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BIANCHI TYPE-VIII INFLATIONARY UNIVERSE WITH FLAT POTENTIAL FOR PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

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Abstract: We have investigated Bianchi type-VIII inflationary universe with flat potential for perfect fluid distribution in general relativity. To obtain the deterministic solution of the model, we assume that the expansion θ is proportional to the shear σ, which leads to $B = A^n$ and potential $V(\phi)$ as constant. The behavior of the model from physical and geometrical aspects is also discussed.

Keywords: Bianchi VIII, Inflationary Universe, Perfect fluid, Cosmology.

1. Introduction

The inflationary scenario explains several mysteries of modern cosmology like homogeneity, isotropy, horizon problem and the flatness of the observed universe. Extremely rapid expansion of early universe by a factor of 10^{78} in volume driven by negative pressure vacuum energy density is termed as inflation. Guth's [12] introduced the concept of inflation and suggested that rapid expansion is due to false vacuum energy and after inflation, the universe is filled with bubbles. Various authors viz. Abbott and Wise [1], Burd and Barow [9], La and Steinhardt [16], Linde [17], Mijic et al. [18], Bassett et al. [8], Earman and Mosterin [11] have investigated inflationary cosmological models using homogeneous and isotropic space-time in different versions.

Bianchi type-VIII inflationary cosmological models play an important role for relativistic studies as these models permits not only expansion but also shear and rotation in general, these models are anisotropic. In recent years, many researchers have taken keen interest to study these models because well-known solution like Robertson Walker space-time, the de-Sitter space-time, the Taub-Nut space-time etc. are specific case of Bianchi type-VIII universe. Bianchi type-VIII cosmological models in different context have been studied by number of authors viz. Panov et al. [19], Kuvshinova et al. [15], Chhajed et al. [10].

Bali and Swati [7] have studied Bianchi type-VIII inflationary universe with massless scalar field in general relativity. Inflationary cosmological models in different context have been studied by number of authors viz. Henriques et al. [13], Adhav et al. [2,3], Sharma et al. [21], Jorwal et al. [14]. Reddy [20] has studied Bianchi Type-V inflationary universe in general relativity. Bali and Goyal [5] investigated inflationary scenario in Bianchi Type-V space-time with variable bulk viscosity and dark energy in radiation dominated phase. Bali and Kumari [4] have studied Chaotic inflation in spatially homogeneous Bianchi type-V space-time. Bali and Singh [6] have discussed Bianchi type-V inflationary universe with decaying vacuum energy (Λ).

Anisotropic cosmological models including the so called Bianchi cosmologies are of great theoretical importance. Bianchi Type-VIII cosmological models are the most general ever-expanding Bianchi cosmologies and are therefore of special interest.

Bianchi type space-time play a vital role in understanding and description of the early stages of evolution of the universe. In particular, the study of Bianchi type II, VIII, and IX universes are important because familiar solution like FRW universe with positive curvature. The de-Sitter universe, the Taub-Nut solutions etc. correspond Bianchi type II, VIII and IX space-times. Shri Ram and Singh [22] have studied Bianchi type-II, VIII and IX cosmological models with matter and electromagnetic fields.

Impelled by the above mentioned studies, we have investigated inflationary scenario in Bianchi type-VIII universe with flat potential for perfect fluid distribution in general relativity. For the complete solution of the field equations, we assume that expansion θ is proportional to the shear σ and V(ϕ) is constant. The physical and geometrical aspects of the model are also discussed.

2. The Metric and Field Equations

We consider Bianchi type-VIII line element in the form

$$ds^2 = dt^2 - B^2dx^2 - A^2dy^2 - \left(A^2\sinh^2 y + B^2\cosh^2 y\right)dz^2 - 2B^2\cosh y dx dz \tag{1}$$

where $A(t)$ and $B(t)$ are function of time t alone.

