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THERMO-DIFFUSION EFFECT OF TRANSIENT MHD CASSON FLUID PAST AN OSCILLATING INCLINED PLATE

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Abstract: The present investigation is to analyse the influence of thermo-diffusion effect, chemical reaction, and thermal radiation on a transient MHD Casson fluid flow through an oscillating inclined plate. The non-linear form of the governing equations are non-dimensionalized and then solved analytically by perturbation technique. The behaviour of flow velocity, concentration, and temperature with the influence of variation of physical parameters like angle of inclination, Casson parameter, Grashof number for mass and heat transfer, radiation effect, chemical reaction, Prandtl number, Hartmann number, Schmidt number and thermo-diffusion effect are examined through graphs. The results indicate that the embedded parameters have considerable impact on fluid properties. The rate of heat and mass transfer and skin friction also has significant impact under the influence of parameters.

Keywords: Casson, MHD, chemical reaction, thermal radiation, and Soret effect.

1. Introduction

Magnetohydrodynamics (MHD) is the science which deals with the study of the magnetic properties and behaviour of electrically conducting fluids. This MHD field was initiated by Hannes Alfven. The equations which govern MHD are a combination of the Maxwell's equations of electromagnetism and Navier-Stokes equations of fluid dynamics. The presence of magnetic fields leads to the forces that in turn act on the fluid, thereby potentially altering the geometry and strength of the magnetic fields themselves. For a particular MHD conducting fluid,the key issue is the relative strength of the advecting motions in the fluid, compared to the diffusive effects caused by the electrical resistivity. Its wide application in many branches of science and technology such as Astrophysics, Engineering, Geophysics, Medical sciences etc. has attracted numerous scientist and engineers for the last several decades.The study of boundary layer ow of viscous and non-Newtonian fluids has attracted large number of researchers due to its importances. Convective flow with heat transfer plays an important rule in non-Newtonian fluid. The heat and mass transfer involving in chemical reaction on MHD ow has received considerable attention in recent years due to its wide applications. Due to heat current and difference in salinity etc, the effects of heat and mass transfer on flow near vertical circular cylinder has been applicable in many engineering and physical problems. Raptis and Soundalgekar [11] studied the effect of mass transfer on the ow of an electrically conducting fluid past a steadily moving infinite vertical porous plate. Agarwal et al. [1] studied the effect of MHD free convection and mass transfer on the ow past a vibrating infinite vertical circular cylinder. Thermal radiation effects on MHD flow has achieved considerable interest due to its application in various industries, nuclear power plants, gas turbines and various devices for missiles, satellites and space vehicles. Sarma et al. [12] presented the effect of thermal radiation and chemical reaction on a unsteady MHD flow past an accelerated infinite vertical porous plate. Ahmed [4] has analyzed the influence of thermal radiation and magnetic Prandtl number on steady MHD heat and mass transfer of viscous, incompressible mixed convection flow. Ahmed and Chamkh [2] studied the effects of radiation, chemical reaction, heat and mass transfer along a vertical porous wall in the presence of induced magnetic field. Makinde and Ogulu [7] studied the effect of temperature-dependent viscosity on free convective flow in the presence of chemical reaction, thermal radiation and magnetic field. Ahmed [5] studied the effect of thermal radiation for a gray, absorbing-emitting radiating but non-scattered medium. In Newtonian fluid model many flow characteristics are not explicable, so in most of the fluid problem we consider non-Newtonian fluid. Casson fluid which is well-known for its various characteristics is an example of a non-Newtonian fluid. Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear. Few examples of Casson fluid are: tomato sauce, honey, jelly, soup, etc. Contributions of many researchers on the study of Casson fluid flow are worth mentioning. The flow characteristics of a Casson fluid in a tube filled with a homogeneous porous medium is investigated by Dash et al [6]. Shehzad et al [13] have studied the effects of mass transfer on MHD flow of a casson fluid with chemical reaction and suction. Mukhophadhyay [8] has investigated the heat transfer of Casson fluid over a nonlinear stretching surface. Mukhophadhyay and Vajravelu [9] have studied the chemical diffusion in Casson fluid flow over an unsteady permeable stretching surface. Pushpalatha et al [10] has investigated heat and mass transfer in unsteady MHD Casson fluid with convective boundary conditions. Vijayaragavan et al [16] investigated the heat and mass transfer of MHD casson fluid flow past an inclined porous plate with the effects of Dufour and chemical reaction. Talukdar and Nath [14] have studied the unsteady MHD free convective ow of a casson fluid past over an oscillating vertical plate. Talukdar and Nath [15] examined a transient MHD Casson fluid flow past an inclined moving plate. The mass transfer caused by the temperature gradient is called the Soret effect. Such effects become crucial when the density difference exists in the flow regimes. The objective of the present work is to investigate the influence of chemical reaction, thermal radiation and Soret effect on a transient MHD Casson fluid flow past an inclined moving plate. This work is an extension of the problem studied by Talukdar and Nath [15] to the case when there is an effect of thermal diffusion.

