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NUMERICAL SOLUTION OF STEADY HEAT FLOW PROBLEM USING FINITE ELEMENT METHOD

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Abstract: In this paper, finite element numerical technique has been used to solve two dimensional steady heat flow problem with dirichlet boundary conditions in a rectangular domain. Finite element solution with triangular grid using spreadsheets is implemented here. Finally comparisons are made between the numerical solution obtained from the finite element method and analytical solution.

Keywords: Finite element, Heat flow, Numerical solution, Triangular grid, Boundary conditions.

1. Introduction

There are variety of mathematical models are available which are based on the numerical methods such as Method of Characteristics, Finite Difference Method (FDM), Finite Volume Method (FVM), Finite Element Method (FEM), Boundary Element Method (BEM) and other have been developed by mathematician and scientists for solution of engineering problems[1]. Out of many available numerical methods, FDM, FVM, FEM and BEM are most popular numerical grid techniques [4]. Most of the development of these techniques will follow those found in electromagnetic book [5]. When numerical grid methods are used in mathematical modelling, the ensuring computer model generally consists of pre-processing, processing and post-processing. In the pre-processing part of the model, input data required to solve the problem include domain geometry, initial and boundary conditions, coefficients and constants for the particular problem, value of universal constants, the grid information such as triangular grid, rectangular grid and so on and various options for the concerned problem like 1D, 2D and 3D steady or unsteady state analysis. Grid generation is an important task of pre-processing. The grid generation includes discretization of the domain into elements and determining grid points and it depends on the type of numerical method used and dimension of the model. Grid can be generated manually or computer programming can be written or separate grid generating

packages can be used. In the processing part, real simulation of the problem with the concerned numerical model is done which includes generation of element matrices, assembly of element equations, and imposition of boundary conditions and solution of system of equation. In the post-processing part, results obtained are processed in terms of tables, charts, graphs and so on. Computer program can be written such that results are displayed graphically or separate post-processing packages can be used.

A solution of the heat equation and electromagnetics problem can be found in [6], besides paper introduces finite element analysis technique using spreadsheet. The spreadsheet are used for solving electrostatic boundary-value problem and spreadsheet can offer a reasonable tradeoff between user-defined programming and specific purpose software is presented in [2]. In the paper [3], the researcher discussed the spreadsheet implementation and advantages of spreadsheet.

The finite element method (FEM) is a numerical grid technique for solving PDE. FEM is used widely for solving engineering problems in solid mechanics, heat transfer, structural mechanics, aerospace, automobiles, biomechanics and fluid mechanics and so on. FEM is one of the most flexible and versatile grid method for solving engineering problems.

The remainder of a paper is organized as follows. The section 2 gives the idea about the problem formulation. In section 3, the spreadsheet implementation and the essential basic steps have discussed. We have discussed the results in section 4. Finally, section 5 concludes a paper.

2. Problem Formulation

Consider steady state heat conduction in a rectangular region $0 \leq x \leq 4$, $0 \leq y \leq 4$ subjected to the boundary conditions shown in the figure 1.

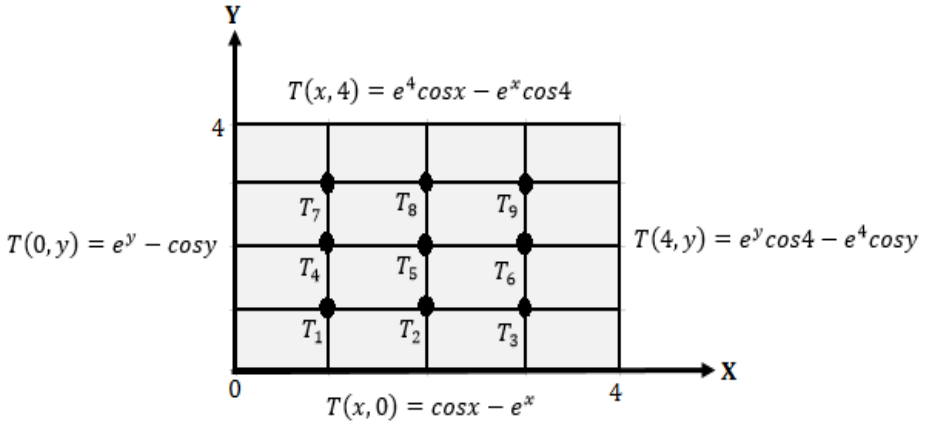

Figure 1: The Geometry and boundary conditions

The analytic solution of this heat conduction problem for the temperature distribution within the region is

$$T(x,y) = e^y \, cosx - \, e^x \, cosy$$

3. The spreadsheet Implementation

The spreadsheet implementation of the finite element numerical method consists of the following steps:

- Divide the solution region into a triangular 32 number of elements.

