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ON RARELY NEUTROSOPHIC REGULAR SEMI CONTINUOUS FUNCTIONS

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Abstract: In this paper, we introduce the concept of rarely neutrosophic regular semi continuous functions in neutrosophic topological spaces. Some interesting properties and characterizations of rarely neutrosophic regular semi continuous and weakly neutrosophic regular semi continuous are investigated. Also, neutrosophic regular semi $T_{\frac{1}{2}}$-space, neutrosophic regular semi T_2-space and some applications to neutrosophic compact spaces are established.

Keywords: RNRC, RNRSC, NRS-compact space, rarely NRS-compact space and rarely NRS- T_2-spaces.

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1. Introduction

The study of fuzzy set was initiated by Zadeh [4] in 1965. Thereafter the paper of Chang[3] paved the way for the subsequent tremendous growth of the numerous fuzzy topology concepts. Currently Fuzzy Topology has been observed to be very beneficial in fixing many realistic problems. Several mathematicians have tried almost all the pivotal concepts of General Topology for extension to the fuzzy settings. In 1983, Atanassov[1] introduced the concept of intuitionitic fuzzy set which was generalization of fuzzy set, where besides the degree of membership there was considered a degree of non-membership of each element x∈X. Later, Coker [4] introduced the concept of intuitionitic fuzzy topological spaces, by using the notion of the intuitionitic fuzzy set. Smarandache [15, 16, 17] introduced the concept of Neutrosophic set. Neutrosophic set is classified into three independent functions namely, membership function, indeterminacy and non-membership function that are independently related. In 2012, Salama and Alblowi [12, 13] introduced the concept of Neutrosophic topology. Neutrosophic topological space are natural generalizations of fuzzy topological space allow more general functions to be member of fuzzy topology. In 2014, Salama et.al., [14] introduced the concept of Neutrosophic closed sets and Neutrosophic continuous functions.Popa [11] introduced the notion of rarely continuity as a generalization of weak continuity [14] which has been further investigated by Long and Herrington [10] and Jafari [7, 8]. In general toology, the concept of regular semiopen set was introduced by Cameron [2] in 1978. Recently Vijayalakshmi and Praveena [18, 19] introduced the concept of neutrosophic regular semiopen, neutrosophic regular semiclosed, neutrosophic regular semicontinuous, neutrosophic regular semi irresolute, neutrosophic regular semi homeomorphisms neutrosophic regular semi C-homeomorphisms in neutrosophic topological spaces. In this paper, we introduce the concepts of rarely neutrosophic regular semi continuous functions in neutrosophic topological spaces. Some interesting properties and characterizations of them are investigated. Also, some applications to neutrosophic compact spaces are established.

2. Preliminaries

Definition 2.1. [1] Let X be a non-empty fixed set. A neutrosophic set [for short, Ns] A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.1. [1] A Ns $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ can be identified to ordered triple $A = \langle x,\ \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ in $]^-0, 1^+[$ on X.

Remark 2.2. [1] For the sake of simplicity, we shall use the symbol $A = \langle \mu_A,\ \sigma_A,\ \gamma_A\rangle$ for the Ns $A = \{\langle x, \mu_A(x),\ \sigma_A(x),\ \gamma_A(x)\rangle : x \in X\}$.

Example 2.1 [1] Every intuitionsistic fuzzy set A is a non-empty set in X is obviously on Ns having the form $A = \{\langle x,\ \mu_A(x), 1 - \mu_A(x) + \gamma_A(x)\rangle : x \in X\}$. Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the neutrosophic sets 0_N and 1_N in X as follows:

$0_N = \{\langle x, 0,0,1\rangle : x \in X\}$ $1_N = \{\langle x, 1,1,0\rangle : x \in X\}$.

Definition 2.2. [1] Let $A = \langle(\mu_A,\ \sigma_A,\ \gamma_A)\rangle$ be a Ns on X, then the complement of set A (A^c or C(A) for short) may be defined as $C(A) = \{\langle x,\ \gamma_A(x),\ 1 - \sigma_A(x) + \mu_A(x)\rangle : x \in X\}$.

