Journal of Rajasthan Academy of Physical Sciences
ISSN: 0972-6306; URL: http://raops.org.in
International Conference on Mathematical and Statistical Computation (ICMSC-2022)
Swami Keshvanand Institute of Technology (SKIT), Jaipur, Rajasthan, (India) 3rd-5th March 2022
October, 2022, 47-56

BUSINESS ANALYSIS USING QUEUING MODEL WITH MIXED ARRIVALS AND RENEGING OF CUSTOMERS

Bhavtosh Awasthi[1] and **Seema Sharma**[2]
[1]Department of Mathematics, Eklavya University, Damoh-470661, M.P. (India)
E-mail: bhavtosh123@gmail.com (Corresponding Author)
[2]Department of Computer Sciences, JECRC University, Jaipur-303905, Rajasthan (India)
E-mail: seemasharmacg@gmail.com

Abstract: Stochastic models have vital role in describing company problems. In current business scenarios of sales and service industries to maintain faith of company among customers and connect new customers is a great task for planners of the business. Company can launch special schemes on product and services to attract new customers and also launch some other schemes for old customers to reside the old customers in the system. So, as a result there may be heavy rush in system. Customers get impatience and may leave the system without getting services due to high waiting time. Reneged customers can be considered as a loss in the business and so it is the requirement for the executive to design new and efficient strategies to retain the reneging customers. In this paper authors have developed multi-server Markovian model for the problem. The reneging behavior of the customers is incorporated with motivational arrival of customers. Simulation results analyzed for some for performance measures of the model developed.

Keywords: Stochastic Model, Motivated Arrivals, Reneging, Iterative Procedure

2010 MSC code: 44A10, 90B22, 60K25, 68M20, 60GXX6

1. Introduction

Haldane [11] did lot of contribution in queuing theory. Stochastic models with reneging and balking behaviors of customers studied by Haight [10]. He also discussed some performance measures and observed that the reneging rate depends on waiting time of the customers. Ancker & Gafarian [5,6] discussed about stochastic model with the reneging & balking behaviors of customers. Rao [14] did lot of work on queuing models. He also developed models with balking and reneging and interruptions. A general type of M/M/C multi-server operations research model with reneging & balking behaviors of customers were studied by Abou-El-Ata and Ibrahim [1]. To model more practical cases, Singh [15] did lot of work on stochastic models.ms. Scope of stochastic models appears in different industrial problems like impatient mobile users in cellular radio system, mobile communication networks, and rooms handling in case of emergency in hospital for different critical analysis of patients discussed by Sundarapandian [16]. Al-Seedy and Kotb [3] proposed the transient solution of the stochastic model of a single server with reneging & balking behavior of customers. The M/M/C multi-server queuing model with customers reneging behavior had proposed by Al-Seedy et al. [4]. The effects of balking and reneging behavior in the M/M/1/N queuing model were proposed by [6] and the M/M/1/N queuing model with server vacation and balking, reneging behavior of customers was discussed by [11]. Jain et al. [13] discussed about M/M/2/K unreliable queuing model with multi optional phase repair problem. The strategic behaviors of customers were examined by Wang et al. [17] in an M/M/1 constant retrial queue with the N-policy. Busy period analysis for M/Mn/1 and Mn/M/1 single server stochastic modals with state dependent service have discussed by Hadidi [9]. Wang & Yang [17] did done lot of work on controlling arrivals with unreliable server. Awasthi & Sharma [16] proposed encouraged model with heterogeneous servers. Performance analysis in presence of encouraged arrivals for multiple servers did by Awasthi & Sharma [7,8]. A single server model with finite capacity have discussed by Ahmad & Jayalalitha [2].

All above discussed stochastic models deal with arrivals of same type of customers with reneging & balking behavior of customers in the system. A company has a negative impact if the customers get impatience and leave the system without getting service due to bed quality of service or may be long waiting time for service. Stochastic model with impatience of customer has vast application in service industries like medical industries, relays reservation system, and in cellular radio system etc. It is a big challenge for managers and planners to retain impatient customers in current era of business. So, executives can make and apply some strategies as per current scenario and mood of customers to motivate new customers and retain impatient customers in the system. Taking all these concepts into consideration, a two server queuing system with mixed type of arrivals with reneging behavior of customers have been developed in this paper. The result has been verified by simulation analysis of the developed model.

