the cracks middle to middle of cracks as 2a, then the width of strip is 2a.

We have discussed separately :

[a] Symmetrical problem and [b]Anti-symmetrical problem

The solution of equations of equilibrium is obtained by using Airy's stress- function method or the method of [2]. We used appropriate Fouier transform, finite and infinite. The conditions of continuity were observed out of crack- axis.

The physical problem was reduced to mixed boundary value problem. Assuming the solution and satisfying the boundary conditions, continuity conditions, the problem is reduced to different types of dual series relations.

The closed form solutions of these dual series are obtained by method of Kushwah [3]. The physical quantities are obtained in closed form in terms of these solutions. One special case of loading is considered.

We observe that

[i] The present method can be extended to the problem of infinite strip having two interior Griffith-cracks or two exterior Griffith-cracks. This problem will be reduced to triple series relations. The physical quantities will be obtained in closed form.

[ii] Instead of having loading at crack faces we can handle the problem with body forces in the medium.

[iii] We can solve the problem of infinite strip with stress-free edges. The problem of (i) and (ii) can be solved with stress-free edges.

[iv] The method can be extended to orthotropic medium also.

[v] In the paper we took one special case of loading defined by (52) and got the physical quantities. We get exactly same expression for ${K_b}^{(s_3)} = {K_b}^{(s_4)} = K_b$ and $u_y^{(s_3)}(x,0) = u_y^{(s_4)}(x,0) = u_y(x,0)$ as obtained in [3].

[vi] If we take q_0 =0, then the problem is reduced to one solved by Kushwah [3].

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References

[1] Burniston, E.E.(1969). An example of partially closed Griffith crack, *Int. J. Fracture Mech.*, **5(1)**, 17-24

[2] Kushwaha, P.S. (1974). A new-method for solving elasticity problems, *Proc. of Conference of Indian Mathematical Society* at IIT, Bombay.

[3] Kushwaha, P.S. (1975). *Some Two Dimensional Crack Problems in the Mathematical Theory of Elasticity*, Ph.D. Thesis, IIT Bombay.

[4] Muttoni, A. and Suah Wartz, J. (1991). Behavior of beams and punching in slabs without shear reinforcement, *IABSE Colloquium Zurich*, Switzerland **6**, 703-708.

[5] Muttoni, A.(2008). Punching shear strength of reinforced concrete slabs without transverse reinforcement, *ACI Struct. J.* **105**(4), 440-450.

[6] Muttoni, A. and Fernandez R.M. (2008). Shear strength of members without transverse reinforcement as a function of critical shear crack width, *ACI Struct. J.* **105**(2), 163-172.

[7] Parihar, K.S.(1971). Some triple trigonometrical series equations and their application, *Proc. Roy. Soc. Edinburg* **69A**, 255-265.

[8] Regan P.E. and Samadian, F.(2001). Shear reinforcement against punching in reinforced concrete flat slabs, *The Struct. Engineer* **79**(10), 24-31.

[9] Ruiz Mignel Fernandez and Aurelio Muttoni(2009). Applications of critical shear crack theory to punching of reinforced concrete slabs with transverse reinforcement, *ACI Structural Journal* **106**(1), 485-494.

[10] Sneddon, I.N.(1946), *Fourier Transforms*, Oxford University Press.

[11] Sneddon, I.N. and Lowengrub I.N. (1972), *The Crack Problems in Mathematical Theory of Elasticity*, John Wiley & Sons, New York.

[12] Stein T, Ghali, A. and Dilger, W. (2007). Distinction between punching and flexural failure modes of flat plates, *ACI Structural J.* **104**(3), 357-365.

[13] Zhou Shan, Z. G., Lin Duand Yi and Wu, Zhi (2006), Investigation of antiplane shear behavior of a Griffith crack / permeable crack in functionally graded piezoelectric materials by use of the non-local theory, Research Report *Centre for Composite Materials and Electro-optics Technology Centre, Harbin Institute*, *Harbin* (Pr China).