We assume the co-ordinates to be co-moving so that

$$v^1 = 0 = v^2 = v^3,\ v^4 = 1$$

In case of gravity minimally coupled to a scalar field V(ϕ), as given by Stein-Schabes [23], we have

$$S = \int\sqrt{-g}\left[R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right]\mathrm{d}^4X \tag{2}$$

The Einstein's field equation (in gravitational units $8\pi G = c = 1$), in the case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{3}$$

with

$$T_{ij} = (\rho + p) v_i v_j - p g_{ij} + \partial_i \phi \partial_j \phi - \left[\frac{1}{2} \partial_r \phi \partial^r \phi + V(\phi) \right] g_{ij} \tag{4}$$

Here ρ is the energy density, p the isotropic pressure, ϕ is Higgs field, V the potential.

The conservation relation leads to

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} \partial^\mu \phi \right) = -\frac{dV}{d\phi} \tag{5}$$

The Einstein's field equation (3) for the line-element (1) leads to non-linear differential equations as follows

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{1}{A^2} - \frac{3B^2}{4A^4} = -p - \frac{1}{2}\phi_4^2 - V(\phi) \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} = -p - \frac{1}{2}\phi_4^2 - V(\phi) \tag{7}$$

$$\frac{2A_4 B_4}{AB} + \frac{A_4^2}{A^2} - \frac{1}{A^2} - \frac{B^2}{4A^4} = \rho + \frac{1}{2}\phi_4^2 - V(\phi) \tag{8}$$

where the sub-indices *4* in *A*, *B* denotes differentiation with respect to time *t*.

From equation (5) for scalar field (ϕ) leads to

$$\phi_{44} + \left(\frac{2A_4}{A} + \frac{B_4}{B} \right) \phi_4 = -\frac{dV}{d\phi} \tag{9}$$

where suffix '4' indicates derivative with respect to time t.

3. Solution of Field Equations

The field equation (6) - (8) are system of three independent equation's with unknown parameters A, B, ρ, p, ϕ. To obtain the deterministic solution, we assume the following conditions:

(i) V(ϕ) is constant.

i.e. V(ϕ) = K (10)

(ii) Expansion θ is proportional to the shear σ, which leads to

$$B = A^n \tag{11}$$

From equations (9) and (10), we get

$$\phi_4 = \frac{L}{A^2 B} \tag{12}$$

where L is constant of integration.

The scale factor R for line-element (1) is given by

$$R^3 = A^2 B = A^{n+2} \tag{13}$$

Equations (6) and (7) together lead to

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} - \frac{A_4 B_4}{AB} + \frac{A_4^2}{A^2} - \frac{1}{A^2} - \frac{B^2}{A^4} = 0 \tag{14}$$

From equations (11) and (14), we obtain

$$A_{44} + (n+1)\frac{A_4^2}{A} = \frac{1}{1-n}\left[\frac{1}{A} + \frac{1}{A^{3-2n}}\right], \quad n \neq 1 \tag{15}$$

Now let us consider $A_4 = f(A)$ then $A_{44} = ff'$, where $f' = \frac{df}{dA}$.

Equation (15) leads to

$$\frac{df^2}{dA} + \frac{2(n+1)}{A} f^2 = \frac{2}{1-n}\left[\frac{1}{A} + \frac{1}{A^{3-2n}}\right] \tag{16}$$

After integration (16) leads to

$$f^2 = \frac{a}{A^{2(1-n)}} + b + \frac{M}{A^{2(n+1)}} \tag{17}$$

where M is the integrating constant and $a = \frac{1}{2n(1-n)}$, $b = \frac{1}{(1-n^2)}$

Equation (17) can be rewrite as

$$\frac{dA}{dt} = \sqrt{\frac{a}{A^{2(1-n)}} + b + \frac{M}{A^{2(n+1)}}} \tag{18}$$

Equation (18) leads to

$$\int \frac{dA}{\sqrt{\frac{a}{A^{2(1-n)}} + b + \frac{M}{A^{2(n+1)}}}} = \int dt + N = t + N \tag{19}$$

where N is constant of integration and value of A can be obtained from (19).