2. Formulation pf Queuing Model

In this section, we formulate the M/M/C/K finite capacity queuing model for the problem with reneging of impatient customers. The stochastic queuing model derived here is based on the following assumptions:

1. Two types of customers arrive in the queuing system. First types of customer are normal customer, who arrives in the system in a Poisson fashion with arrival rate λ_1. Second types of customers are motivated customers who join the system to take the service due to some motivational schemes launched by the organization or service industries. These customers also arrive in a Poisson fashion with arrival rate λ_2. C servers are available in the service facility to serve the customers. The service times of the customers are distributed independently, identically and exponentially with parameters μ.

2. The discipline of the queuing system is FCFS (First Come First Service).
3. The capacity of the system is finite and is K.
4. When customers join the queue they wait for service for a certain time. The customer becomes impatient and reneged from the queue with rate υ.

3. Solution of the Queuing Model

Let $P_k(t)$ be the probability that there are k customers in the system at time t and P_k be the steady-state probability of having n customers in the system. Birth-death process has been used to analyze the system. The system equations are as follows:

$$\frac{dP_0(t)}{dt} = -(\lambda_1 + \lambda_2)P_0(t) + \mu P_1(t) \quad (1)$$

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2)P_0(t) - 2\mu P_2(t) + [\mu + (\lambda_1 + \lambda_2)]\, P_1(t) \quad (2)$$

$$\frac{dP_2(t)}{dt} = -(\lambda_1 + \lambda_2)P_1(t) - 3\mu P_3(t) + [2\mu + (\lambda_1 + \lambda_2)]\, P_2(t) \quad (3)$$

$$\frac{dP_C(t)}{dt} = -(\lambda_1 + \lambda_2)P_{C-1}(t) - (2\mu + \upsilon)\, P_{C+1}(t) + [C\mu + (\lambda_1 + \lambda_2)]\, P_C(t) \quad (4)$$

$$\frac{dP_k(t)}{dt} = -(\lambda_1 + \lambda_2)P_{k-1}(t) - (C\mu + (k+1-C)\upsilon)\, P_{k+1}(t) + [\{C\mu + (k-C)\upsilon\} + (\lambda_1 + \lambda_2)]\, P_k(t) \quad (5)$$

$$\frac{dP_K(t)}{dt} = -(\lambda_1 + \lambda_2)P_{K-1}(t) + \{C\mu + (K-1)\,\upsilon\}P_K(t) \quad (6)$$

For steady state situation of stochastic model,

$$\underset{t\to\infty}{Limit}\, P_k(t) = P_k$$

So, $\dfrac{dP_k(t)}{dt} = 0 \, as \;\; t \to \infty$

After applying the steady state condition in equations (1)-(6), we get the following equation in steady state condition

$$0 = -(\lambda_1 + \lambda_2)P_0 + \mu P_1 \quad (7)$$

$$0 = -(\lambda_1 + \lambda_2)P_0 - 2\mu P_2 + [\mu + (\lambda_1 + \lambda_2)]\, P_1 \quad (8)$$

$$0 = -(\lambda_1 + \lambda_2)P_1 - 3\mu P_3 + [2\mu + (\lambda_1 + \lambda_2)]\, P_2 \quad (9)$$

$$0 = -(\lambda_1 + \lambda_2)P_{C-1} - (2\mu + \upsilon)\, P_{C+1} + [C\mu + (\lambda_1 + \lambda_2)]\, P_C \quad (10)$$

$$0 = -(\lambda_1 + \lambda_2)P_{k-1} - (C\mu + (k+1-C)\upsilon)\, P_{k+1} + [\{C\mu + (k-C)\upsilon\} + (\lambda_1 + \lambda_2)]\, P_k \quad (11)$$

$$0 = -(\lambda_1 + \lambda_2)P_{K-1} + \{C\mu + (K-1)\,\upsilon\}P_K \quad (12)$$

Solving equations (7) to (12) iteratively, we get the following general solution

$$P_k = \begin{cases} \dfrac{(\lambda_1+\lambda_2)^k}{k!\mu} P_0, \; 1 \le k \le C \\ \dfrac{(\lambda_1+\lambda_2)^k}{\prod_{i=C+1}^{K}[C\mu + (i-C)\upsilon]}, \; C \prec k \le K \end{cases} \tag{13}$$

Using the condition of normality, we get

$$P_0 = \frac{1}{\left[\sum_{k=0}^{C} \dfrac{(\lambda_1+\beta\lambda_2)^k}{k!\mu} + \sum_{k=C+1}^{K} \dfrac{(\lambda_1+\lambda_2)^k}{\prod_{i=C+1}^{k}[C\mu+(i-C)\upsilon]} \right]} \tag{14}$$

4. Performance Measures of the Queuing Model

In this section, some performance measures of queuing model are derived. These are useful to study the performance of the queuing system under consideration.