2. Mathematical Analysis

We consider an incompressible one-dimensional unsteady MHD free convection flow with heat and mass transfer of a Casson fluid flowing through an oscillating inclined plate. The fluid is considered to be viscous with the effect of thermal radiation and chemical reaction. We consider a coordinate system in such a way that x_0-axis is considered in vertically upward direction and y_0-axis is normal to the plate in the direction of the fluid flow. All the fluid properties except the influence of density in concentration and temperature are considered to be constant. The induced magnetic field in comparison to the applied magnetic field is assumed to be negligible. About a non-uniform temperature, the temperature and concentration on the plate oscillates with small amplitude.

Taking into account the rheological equation for an incompressible and isotropic Casson fluid with the above assumptions is

$\tau = \tau_0 + \mu\alpha^*$

where τ is the shear stress, τ_0 is the Casson yield stress, μ is the dynamic viscosity and α^* is the shear rate.

Equivalently,

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}}\right) e_{ij}, \pi < \pi_c \end{cases}$$

where p_y denote the yield stress of the fluid, μ_B is the plastic dynamic viscosity $\pi = e_{ij}\, e_{ij}$ and e_{ij} is the (i j)th component of deformation rate, π is the product of component of deformation rate with itself and π_c is the critical value of this product based on the non-Newtonian model.

Keeping in view the assumptions made above and usual Boussinesq's approximation the equations [9] which governs the flow are:

$$\frac{\partial u'}{\partial t'} = \nu\left(1 + \frac{1}{\alpha}\right)\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty)\cos\gamma + g\beta^*(C' - C'_\infty)\cos\gamma - \frac{\sigma}{\rho}B_0^2 u' \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r'}{\partial y'} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - kr'(C' - C'_\infty) + \frac{D_m K_T}{T'_m}\left(\frac{\partial^2 T'}{\partial y'^2}\right) \quad (3)$$

The relevant initial and boundary conditions:

$$u' = U,\; T' = T'_w + \epsilon e^{i\omega t}(T'_w - T'_\infty),\; C' = C'_w + \epsilon e^{i\omega t}(C'_w - C'_\infty) \text{ at } y'=0 \quad (4)$$

$$u' \to 0, T' \to 0, C' \to 0 \text{ at } y' \to \infty \quad (5)$$

where u' is the velocity component in x'-direction, t' is the time, υ is the kinematic viscosity, g is the acceleration due to gravity, α is the Casson parameter, γ is the angle of inclination, β is the thermal expansion coefficient, β^* is the mass expansion coefficient, $T'$is the fluid temperature, T'_∞ is the temperature away from the plate, T'_∞ is the temperature near the plate, C' is the concentration of the fluid, C'_∞ is the concentration away from the plate, C'_w is the concentration near the plate, σ is the magnetic permeability of the fluid, ρ is the density of the fluid, B_0 is the coefficient of magnetic field, k is the thermal conductivity, C_p is the specific heat at constant pressure, q'_r is the thermal radiation flux, D is the chemical molecular diffusivity, k'_r is the chemical reaction rate constant, K_T is the thermal diffusion ratio, D_m is the coefficient of mass diffusion, T'_m is the mean fluid temperature, ϵ is the scalar constant, ω is the dimensionless exponential index.