Definition 2.3. [1] Let X be a non-empty set and Ns's A and B in the form $A = \{\langle x, \mu_A,\ \sigma_A, \gamma_A\rangle : x \in X\}$ and $B = \{\langle x, \mu_B, \sigma_B,\ \gamma_B\rangle : x \in X\}$. Then $(A \subseteq B)$ may defined as:$(A \subseteq B) \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x)\ \forall\ x \epsilon X$.

Definition 2.4. [1] Let X be a non-empty set and $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \epsilon X\}$, $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \epsilon X\}$ are Ns's. Then $A \cap B$ and $A \cup B$ may defined as:

(i) $$A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle \quad (1)$$

(ii) $$A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle \quad (2)$$

Definition 2.5. [1] A neutrosophic topology (for short, NT or nt) is a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

(i) $0_N, 1_N \in \tau_N$, (3)

(ii) $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$, (4)

(iii) $\cup G_i \in \tau_N$ for every $\{G_i : i \in J\} \subseteq \tau_N$ (5)

Throughout this paper, the pair of $(X,\ \tau_N)$ is called a neutrosophic topological space (for short, nts). The elements of τ_N or τ are called neutrosophic open set (for short, nos). A neutrosophic set F is neutrosophic closed set (for short, ncs) if and only if F^c is nos.

Definition 2.6. [1] Let (X, τ_N) be nts and $A = \langle(\mu_A, \sigma_A, \gamma_A)\rangle$ be a Ns in X, Then the neutrosophic closure and neutrosophic interior of A are defined by $NCl(A) = \cap \{K : K\ is\ a\ ncs\ in\ X\ and\ A \subseteq K\}$, $NInt(A) = \cup \{G : G\ is\ a\ nos\ in\ X\ and\ G \subseteq K\}$. It can be also shown that $NCl(A)$ is ncs and $NInt(A)$ is a nos in X. A is nos if and only if $A = NInt(A)$, A is ncs if and only if $A = NCl(A)$.

Definition 2.7. [7] Let X be a nonempty set. If r, t, s be real standard or non standard subets of $]0^-, 1^+[$, then the neutrosophic set $x_{r,t,s}$ is called a neutrosophic point (briefly NP) in X given by

$$x_{r,t,st}(x_p) = \begin{cases} (r,t,s), if\ x = x_p \\ (0,0,1) if\ x\ \neq x_p \end{cases} \quad (6)$$

for $x_p \in X$ is called the support $x_{r,t,s}$,where r denotes the degree of membership value, t the degree of indeterminacy and s the degree of non-membership value of $x_{r,t,s}$.

Definition 2.8. [21] Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x\epsilon X\}$ be a Ns on a nts (X, τ_N) then A is called:

(i) neutrosophic regular open (for short, nro) iff $A = NInt(NCl(A))$.

(ii) neutrosophic regular closed (for short, nrc) iff $A = NCl(NInt(A))$.

Definition 2.9. [21] Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x\epsilon X\}$ be a Ns and $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x\epsilon X\}$ be a Ns on a nts (X, τ_N) then A is called neutrosophic semi-open (for short, nso) iff $A \subseteq NCl(NInt(A))$.

Definition 2.10. [15] Let (X, τ_N) be a nts. Then A is called

(i) neutrosophic regular semi open (for short, nrso) if there exist an nro set B in X such that $B \subseteq A \subseteq NCl(B)$.

(ii) neutrosophic regular semi closed (for short, nrsc) if there exist an nrc set B in X such that $NCl(B) \subseteq A \subseteq B$.

Definition 2.11. [15] Let (X, τ_N) be a nts. Then

(i) the neutrosophic regular closure of A, denoted by $nrcl(A)$, and is defined by $nrcl(A) = \bigcap\{B | B \supseteq A, B\ is\ nrc\}$.

(ii) the neutrosophic regular interior of A, denoted by $nrint(A)$, and is defined by $nrint(A) = \bigcup\{B | B \subseteq A, B\ is\ nro\}$.