4.1 Expected System Size (Ls)

Expected no. of customer in the queuing system is given by

$$L_s = \sum_{k=1}^{K} kP_k \tag{15}$$

4.2 Expected Queue Length (L_q)

Mean number of customer waiting for service is defined as

$$L_q = \sum_{k=C+1}^{K} (k-C)\, P_k \tag{16}$$

4.3 Average rate of reneging (R_{reg})

Average rate of reneging of customers is given by

$$R_{reg} = \sum_{k=C+1}^{K} (k-C)\upsilon\, P_k \tag{17}$$

4.4 Expected Waiting Time of a customer in the System (Ws)

Expected waiting Time in the system for slow service rate is given by

$$W_s = \frac{L_s}{\lambda_1+\lambda_2} \tag{18}$$

4.5 Expected Number of customers served

Expected number of customer served is given by

$$E_{CS} = \sum_{k=1}^{K} k\mu P_k$$

$$= \left[\sum_{k=1}^{2} k.\frac{(\lambda_1+\beta\lambda_2)^k}{k!\,\mu} + \sum_{k=2}^{K} k(\lambda_1+\beta\lambda_2)^k \prod_{i=1}^{k} \frac{1}{[2\mu+(i-2)\upsilon]}\right]\mu P_0 \quad (19)$$

4.6 Expected waiting time in the queue

Expected waiting time of a customer in the queue is given by

$$W_q = \frac{L_q}{\lambda_1+\lambda_2} \quad (20)$$

Simulation Analysis of Stochastic model

In this section, we analyzed some numerical results using simulation techniques.

Table 1: Effect of normal Arrival rate of customers λ_1 on Ls, L_q, Ws, Wq

$K = 20,\ c = 5,\ \lambda_2 = 1.35,\ \mu = 03.2, \theta = 0.3$

Normal Arrival rate (λ_1)	Expected System Size (Ls)	Expected Queue Length (L_q)	Expected Waiting Time of customer in the System (Ws)	Expected waiting time in the queue (Wq)
0.2	9.87E-20	7.27E-20	6.37E-20	4.69E-20
0.4	1.12E-20	8.22E-21	6.37E-21	4.70E-21
0.6	1.59E-21	1.18E-21	8.18E-22	6.03E-22
0.8	2.76E-22	2.03E-22	1.28E-22	9.45E-23
1	5.57E-23	4.11E-23	2.37E-23	1.75E-23
1.2	1.28E-23	9.46E-24	5.03E-24	3.71E-24
1.4	3.30E-24	2.43E-24	1.20E-24	8.85E-25
1.6	9.34E-25	6.88E-25	3.17E-25	2.33E-25
1.8	2.87E-25	2.12E-25	9.12E-26	6.72E-26
2	9.49E-26	6.99E-26	2.83E-26	2.09E-26
2.2	3.34E-26	2.46E-26	9.42E-27	6.94E-27
2.4	1.25E-26	9.20E-27	3.33E-27	2.45E-27
2.6	4.90E-27	3.61E-27	1.24E-27	9.14E-28
2.8	2.02E-27	1.49E-27	4.86E-28	3.58E-28
3	8.65E-28	6.37E-28	1.99E-28	1.46E-28
3.2	3.85E-28	2.84E-28	8.47E-29	6.24E-29
3.4	1.78E-28	1.31E-28	3.74E-29	2.76E-29

We can see in table 1 that with increase in normal arrival rate of customers, the expected system size, expected queue length, expected waiting time of customers in the system and expected waiting time in the queue going downwards.

Table 2: Effect of Motivated Arrival rate of customers a_2 on $Ls,\ L_q,\ Ws,\ Wq$