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = \left(\frac{1}{aT^{2(n-1)} + b + MT^{-2(n+1)}}\right) dT^2 - T^{2n} dX^2 - T^2 dY^2 - \left(T^2 \sinh^2 Y + T^{2n} \cosh^2 Y\right) dZ^2 - 2T^{2n} \cosh Y dX\, dZ \tag{20}$$

where $x = X$, $y = Y$, $z = Z$ and $A = T$

4. Special Model

For $n = \frac{1}{2}$ **and** $M = 0$

When, we put $n = \frac{1}{2}$ and $M = 0$ in equation (17) it leads to

$$f^2 = \frac{2}{A} + \frac{4}{3} \tag{21}$$

Equation (21) leads to

$$\int \frac{dA}{\sqrt{\frac{2}{A} + \frac{4}{3}}} = \int dt + Q = t + Q \tag{22}$$

where Q is integrating constant.

Let $\xi^2 = A + \frac{3}{2}$, equation (22) becomes

$$\int\left(\sqrt{\xi^2-\frac{3}{2}}\right)d\xi=\frac{1}{\sqrt{3}}(t+Q) \tag{23}$$

From equation (23), we get

$$\frac{\xi}{2}\sqrt{\xi^2-\frac{3}{2}}-\frac{3}{4}\log_e\left|\xi+\sqrt{\xi^2-\frac{3}{2}}\right|=\frac{1}{\sqrt{3}}(t+Q) \tag{24}$$

Equation (24) leads to

$$\frac{\sqrt{3}}{2}\left(\sqrt{A+\frac{3}{2}}\right)\left(\sqrt{A}\right)-\frac{3\sqrt{3}}{4}\log_e\left|\sqrt{A}+\sqrt{A+\frac{3}{2}}\right|=t+Q \tag{25}$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2=\left(\frac{3\tau}{4\tau+6}\right)d\tau^2-\tau dX^2-\tau^2 dY^2-\left(\tau^2\sinh^2 Y+\tau\cosh^2 Y\right)dZ^2$$
$$-2\tau\cosh Y dX\, dZ \tag{26}$$

where $x=X$, $y=Y$, $z=Z$ and $A=\tau$

5. Physical and Geometrical Aspects

For the model (20), the rate of Higgs field

$$\phi=L\int\frac{1}{\sqrt{aT^{4n+2}+bT^{2(n+2)}+MT^2}}dT+P \tag{27}$$

where P is constant of integration.

For the model (20), pressure (p), density (ρ), the spatial volume (R^3), the expansion (θ), shear (σ), decelerating parameter (q), directional Hubble parameters $(H_x,\ H_y,\ H_z)$, Hubble parameter (H) and the anisotropic parameter of the expansion Δ are given by

$$p=-K+\left[\frac{3}{4}-a(2n-1)\right]\left(\frac{1}{T^{4-2n}}\right)+(1-b)\left(\frac{1}{T^2}\right)-\left[\frac{1}{2}L^2+M(2n+3)\right]\left(\frac{1}{T^{2n+4}}\right) \tag{28}$$

$$\rho=K+\left[a(2n+1)-\frac{1}{4}\right]\left(\frac{1}{T^{4-2n}}\right)+\left[b(2n+1)-1\right]\left(\frac{1}{T^2}\right)+\left[M(2n+1)-\frac{1}{2}L^2\right]\left(\frac{1}{T^{2n+4}}\right) \tag{29}$$

$$R^3 = T^{n+2} \tag{30}$$

$$\theta = (n+2)\sqrt{\frac{a}{T^{4-2n}} + \frac{b}{T^2} + \frac{M}{T^{2n+4}}} \tag{31}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\sqrt{\frac{a}{T^{4-2n}} + \frac{b}{T^2} + \frac{M}{T^{2n+4}}} \tag{32}$$

$$q = -1 + \frac{3}{2(n+2)}\left\{\frac{\frac{a(4-2n)}{T^{4-2n}} + \frac{2b}{T^2} + \frac{M(2n+4)}{T^{2n+4}}}{\frac{a}{T^{4-2n}} + \frac{b}{T^2} + \frac{M}{T^{2n+4}}}\right\} \tag{33}$$

$$H_x = H_y = \sqrt{\frac{a}{T^{4-2n}} + \frac{b}{T^2} + \frac{M}{T^{2n+4}}} \tag{34}$$

and

$$H_z = n\sqrt{\frac{a}{T^{4-2n}} + \frac{b}{T^2} + \frac{M}{T^{2n+4}}} \tag{35}$$

$$H = \left(\frac{n+2}{3}\right)\sqrt{\frac{a}{T^{4-2n}} + \frac{b}{T^2} + \frac{M}{T^{2n+4}}} \tag{36}$$

$$\Delta = \frac{2(n-1)^2}{(n+2)^2} \tag{37}$$

From equations (31) and (32), we get

$$\frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(n+2)} = \text{constant}, \quad (n \neq -2) \tag{38}$$