(iii) the neutrosophic regular semi closure of A defined by $nrscl(A) = \bigcap\{B | A \subseteq B\ and\ B \in\ NRSCS(X, \tau)\}$ is a neutrosophic set.

(iv) the neutrosophic regular semi interior of A defined by $nrsint(A) = \bigcup\{B | B \subseteq A\ andB \in\ NRSOS(X, \tau)\}$ is a neutrosophic set.

Definition 2.12. [3] Let (X, τ) and (Y, σ) be any two nts's. A map $f: (X, \tau) \to (Y, \sigma)$ is neutrosophic continuous (for short, NC) if the inverse image of every neutrosophic closed set in (Y, σ) is neutrosophic closed set in (X, τ).

Definition 2.13. [16] Let (X, τ) and (Y, σ) be two nts's. A neutrosophic function $f: X \to Y$ is said to be

(i) neutrosophic regular continuous (for short, NRC) if for each nos A of Y, the inverse image $f^{-1}(A)$ is a nro set of X.

(ii) neutrosophic regular semi continuous (for short, NRSC) if for each nos A of Y, the inverse image $f^{-1}(A)$ is a nrso set of X.

(iii) neutrosophic regular semi irresolute (for short, NRSI) if for each nrso set A of Y, the inverse image $f^{-1}(A)$ is a nrso set of X.

(iv) neutrosophic regular semi open function (for short, NRS-O) if for each nos set B of X, the image $f(B)$ is a nrso set of Y.

(v) neutrosophic regular semi closed function (for short, NRS-C) if for each ncs set B of X, the image $f(B)$ is a nrsc set of Y.

Definition 2.14. [5] Let A be a neutrosophic set in the nts (X, τ). Then A is called an neutrosophic rare set (for short Nr-set) if $NInt(A) = 0_N$.

Proposition 2.1. [5] Let (X, τ) and (Y, σ) be two nts's and $f: (X, \tau) \to (Y, \sigma)$ is neutrosophic open and one-to-one, then f preserves neutrosophic rare set.

3 Rarely Neutrosophic Regular Semi Continuous Functions

Definition 3.1. Let (X, τ) and (Y, σ) be a nts's and $f: (X, \tau) \to (Y, \sigma)$ be a function. Then f is called :

(i) rarely neutrosophic regular continuous (for short, RNRC) if for each nos B of Y, there exists an Nr-set R with $B + NCl(R) \geq 1$ and nro set A of X such that $f(A) \leq B \cup R$.

(ii) rarely neutrosophic regular semi continuous (for short, RNRSC) if for each nos B of Y, there exists an Nr-set R with $B + NCl(R) \geq 1$ and an nrso set A of X such that $f(A) \leq B \cup R$.

Remark 3.1.

(i) Every NRC function is NRSC but converse need not be true.

(ii) Every RNRC function is RNRSC but converse need not be true.

(iii) Every NRSC function is RNRSC but converse need not be true.

(iv) Every RNRC function is RNC but converse need not be true.

Example 3.1. Let $X = \{a, b\}, \tau = \{0_N, 1_N, A, B\}, Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, C\}$, where A and B are Ns's of X and C is Ns of Y, defined as follows:

$$A = \langle\left(\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}\right), \left(\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}\right)\rangle, \tag{7}$$

$$B = \langle\left(\frac{\mu_a}{0.4}, \frac{\mu_b}{0.5}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}\right), \left(\frac{\gamma_a}{0.4}, \frac{\gamma_b}{0.5}\right)\rangle, \tag{8}$$

$$C = \langle\left(\frac{\mu_a}{0.5}, \frac{\mu_b}{0.5}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}\right), \left(\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.5}\right)\rangle. \tag{9}$$

Clearly τ and σ are NT on X and Y. If we define the function $f: X \rightarrow Y$ as $f(a) = p$ and $f(b) = q$, then f is NRSC function but not NRC functions, the Ns C is nrso set of X, since $\exists$ a nro set B such that $B \subseteq C \subseteq NCl(B)$ but not nro.