The thermal radiation flux q'_r gradient under Rosseland approximation is expressed as follows:

$$-\frac{\partial q'_r}{\partial y'} = 4\, a\, \sigma^*(T'_\infty - T'_4) \tag{6}$$

Where σ^* is the Stefan-Boltzmann constant. The temperature differences within the flow are assumed to be sufficiently small, so we can expand T'_4into Taylor's series about the free stream temperature. Hence neglecting the higher order terms the result of the approximation is as follows:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty$$

We now introduce the following non-dimensional quantities:

$$y = \frac{y'U}{\nu}, u = \frac{u'}{U}, t = \frac{t'U^2}{\nu}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \ \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \ Gr = \frac{g\beta\nu(T'_w - T'_\infty)}{U^3}, Gm = \frac{g\beta^*\nu(C'_w - C'_\infty)}{U^3},$$
$$Pr\frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho U^2}, R = \frac{16 a\sigma^* \nu^2 T'^3_\infty}{U^2 \rho C_p}, Kr = \frac{\nu Kr'}{U^2}, Sr = \frac{D_m K_T(T'_w - T'_\infty)}{T_m\, \upsilon(C'_w - C'_\infty)}$$

Where Gm is the Grashof number for mass transfer, Gr is the Grashof number for heat transfer, Pr is the Prandtl number, Sc is the Schmidt number, Kr is the chemical reaction parameter, M is the Hartmann number and R is the radiation parameter and Sr is the Soret number.

Using the non-dimensional quantities, the equations (1) to (3) reduces to:

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\alpha}\right)\frac{\partial^2 u}{\partial y^2} + G_1\theta + G_2\phi - Mu \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - R\theta \tag{8}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - Kr\phi + Sr\frac{\partial^2 \theta}{\partial y^2} \tag{9}$$

Here,

$$G_1 = Gr\cos\gamma, G_2 = Gm\cos\gamma$$

The non-dimensional form of the corresponding boundary conditions is:

$u = 1, \theta = 1 + \epsilon e^{i\omega t}, \phi = 1 + \epsilon e^{i\omega t}$ at $y = 0$

$u \to 0, \theta \to 0, \phi \to 0$ as $y \to \infty$

3. Method of Solution

In order to solve the equations (7) to (9), we assume u, θ and ϕ to be of the form:

$$u = u_0 + \epsilon e^{i\omega t} u_1 + O(\epsilon^2)$$

$$\theta = \theta_0 + \epsilon e^{i\omega t} \theta_1 + O(\epsilon^2)$$

$$\phi = \phi_0 + \epsilon e^{i\omega t} \phi_1 + O(\epsilon^2)$$

Substituting the above relations in equations (7) to (9) and equating the harmonic terms and neglecting ϵ^2 we get the following set of zeroth and first order ordinary differential equations:

3.1 Zeroth order equations

$$\theta_0'' - RPr\theta_0 = 0 \quad (10)$$

$$\left(1 + \frac{1}{\alpha}\right) u_0'' - Mu_0 = -G_1\theta_0 - G_2\phi_0 \quad (11)$$

$$\frac{1}{Sc}\phi_0{}'' - kr\phi_0 = -Sr\theta_0{}'' \quad (12)$$

3.2 First order equations

$$\theta_1{}'' - Pr(R + i\omega)\theta_1 = 0 \quad (13)$$

$$\left(1 + \frac{1}{\alpha}\right) u_1'' - (M + i\omega)u_1 = G_1\theta_1 - G_2\phi_1 \quad (14)$$

$$\frac{1}{Sc}\phi_1{}'' - (kr + i\omega)\phi_1 = -Sr\theta_1{}'' \quad (15)$$

The corresponding boundary conditions are:

$y = 0: u_0 = 1, \theta_0 = 1, \phi_0 = 1, u_1 = 0, \theta_1 = 1, \phi_1 = 1$

$y \to \infty: u_0 = 0, \theta_0 = 0, \phi_0 = 0, u_1 = 0, \theta_1 = 0, \phi_1 = 0$