$K = 20,\ c = 5,\ \lambda_1 = 1.85,\ \mu = 3.2,\ \theta = 0.3$

Motivated Arrival rate (λ_2)	Expected System Size (Ls)	Expected Queue Length (L_q)	Expected Waiting Time of customer in the System (Ws)	Expected waiting time in the queue (Wq)
0.1	1.59E-21	1.18E-21	8.18E-22	6.03E-22
0.25	4.21E-22	3.10E-22	2.00E-22	1.48E-22
0.4	1.22E-22	8.97E-23	5.41E-23	3.99E-23
0.55	3.82E-23	2.81E-23	1.59E-23	1.17E-23
0.7	1.28E-23	9.46E-24	5.03E-24	3.71E-24
0.85	4.59E-24	3.38E-24	1.70E-24	1.25E-24
1	1.74E-24	1.28E-24	6.09E-25	4.49E-25
1.15	6.90E-25	5.09E-25	2.30E-25	1.70E-25
1.3	2.87E-25	2.12E-25	9.12E-26	6.72E-26
1.45	1.24E-25	9.17E-26	3.77E-26	2.78E-26
1.6	5.59E-26	4.12E-26	1.62E-26	1.19E-26
1.75	2.60E-26	1.92E-26	7.22E-27	5.32E-27
1.9	1.25E-26	9.20E-27	3.33E-27	2.45E-27
2.05	6.16E-27	4.54E-27	1.58E-27	1.16E-27
2.2	3.13E-27	2.30E-27	7.72E-28	5.69E-28
2.35	1.63E-27	1.20E-27	3.87E-28	2.85E-28
2.5	8.65E-28	6.37E-28	1.99E-28	1.46E-28

It is clearly visible in the table 2 that as motivated arrival rate of customers' increases, the expected system size, expected queue length, expected waiting time of customers in the system and expected waiting time in the queue continuously going downwards.

Table 3: Effect of service rate of customers μ on $Ls,\ L_q,\ Ws,\ Wq$

$K = 20,\ c = 5,\ \lambda_1 = 1.85,\ \lambda_2 = 1.35,\ \ \theta = 0.3$

Service rate (μ)	Expected System Size (Ls)	Expected Queue Length (L_q)	Expected Waiting Time of customer in the System (Ws)	Expected waiting time in the queue (Wq)
0.2	7.81E-14	5.75E-14	2.44E-14	1.80E-14
0.25	1.95E-14	1.44E-14	6.10E-15	4.49E-15
0.3	5.67E-15	4.17E-15	1.77E-15	1.30E-15
0.35	1.85E-15	1.36E-15	5.78E-16	4.26E-16
0.4	6.63E-16	4.89E-16	2.07E-16	1.53E-16
0.45	2.57E-16	1.89E-16	8.03E-17	5.92E-17
0.5	1.06E-16	7.85E-17	3.33E-17	2.45E-17

0.55	4.67E-17	3.44E-17	1.46E-17	1.07E-17
0.6	2.15E-17	1.58E-17	6.72E-18	4.95E-18
0.65	1.03E-17	7.62E-18	3.23E-18	2.38E-18
0.7	5.17E-18	3.81E-18	1.62E-18	1.19E-18
0.75	2.68E-18	1.97E-18	8.36E-19	6.16E-19
0.8	1.43E-18	1.05E-18	4.46E-19	3.29E-19
0.85	7.84E-19	5.78E-19	2.45E-19	1.81E-19
0.9	4.42E-19	3.25E-19	1.38E-19	1.02E-19
0.95	2.55E-19	1.88E-19	7.96E-20	5.86E-20
1	1.50E-19	1.11E-19	4.69E-20	3.46E-20

It can be observed from the table 3 that with the increasesment in service rate of the system, the expected system size, expected queue length, expected waiting time of customers in the system and expected waiting time in the queue gradually going downwards.

Table 4: Effect of reneging of customers θ on Ls, L_q, Ws, Wq

$K = 20, c = 5, \lambda_1 = 1.85, \lambda_2 = 1.35, \mu = 3.2$

Reneging rate (θ)	Expected System Size (Ls)	Expected Queue Length (L_q)	Expected Waiting Time of customer in the System (Ws)	Expected waiting time in the queue (Wq)
0.1	6.01E-25	4.43E-25	1.88E-25	1.38E-25
0.15	4.61E-25	3.40E-25	1.44E-25	1.06E-25
0.2	3.56E-25	2.62E-25	1.11E-25	8.20E-26
0.25	2.77E-25	2.04E-25	8.65E-26	6.37E-26
0.3	2.16E-25	1.59E-25	6.76E-26	4.98E-26
0.35	1.70E-25	1.25E-25	5.31E-26	3.92E-26
0.4	1.34E-25	9.90E-26	4.20E-26	3.09E-26
0.45	1.07E-25	7.87E-26	3.34E-26	2.46E-26
0.5	8.52E-26	6.28E-26	2.66E-26	1.96E-26
0.55	6.84E-26	5.04E-26	2.14E-26	1.57E-26
0.6	5.51E-26	4.06E-26	1.72E-26	1.27E-26
0.65	4.45E-26	3.28E-26	1.39E-26	1.03E-26
0.7	3.62E-26	2.67E-26	1.13E-26	8.33E-27
0.75	2.95E-26	2.17E-26	9.22E-27	6.79E-27
0.8	2.42E-26	1.78E-26	7.55E-27	5.56E-27
0.85	1.98E-26	1.46E-26	6.20E-27	4.57E-27
0.9	1.64E-26	1.21E-26	5.11E-27	3.77E-27