Example 3.2. Let $X = \{a, b\}, \tau = \{0_N, 1_N, A, B\}, Y = \{p, q\}$ and $\sigma = \{0_N, 1_N, C\}$, where A and B are Ns's of X and C is Ns of Y, defined as follows:

$$A = \langle\left(\frac{\mu_a}{0.4}, \frac{\mu_b}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}\right), \left(\frac{\gamma_a}{0.6}, \frac{\gamma_b}{0.6}\right)\rangle, \tag{10}$$

$$B = \langle\left(\frac{\mu_a}{0.4}, \frac{\mu_b}{0.4}\right), \left(\frac{\sigma_a}{0.5}, \frac{\sigma_b}{0.5}\right), \left(\frac{\gamma_a}{0.4}, \frac{\gamma_b}{0.4}\right)\rangle, \tag{11}$$

$$C = \langle\left(\frac{\mu_p}{0.8}, \frac{\mu_q}{0.8}\right), \left(\frac{\sigma_p}{0.6}, \frac{\sigma_q}{0.6}\right), \left(\frac{\gamma_p}{0.8}, \frac{\gamma_q}{0.8}\right)\rangle, \tag{12}$$

$$D = \langle\left(\frac{\mu_p}{0.5}, \frac{\mu_q}{0.5}\right), \left(\frac{\sigma_p}{0.5}, \frac{\sigma_q}{0.5}\right), \left(\frac{\gamma_p}{0.6}, \frac{\gamma_q}{0.6}\right)\rangle, \tag{13}$$

$$R = \langle\left(\frac{\mu_p}{0.6}, \frac{\mu_q}{0.6}\right), \left(\frac{\sigma_p}{0.8}, \frac{\sigma_q}{0.8}\right), \left(\frac{\gamma_p}{0.8}, \frac{\gamma_q}{0.8}\right)\rangle \tag{14}$$

Clearly τ and σ are NT on X and Y. If we define the function $f: X \rightarrow Y$ as $f(a) = p$ and $f(b) = q$, then f is RNRSC but not RNRC, the nos C of Y, since $\exists$ a Nr-set R and a nrso set D of X such that $f(D) \leq C \cup R$.

Example 3.3. In Example 3.2, f is rarely neutrosophic continuous but not RNRC. Since C is a nos in Y, R be an neutrosophic rare set and a nos A, $f(A) \leq C \cup R$ and also nos B of X, $f(B) \leq C \cup R$.

Example 3.4. In Example 3.2, f is RNRSC but not NRSC. Since nos C in, there exist a nro set A of X such that $A \leq C \nleq NCl(A), f^{-1}(C)$ is not nrso set in X.

Definition 3.2. Let $f: (X, \tau) \to (Y, \sigma)$ be a function. Then f is called RNRSC at $x_{r,t,s}$ in X if for each nos B of Y containing $f(x_{r,t,s})$, there exists an Nr-set R with $B + NCl(R) \geq 1_N$ and an nrso set A of X such that $f(A) \leq B \cup R$.

Theorem 3.1. Let $f: (X, \tau) \to (Y, \sigma)$ be a RNRSC at $x_{r,t,s}$ in X. Then for each nos G in (Y, σ) containing $f(x_{r,t,s})$, there exists an Nr-set R with G+$NCl(R) \geq 1_N$ such that $x_{r,t,s} \in NRSInt(\left(f^{-1}(G \cup R)\right)$.

Proof. Suppose that G be a nos in Y containing $f(x_{r,t,s})$. Then there exists an Nr-set R with G+$NCl(R) \geq 1_N$ and U be a nrso set in X containing $x_{r,t,s}$ such that $f(U) \leq G \cup R$. It follows that $x_{r,t,s} \in U \leq f^{-1}(G \cup R)$. This implies that $x_{r,t,s} \in NRSInt(\left(f^{-1}(G \cup R)\right)$

Theorem 3.2. Let (X, τ) and (Y, σ) be any two nts's. Then a function $f: (X, \tau) \to (Y, \sigma)$ is a RNRSC if and only if $f^{-1}(G) \leq NRSInt\left(f^{-1}(G \cup R)\right)$ for every nos G in Y, where R is a Nr-set with $NCl(R) + G \geq 1_N$.