The solutions of the above equation subject to the boundary conditions are:

$$\theta = e^{-a_1 y} + \epsilon e^{i\omega t} e^{-a_2 y}$$

$$\phi = (1 - a_3)e^{-a_4}y + a_3 e^{-a_1 y} + \epsilon e^{i\omega t}\left((1 + a_6)e^{-a_5 y} - a_6 e^{-a_2 y}\right)$$

$$u = (1 + a_9 + a_{10})e^{-a_8 y} - a_9 e^{-a_1 y} - a_{10} e^{-a_4 y} + \epsilon e^{i\omega t}\left((a_{15} - a_{14})e^{-a_{13} y} + a_{14} e^{-a_2 y} - a_{15} e^{-a_5 y}\right)$$

4. Skin Friction

The skin friction τ at the plate y = 0 is given by:

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0},$$

$$\tau = (-a_8 - a_8a_9 - a_8a_{10} + a_9a_1 + a_{10}a_4) + \epsilon e^{i\omega t}(-a_{13}a_{15} + a_{13}a_{14} - a_2a_{14} + a_{15}a_5)$$

5. Rate of Heat Transfer

The rate of heat transfer in terms of Nusselt number is given by:

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$\frac{\partial \theta}{\partial y} = -a_1e^{-a_1y} - \epsilon e^{i\omega t}a_2e^{-a_2y}$$

$$-\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = a_1 + \epsilon e^{i\omega t}a_2$$

6. Rate of Mass Transfer

The rate of mass transfer on the wall in terms of Sherwood number is given by:

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

$$\frac{\partial \phi}{\partial y} = -a_4(1 - a_3)e^{-a_4y} - a_1a_3e^{-a_1y} + \epsilon e^{i\omega t}(-a_5e^{-a_5y} - a_5a_6e^{-a_5y} + a_6a_2e^{-a_2y})$$

$$-\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = a_4 - a_3a_4 + a_1a_3 + \epsilon e^{i\omega t}(a_5 + a_6a_5 - a_6a_2)$$

7. Result and Discussion

The problem of a transient MHD Casson fluid flow past an inclined moving plate in the presence of chemical reaction, thermal radiation and Soret effect has been investigated and the analytical solution of dimensionless temperature, concentration and velocity has been presented in the previous section. The skin-friction, Nusselt number and Sherwood number are also obtained.

In order to get physical insight into the problem some numerical calculation has been carried out for the velocity field, temperature field, concentration field, skin friction at the plate, coefficient of the rate of heat and mass transfer from the plate to the fluid in terms of Nusselt number and Sherwood number at the plates by assigning some selected values to the parameters involved in the problem. Ignoring the imaginary part, numerical results have been displayed in figures and tables.

Figures 1-9 displays the velocity profile against y under the influence of thermal Grashof number Gr, solutal Grashof number Gm, chemical reaction parameter Kr, Prandtl number Pr, Hartmann number M, Schmidt number Sc, angle of inclination parameter α, Soret number Sr. In most of the cases of our investigation, the values of the parameter Sc is considered to be 0.60 which represent water vapour diffused in air, Pr is chosen to be 0.71 which corresponds to air at 25^0C and the plate is considered to be inclined at an angle of $\pi/6$ with the vertical axis. The thermal Grashof number is taken to be externally cool, the radiation parameter R is considered positive and all the other parameters are chosen arbitrarily positive.

Figures 1 and 2 displays the effect of Grashof number for mass transfer Gm and Grashof number for heat transfer Gr on the velocity profile with other parameters to be fixed.

Gm for mass transfer is defined as the ratio of buoyancy force due to concentration gradient to the viscous force. It is observed that as the Grashof number Gm increases velocity also increases. The peak value i.e the maximum value is more distinctive due to an increase in the species buoyancy force. The velocity in presence of Gm once attaining the maximum value decreases gradually to approach the free stream value.

The Grashof number for heat transfer Gr is the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As seen in the figure 2 velocity increases with the increase in Gr. It is also observed that as the Gr value increases, the peak values of the velocity increases near the porous plate and then decreases slowly to the free stream velocity.