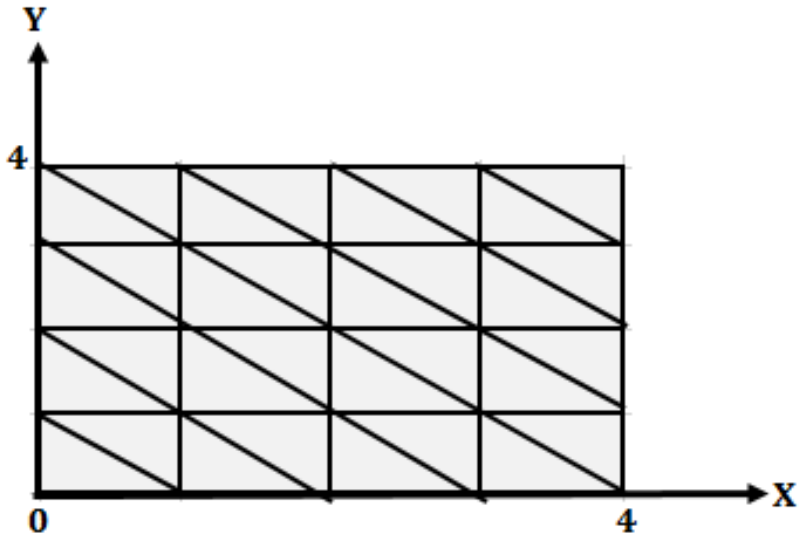

Figure 2: Finite element arrangement for heat flow problem.

- Generate the input data and consists of three tables: global node x and y coordinates; global and local node correspondence for each element; and list of nodes with fixed temperature.
- For each element, the following quantities are computed and obtain the element coefficient matrix.

$$(x_3 - x_2, y_2 - y_3), \qquad (x_1 - x_3, y_3 - y_1), \qquad (x_2 - x_1, y_1 - y_2)$$

- The VLOOKUP function is invoked to retrieve global node coordinates and assembled for the global coefficient matrix C.
- The matrices C_f (free nodes) and C_{ff} (free and fixed nodes) are formed by extracting the appropriate rows and columns from the global coefficient matrix C.
- The final solution is obtained by using the matrix capabilities of Microsoft Excel.

4. Discussion and Result

As indicated in Table 1, the temperature at the free nodes computed by finite element numerical triangular grid method and given analytical solution of this heat conduction problem compared fairly well.

Nodes	FE Numerical Solution	Analytical Solution
1	0.32930515	0.0000
2	-4.475653452	-5.1235
3	-11.08532632	-13.5433
4	5.792874047	5.1235
5	0.6586103	0.0000
6	1.94840679	1.0434
7	14.3783778	13.5433
8	-0.631186187	-1.0434
9	0.329305146	0.0000

Table 1: Numerical solution by finite element method and Analytical solution

The accuracy of the finite element numerical solution can be improved if a finer triangular grid is used.

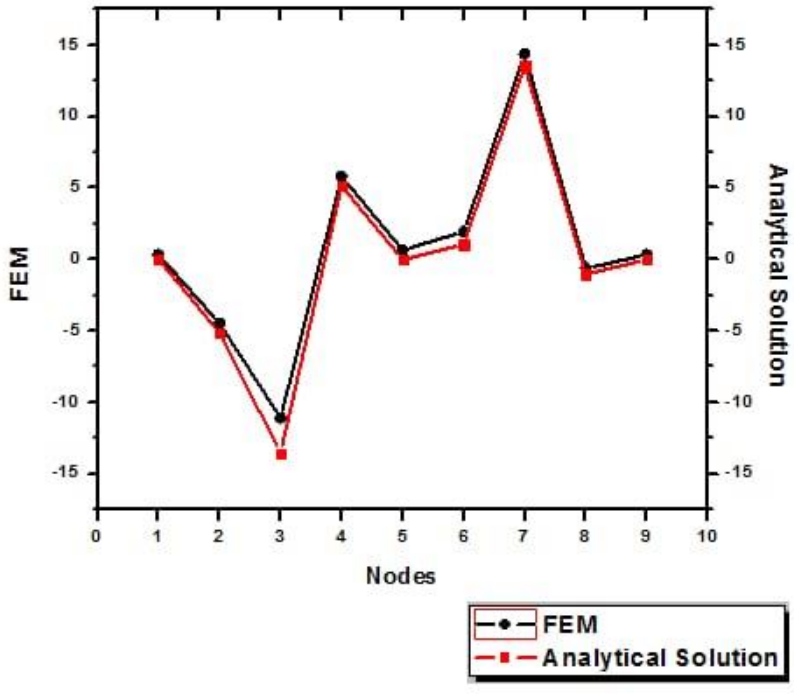

Figure 3: Compare the FE solution with the analytic solution.

5. Conclusion

This paper presented the finite element numerical solution using MS Excel for solving steady state heat conduction problem. The spreadsheet approach is ideal if the emphasis is on understanding of finite element numerical grid technique. After comparing with the given analytical solution, the obtained the Finite element numerical solution is fairly well.

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