Proof. Suppose that G be a nos in Y containing $f(x_{r,t,s})$. Then $NCl(R) + G \geq 1_N$ and U be a nrso set in X containing $x_{r,t,s}$ such that $f(U) \leq G \cup R$. It follows that $x_{r,t,s} \in U \leq f^{-1}(G \cup R)$. This implies that $f^{-1}(G) \leq NRSInt\left(f^{-1}(G \cup R)\right)$.

Definition 3.3. Let (X, τ) and (Y, σ) be nts's and $f: (X, \tau) \to (Y, \sigma)$ be a function. Then f is called weakly neutrosophic regular semi continuous (for short, WNRSC) if for each nrso set B of Y, $f^{-1}(B) \leq NInt\left(f^{-1}(NCl(B))\right)$.

Definition 3.4. A nts (X, τ) is said to be NRS$T_{\frac{1}{2}}$-space if every nrso set A of X is nro set.

Theorem 3.3. Let (X, τ) and (Y, σ) be any two nts's. If $f: (X, \tau) \to (Y, \sigma)$ is both NRS-O, NRSI and (X, τ) is NRS$T_{\frac{1}{2}}$-space, then it is WNRSC.

Proof. Let A be a nos in X. Since f is NRS-O, $f(A)$ is nrso set in Y. Also, since f is NRSI, $f^{-1}(f(A))$ is a nrso set. Since (X, τ) is NRS$T_{\frac{1}{2}}$-space, every nrso set is nro set and also every nro set is nos, now, $f^{-1}(f(A))$ is nos. Consider $f^{-1}(f(A)) \leq f^{-1}(NCl(f(A)))$ from which $NInt(f^{-1}(f(A))) \leq NInt(f^{-1}(NCl(f(A))))$. Since $f^{-1}(f(A))$ is nos, $f^{-1}(f(A)) \leq NInt(f^{-1}(NCl(f(A))))$. Thus f is WNRSC.

Definition 3.5. Let (X, τ) be a nts. A NRS-open cover of (X, τ) is the collection $\{A_i : i \in J\}$ of X, A_i is nrso such that $\cup_{i \in J} A_i = 1_N$.

Definition 3.6. A nts (X, τ) is said to be NRS-compact space if every NRS-open cover of (X, τ) has finite sub cover.

Definition 3.7. A nts (X, τ) is said to be rarely NRS-almost compact if every NRS-open cover $\{A_i : i \in J\}$ of X, A_i is nrso of (X, τ), there exists a finite subset J_0 of J such that $\bigcup_{i \in J} A_i \cup R_i = 1_N$ where R_i of X are Nr-set.

Theorem 3.4. Let (X, τ) and (Y, σ) be any two nts's and $f: (X, \tau) \to (Y, \sigma)$ be RNRSC. If (X, τ) is NRS-compact then (Y, σ) is rarely NRS-almost compact.

Proof. Let $\{A_i : i \in J\}$ be NRS-open cover of Y. Then $1_N = \cup_{i \in J} A_i$. Since f is RNRSC, there exists an Nr-sets R_i of Y such that $A_i + NCl(R_i) \geq 1_N$ and a nrso set B_i of X such that $f(B_i) \leq A_i \cup R_i$. Since (X, τ) is NRS-compact, every NRS-open cover of (X, τ) has a finite sub cover. Thus $1_N \leq \cup_{i \in J_0} B_i$. Hence $1_N = f(1_N) = f\left(\cup_{i \in J_0} B_i\right) = \cup_{i \in J_0} f(B_i) \leq \cup_{i \in J_0} A_i \vee R_i$. Therefore (Y, σ) is rarely NRS-almost compact.

Theorem 3.5. Let (X, τ) and (Y, σ) be any two nts's and $f: (X, \tau) \to (Y, \sigma)$ be RNRC. If (X, τ) is NRS-compact then (Y, σ) is rarely NRS-almost compact.