Figure-3 displays that the velocity with the influence of chemical reaction parameter Kr decreases with the increase in parameter Kr. Figure-4 shows that velocity profile decreases with the increase in Prandtl number Pr. Hartmann number M is defined as the ratio of the electromagnetic forces to inertia forces. It is observed that in the figure-5 the velocity profile with the influence of Hartmann decreases as the M increases. The Schmidt number Sc compares the relative thickness of velocity and concentration boundary layers. The figure-6 depicts that as the Schmidt number increases the velocity profile decreases. The figure-7 shows that as the angle of inclination increases the velocity profile decreases. The thermo-diffusion or Soret effect Sr may take place due to the presence of temperature gradient.It is evident from the figure-8 that the velocity profile increases with the increase in Soret number and the figure-9 shows that as the radiation parameter increases the velocity profile decreases.

Figures 10-14 demonstrate the change in concentration profile with the influence of different parameters. It is observed that in the figure-10 and 11 the concentration decreases with the increase in Schmidt number and chemical reaction parameter respectively. The figure-12 depicts that the concentration increases with the increase in Soret number. Figure-13 and 14 shows that concentration increases with the increase in radiation parameter and Prandtl number respectively in the region near to the wall.

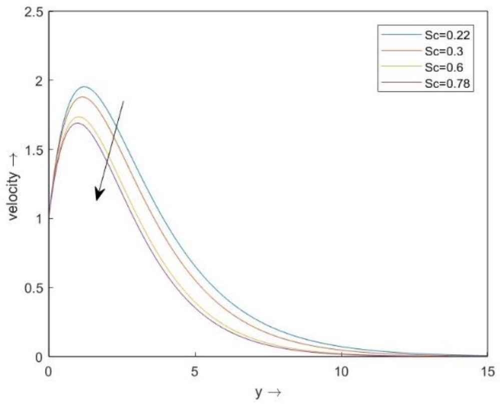

Figure 1: Schmidt number influence on velocity profile.

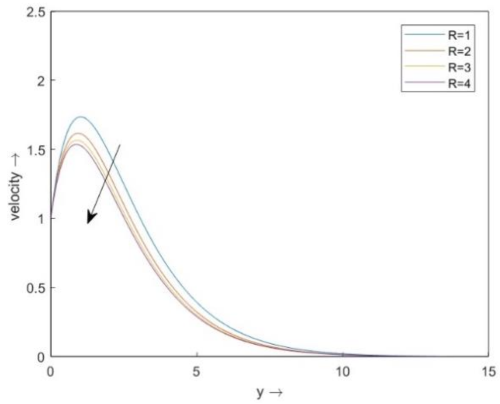

Figure 2: Radiation parameter influence on velocity profile.

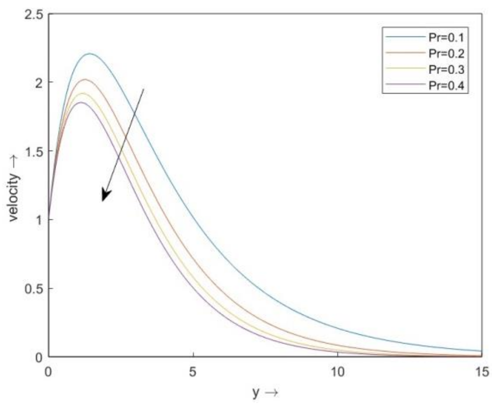

Figure 3: Prandtl number influence on velocity profile.

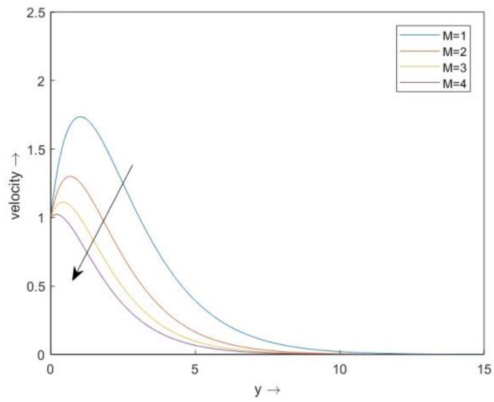

Figure 4: Hartmann number influence on velocity profile.

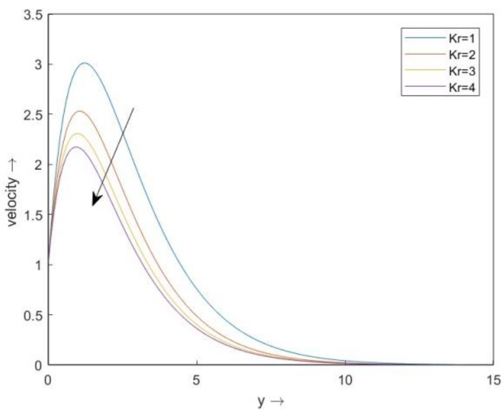

Figure 5: Chemical reaction parameter influence on velocity profile.