We can observe from the table 4 that as reneging rate of customers increases, as a result the expected system size, expected queue length, expected waiting time of customers in the system and expected waiting time in the queue continuously going downwards. This simulation result establishes that the performance measures developed in the model working properly.

5. Conclusion and Future Work

Here we study a C server stochastic model with mixed type of arrival of customers. The reneging behaviours of customers has been taken into consideration with finite capacity of the system. Steady state solution of the model has been derived here with iterative procedure. The stochastic model has been used to study how the performance of the sales and service industries can achieve optimal business. Simulation analysis has also done here to establish validity of the model developed. Cost analyses can also be done for the developed queuing model to visualize and analyze the result in better way to predict fruitful results using simulation.

Acknowledgement: The authors are thankful to the Referee for valuable comments and suggestions.

References

[1] Abou-El-Ata, M. and & S. Ibrahim, S. (1992). Two-server Heterogeneous Overflow Queues, J. King Abd. Univ. Sci. **4**, 159-168.

[2] Ahmad, M. and Jayalalitha, G. (2021). Single Server Finite Capacity Markovian Queuing Model with Encouraged Arrival, Nat. Volatiles & Esent. Oils **8**(5), 9417-9424.

[3] Al-Seedy, R. O. and K. A. M. Kotb, K. A. M. (1992). Transient solution of the state-dependent queue M /M/1 with balking, AMSE. **35**, 55-64.

[4] Al-Seedy, R. O., El-Sherbiny, A. A., El-Shehawayand, S. A. and Ammar, S. L. (2009). Transient solution of the M/M/c queue with balking and reneging, Computers & Mathematics with Applications **57**, 1280-1285.

[5] Ancker, C. J. Jr. and A.V., Gafarian, A.V. (1963b). Some queuing problems with balking and reneging II. Operations Research **11**, 928-937.

[6] Ancker, C. J. Jr. and Gafarian, A. V. (1963a). Some queuing problems with balking and reneging. I, Operations Research **11**, 88-100.

[7] Awasthi, B. and Sharma, S. (2018). Performance Analysis of Markovian Queueing Model in presence of Encouraged Arrivals, International Journal of Engineering Sciences Paradigms and Researches **47**(04), 43-47.

[8] Awasthi, B. and Sharma, S. (2018) M/M/2/K Queuing Model with Encouraged Arrivals, Heterogeneous Service and Reneging, International Journal of Engineering, Applied and Management Sciences Paradigms, **52**(04), 7-12.

[9] Hadidi, N. (1974). Busy periods of queues with state dependent arrival and service rates, Journal of Applied Probability **11**, 842-848.

[10] Haight, F. A. (1959). Queuing with reneging, Metrika **2**, 186-197.

[11] Haldane, J. B. S. (1957). Queuing with balking, Biometrika **44**, 360-369.

[12] J. Wang, J., X. Zhang, X. and Huang, P. (2017). Strategic behavior and social optimization in a constant retrial queue with the N-policy, European Journal of Operational Research **256**, 841-849.

[13] Jain, M., Sharma, G. C. and Sharma, R. (2012). Optimal control of (N,F) policy for unreliable server queue with multi optional phase repair and start up, International Journal of Mathematics in Operational Research **4**, 152-174.

[14] Rao, S. S. (1965). Queuing models with balking and reneging and interruptions, Operations Research **13**, 596-608.

[15] Singh, V. P. (1970). Two-server markovian queues with balking: heterogeneous vs. homogeneous servers, Oper. Res. **18**, 145-159.

[16] Sundarapandian, V. (2009), Probability, Statistics and Queuing Theory, PHI Learning, New Delhi, India, ISBN 8120338448.

[17] Wang, K. H. and Yang, D. Y. (2009). Controlling arrivals for a queuing system with an unreliable server: Newton-Quasi method, Applied Mathematics and Computation **213**, 92-101.

[18] Y. Zhang, Y., Yue, D. and W. Yue, W. (2005). Analysis of an M/M/1/N queue with balking, reneging and server vacations, Proc. of the Fifth International Symposium 37-47.