Proof. Since every RNRC function is RNRSC, then proof follows immediately from Theorem 3.2.

Theorem 3.6. Let (X, τ), (Y, σ) and (Z, η) be any two nts's. If $f: (X, \tau) \to (Y, \sigma)$ be RNRSC, NRS-O and $g: (Y, \sigma) \to (Z, \eta)$ is neutrosophic open and one-to-one, then $g \circ f: (X, \tau) \to (Z, \eta)$ is RNRSC.

Proof. Let A be a nos in. Since f is NRS-O, $f(A)$ is nrso set. Since f is RNRSC and there exists a Nr-set R of Y with $f(A) + NCl(R) \geq 1_N$ and a nrso set B in X such that $f(B) \leq f(A) \cup R$. By proposition 2.1., $g(R)$ of Z is also an Nr-set. Since R of Y is such that $R < C$ for all nos C of Y and g is injective, it follows that $(g \circ f)(A) + NCl(g(R)) \geq 1_N$. Then $(g \circ f)(B) = g\big(f(B)\big) \leq g(f(A) \cup R) \leq g\big(f(A)\big) \cup g(R) \leq (g \circ f)(A) \cup g(R)$. Hence the result.

Theorem 3.7. Let (X, τ) , (Y, σ) and (Z, η) be any nts's. If $f: (X, \tau) \to (Y, \sigma)$ be NRS-O and onto and $g: (Y, \sigma) \to (Z, \eta)$ be a function such that $g \circ f: (X, \tau) \to (Z, \eta)$ is RNRSC, then g is RNRSC.

Proof. Let A and B are two Ns's of X and Y such that $f(A) = B$. Let $(g \circ f)(A) = C$ is nos in Z. Since $(g \circ f)$ is NRSC, there exists a Nr-set R of Z with $C + NCl(R) \geq 1_N$ and a nrso set D of X such that $(g \circ f)(D) \leq C \cup R$. Since f is NRS-O, $f(D) \in Y$ is an nrso set. Thus there exists a Nr-set R of Z with $C + NCl(R) \geq 1_N$ and a nrso set $f(D) \in Y$ such that $g(f(D)) \leq C \cup R$. Hence g is RNRSC.

Theorem 3.8. Let (X, τ) and (Y, σ) be any two nts's. If $f: (X, \tau) \to (Y, \sigma)$ is RNRSC and (X, τ) is NRS$T_{\frac{1}{2}}$-space, then f is RNRC.

Proof. The proof is trivial.

Definition 3.8. A nts (X, τ) is said to be rarely NRS-T_2-space if for each pair of disjoint nonempty Ns's A, B of X there exists disjoint nrso sets R_1, R_2 of X and a Nr-set R of X with $R_1 + NCl(R) \geq 1_N$ and $R_2 + NCl(R) \geq 1_N$ such that $A \leq R_1 \cup R$ and $B \leq R_2 \cup R$.

Theorem 3.9. Let (X,τ) and (Y,σ) be any two nts's. If $f:(X,\tau)\to(Y,\sigma)$ is NRS-O and injective and (X,τ) be a rarely NRS-T_2space, then (Y,σ) is also a rarely NRS-T_2 space.

Proof. For any two disjoint nonempty Ns's A and B in X. Since f is injective, $f(A)$ and $f(B)$ are disjoint. Since (X,τ) is rarely NRS -T_2-space, there exists disjoint nrso sets R_1, R_2 in X and Nr-set R in X with $R_1+NCl(R)\geq 1_N$ and $R_2+NCl(R)\geq 1_N$ such that $A\leq R_1\cup R$ and $B\leq R_2\cup R$. Since f is NRS-O, $f(R_1)$, $f(R_2)$ are disjoint nrso sets. Since f is NRS-O and injective, $f(R)$ is also an Nr-set with $f(R_1)+NCl(R)\geq 1_N$ and $f(R_2)+NCl(R)\geq 1_N$ such that $f(A)\leq f(R_1\cup R)$ and $f(B)\leq f(R_1\cup R)$. Thus (Y,σ) is a rarely NRS-T_2-space.

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