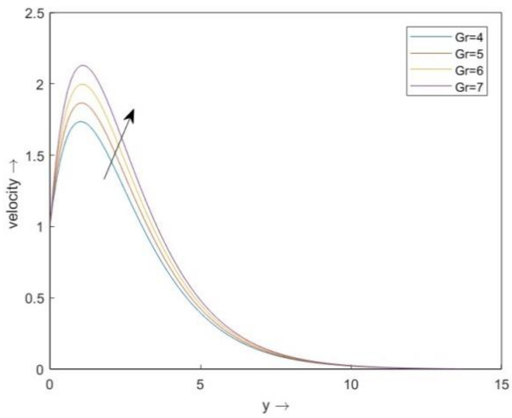

Figure 6: Grashof number for heat transfer influence on velocity profile.

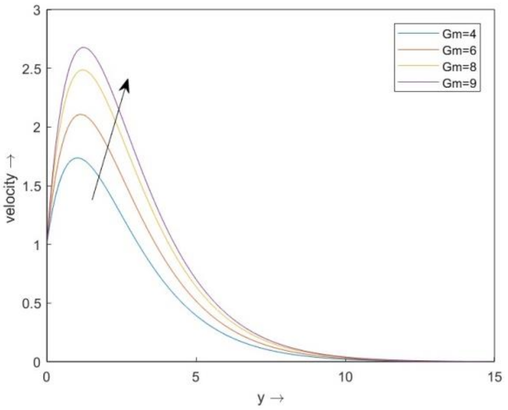

Figure 7: Grashof number for mass transfer influence on velocity profile.

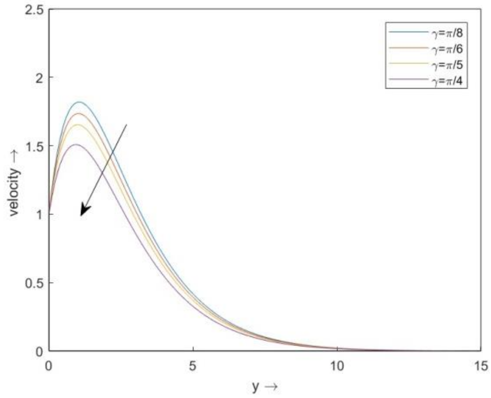

Figure 8: Angle of inclination parameter influence on velocity profile.

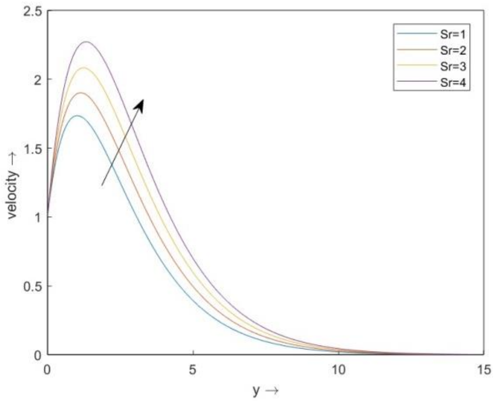

Figure 9: Soret number influence on velocity profile.

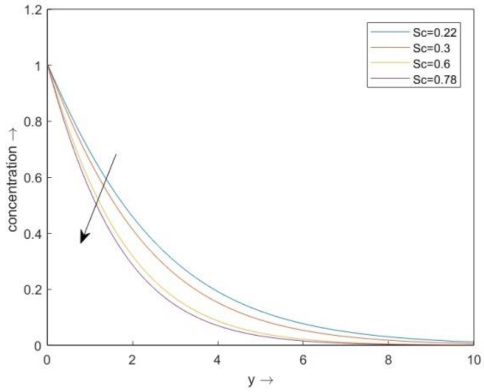

Figure 10: Schmidt number influence on concentration profile.