For the special model (26), the rate of Higgs field

$$\phi = L\int \frac{1}{\sqrt{2\tau^4 + \frac{4}{3}\tau^5}}\, d\tau + U \tag{39}$$

where U is constant of integration.

For the special model (26), pressure (p), density (ρ), the spatial volume (R^3), the expansion (θ), shear (σ), decelerating parameter (q), directional Hubble parameters $(H_x,\ H_y,\ H_z)$, Hubble parameter (H) and the anisotropic parameter of the expansion Δ are given by

$$p = -K + \frac{3}{4\tau^3} - \frac{1}{3\tau^2} - \frac{L^2}{2\tau^5} \tag{40}$$

$$\rho = K + \frac{15}{4\tau^3} + \frac{5}{3\tau^2} - \frac{L^2}{2\tau^5} \tag{41}$$

$$R^3 = \tau^{\frac{5}{2}} \tag{42}$$

$$\theta = 5\sqrt{\frac{1}{2\tau^3} + \frac{1}{3\tau^2}} \tag{43}$$

$$\sigma = -\frac{1}{\sqrt{3}}\sqrt{\frac{1}{2\tau^3} + \frac{1}{3\tau^2}} \tag{44}$$

$$q = -1 + \frac{3}{5}\left\{\frac{\frac{3}{\tau^3} + \frac{4}{3\tau^2}}{\frac{1}{\tau^3} + \frac{2}{3\tau^2}}\right\} \tag{45}$$

$$H_x = H_y = \sqrt{\frac{2}{\tau^3} + \frac{4}{3\tau^2}} \tag{46}$$

and

$$H_z = \frac{1}{2}\sqrt{\frac{2}{\tau^3} + \frac{4}{3\tau^2}} \tag{47}$$

$$H = \left(\frac{5}{3}\right)\sqrt{\frac{1}{2\tau^3} + \frac{1}{3\tau^2}} \tag{48}$$

$$\Delta = \frac{2}{25} \tag{49}$$

From equations (43) and (44), we get

$$\left|\frac{\sigma}{\theta}\right| = \frac{1}{5\sqrt{3}} = \text{constant} \tag{50}$$

6. Conclusion

The model (20) starts expanding with big-bang at $T = 0$. The expansion decreases as time increases for $-2 < n < 2$ and approaches to zero as $T \to \infty$.

The Spatial Volume increases as time increases for $n > -2$. It represents inflationary scenario of universe containing massless scalar field with flat potential.

The Hubble parameter decreases as time increases for $-2 < n < 2$. The energy density and pressure of the model is initially large.

When $n = 2$, $T \to \infty$ then deceleration parameter (q) tends to -1, so the model represent accelerating phase of the universe.

The rate of Higgs field is initially large, but decreases as time increases for $n > -2$ and constant for $T \to \infty$. Since $\lim_{T \to \infty} \frac{\sigma}{\theta} \neq 0$, the model represents anisotropic space-time in general. The model has point type singularity at $T = 0$.

For the special model (26), expansion decreases as time increases and approaches to zero as $\tau \to \infty$. Spatial Volume is increasing function of time τ. We also observe that pressure (p), density (ρ), Hubble parameter (H) are decreasing function of τ and approaches to zero as $\tau \to \infty$. When $n = \frac{1}{2}$, $\tau \to \infty$ then deceleration parameter (q) tends to 0.2, so the model represent decelerating phase of the universe. The rate of Higgs field is initially large, but decreases as time increases and constant for $\tau \to \infty$. Since $\lim_{\tau \to \infty} \frac{\sigma}{\theta} \neq 0$, the model represents anisotropic space-time in general. The model has point type singularity at $\tau = 0$.

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