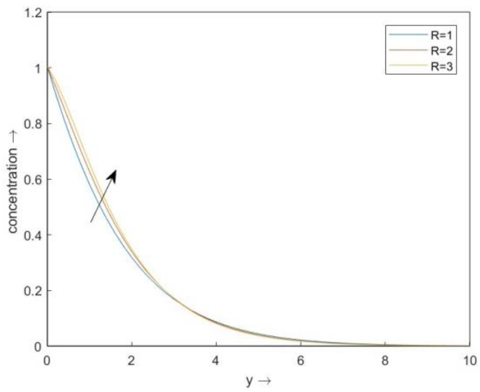

Figure 11: Radiation parameter influence on concentration profile.

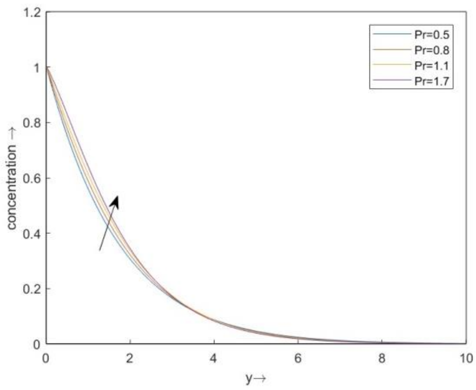

Figure 12: Prandtl number influence on concentration profile.

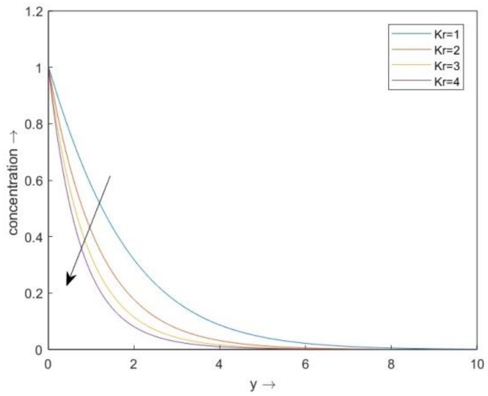

Figure 13: Chemical reaction parameter influence on concentration profile.

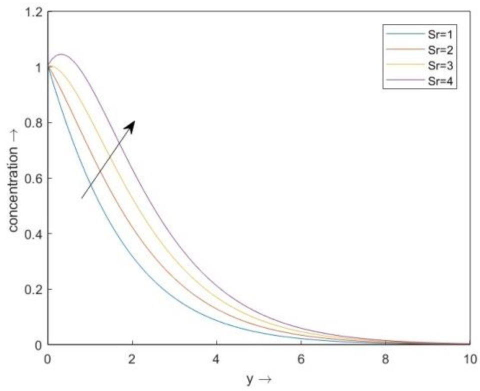

Figure 14: Soret number influence on concentration profile.

From Table.1(a), it is seen that Soret effect decreases the skin friction and from 1(b) and 1(c), it is observed that the Grashof number for mass transfer Gm and the Grashof number for heat transfer Gr increases the skin friction.

From Table.2(c), 2(a) and 2(b), it is observed that Nusselt number decreases with the increase in Soret effect and increases with the increase in radiation parameter and Prandtl number.

From Table.3(c), it is observed that Sherwood number decreases with the increase in Soret effect and from 3(a), 3(b) it is observed that it increases with the increase in radiation parameter and Prandtl number.

TABLE.1(a)

Sr	Gm	Gr	Skin friction
3	4	4	1.0849
4	4	4	0.9771
5	4	4	0.8655
6	4	4	0.7523

TABLE.1(b)

Sr	Gm	Gr	Skin friction
3	4	4	1.0849
3	5	4	1.4291
3	6	4	1.7732
3	7	4	2.1174

TABLE.1(c)

Sr	Gm	Gr	Skin friction
3	4	4	1.0849
3	4	5	1.1887
3	4	6	1.2926
3	4	7	1.3964

TABLE.2(a)

R	Pr	Sr	Nusselt number
1	0.71	3	0.7798
2	0.71	3	1.1748
3	0.71	3	1.4459
4	0.71	3	1.6729

TABLE.2(b)

R	Pr	Sr	Nusselt number
1	0.71	3	0.7798
1	1	3	0.9757
1	1.7	3	1.2888
1	2	3	1.4003

TABLE.3(a)

R	Pr	Sr	Sherwood number
1	0.71	3	0.8033
2	0.71	3	1.2535
3	0.71	3	1.7348
4	0.71	3	2.2510

TABLE.3(b)

R	Pr	Sr	Sherwood number
1	0.71	3	0.8033
1	1	3	1.0005
1	1.7	3	1.4952
1	2	3	1.7142

TABLE.3(c)

R	Pr	Sr	Sherwood number
1	0.71	3	0.8033
1	0.71	4	0.7950
1	0.71	5	0.7864
1	0.71	6	0.7777

8. Conclusion

Following are the conclusion of the present investigation:

- The fluid velocity lowers with the rise in reaction by chemical, Prandtl number, Hartmann number, Schmidt number, angle of inclination and radiation parameter.
- The fluid velocity is directly proportional with the rise in Soret number and Grashof number for heat and mass transfer.
- The concentration level of the fluid rises with the rise in Soret number.
- The concentration level lowers with the rise in Schmidt number, chemical reaction and parameter radiation and Prandtl number.
- The skin friction coefficient falls with the increase in Soret number, whereas it directly proportional with increase in Grashof number for heat and mass transfer.
- The rate of heat transfer boosts up with the rise in radiation parameter and Prandtl number, whereas it falls with the rise in Soret number.
- The rate of mass transfer is directly proportional with the increase in radiation parameter and Prandtl number, whereas it is inversely proportional with the increase in Soret number.

9. Nomenclature

υ = *kinematic viscosity*

α = *Casson parameter*

γ = *angle of inclination*

β = *thermal expansion coefficient*

β^*= *mass expansion coefficient*

g = *acceleration due to gravity*

C' = *species concentration*

C'_∞= *fluid concentration far away from the wall*

T' = *temperature of the fluid*

T'_∞ = *fluid temperature far away from the wall*

σ = *electrical conductivity*

ρ = *fluid density*

B_0 = *magnetic field*

C_p = *specific heat at constant pressure*

D_m = *mass diffusivity*

Kr' = *chemical reaction rate constant*

K_T = *thermal diffusion ratio*

T'_m= *mean fluid temperature*

q'_r = *radiative heat flux*

u = *dimensionless velocity*

θ = *nondimensional temperature*

ϕ = *nondimensional concentration*

Gr = *Grashof number for heat transfer*

Gm = *Grashof number for mass transfer*

Pr = *Prandtl number*

Sc = *Schmidt number*

M = *Hartmann number*

R = *radiation parameter*

Kr = *chemical reaction parameter*

Sr = *Soret number*

τ = *skin friction*

Nu = *Nusselt number*

Sh = *Sherwood number*

10. Appendix

$a_1 = \sqrt{R\,Pr}$,

$a_2 = \dfrac{Sr\,Sc\,R\,Pr}{kr\,Sc} - R\,Pr$,

$a_3 = \sqrt{Pr(R + i\omega)}$,

$a_4 = \sqrt{Kr\,Sc}$,

$a_5 = \sqrt{Sc(Kr + i\omega}$,

$a_6 = \frac{Sr\,Sc\,{a_3}^2}{{a_3}^2 - Sr(kr + i\omega)}$,

$a_7 = M\left(\frac{\alpha}{\alpha+1}\right)$,

$a_8 = \sqrt{a_7}$,

$a_9 = \left(\frac{\alpha}{\alpha+1}\right)\frac{G_1 + G_2 a_2}{{a_1}^2 - a_7}$,

$a_{10} = \left(\frac{\alpha}{\alpha+1}\right)\frac{G_1 - G_2 a_2}{{a_4}^2 - a_7}$,

$a_{11} = -G_1 + G_2 a_6$,

$a_{12} = G_2 + G_2 a_6$,

$a_{13} = \sqrt{\left(\frac{\alpha}{\alpha+1}\right)}(M + i\omega)$,

$a_{14} = \left(\frac{\alpha}{\alpha+1}\right)\frac{a_{11}}{{a_3}^2 - {a_{13}}^2}$,

$a_{15} = \left(\frac{\alpha}{\alpha+1}\right)\frac{a_{12}}{{a_5}^2 - {a_{13}}^2}$,

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