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COSMOLOGICAL MODELS IN 5-DIMENSIONAL SPACE-TIME FOR EQUATION OF STATE DISTRIBUTION WITH TIME VARYING GRAVITATION CONSTANT IN CREATION FIELD COSMOLOGY

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Abstract: Cosmological models in 5-dimensional space-time are investigated for equation of state distribution with time varying gravitation constant in creation field cosmology introduced by Hoyle and Narlikar (1964). The variation in G is assumed as $G = R^n$ where R is scale factor and n is constant. We find that the universe has a singular state at t = 0 and matter density is uniform throughout even both R and A (metric potentials) approach infinity which is consistent with steady state theory where matter creation via creation field maintains the balance. The creation field matches with the result as given in H-N theory (1964). The deceleration parameter q > 0 represents decelerating universe. The formation of structure is better explained by decelerating universe. Thus the model is asymptotically relevant. The other particular cases are also discussed.

Keywords: C-field, cosmology, 5-dimensional space time, barotropic fluid.

1. Introduction

Gravitation plays a significant role on the large scale due to the short range of strong and weak forces and electromagnetic forces become weak due to global neutrality of matter as investigated by Dicke and Peebles.[9] On the basis of large scale hypothesis, Dirac [10] introduced a theory with variable gravitational constant. Demarque et al. [8] considered an ansatz $G\ \alpha\ \bar{t}^n$ with $|n| < 0.1$. Barrow [4] considered $G\ \alpha\ \bar{t}^4$ with helium abundances with $-5.9 \times 10^{-3} < n < 7 \times 10^{-3}$ and $\left|\frac{\dot{G}}{G}\right| < (1.5 \pm 0.7) \times 10^{-12} yr^{-1}$ for flat universe where $\dot{G} = \frac{\partial G}{\partial t}$. Therefore, after native theories of gravity, the other theories were proposed to generalize Einstein's General theory of relativity by assuming variable G and satisfying conservation equation as investigated by Brans and Dicke. [6] The other alternative theory of Steady state theory was proposed by Bondi and Gold. [5] In this theory, the universe does not have any singular beginning nor an end on the cosmic time scale. To maintain the constantancy of mass density, they considered a very slow but continuous creation of matter in contrast to explosive creation at $t = 0$ of standard model (FRW model). But it suffered a serious disqualification for not giving any justified reason of continuous creation of matter and principle of conservation of energy is sacrificed in this formation. To overcome this difficulty, Hoyle and Narlikar [11] adopted a field theoretic approach considering a massless and chargeless Scalar field C in Einstein-Hilbert action for creation of matter. In C-field theory, there is no big bang type of singularity as in steady state theory of Bondi and Gold. Creation theory has solved the problems of horizon, singularity and flatness. Narlikar [13] has investigated that matter creation is accomplished at the expense of negative energy C-field. Creation field cosmological model for radiation of matter is investigated by Narlikar and Padmanabhan. [14]

An important achievement of Hoyle-Narlikar [11] creation field cosmology is that it allows the possibility of an ever existing expanding universe with constant density of matter. The constancy of density is possible due to occurrence of an appropriate creation of C-field with negative energy. Therefore, it is reasonable to study the implication of Hoyle-Narlikar theory in higher dimensional set up which admits the presence of ever existing expanding universe with constant density and implication of time variation of G. The cosmological models in higher dimensional theories play a significant role in the study because these models have been very helpful in attempts to unify the gravity and other forces in nature as pointed out by Chatterjee and Bhui. [7] The other relevant works by Appelquist and Chodos [1], Randjbar-Daemi et al. [15], Singh et al. [16], Mohanty and Sahoo [12] give detailed informations for cosmological models in the frame work of 5-dimensional space-time. In earlier paper, Bali and Kumawat [3] have studied creation field cosmology with variable gravitational constant in 5-dimensional space-time for dust distribution. Recently Bali [2] has investigated creation field cosmological models for dust distribution and cosmological term.

In the present study, we have extended the pioneer work of creation field cosmology by Hoyle and Narlikar (1964) in the frame work of five dimensional space-time and investigated C-field cosmological model with time varying G for equation of state distribution. The physical aspects of the model are discussed. The special cases for dust distribution, stiff fluid and radiation dominated universe are also discussed and comparison is also mentioned.

2. Metric and Field Equations

We consider a spatially flat 5-dimensional homogeneous cosmological model as

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2] - A^2(t) dy^2 \tag{1}$$

where R(t) is the scale factor and A(t) that for the extra dimension.

Einstein's modified field equations by the introduction of C-field are given by

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G [{}^m T_i^j + {}^C T_i^j] \tag{2}$$

where

$$^{m}T_i^j = (\rho + p)\, v_i v^j - p\, g_i^j \tag{3}$$

$$^{C}T_i^j = -f\left(C_i C^j - \frac{1}{2} g_i^j C^{\alpha} C_{\alpha}\right) \tag{4}$$

where f > 0 is a constant between matter and creation field and $C_i = \frac{dC}{dx^i}$. The field equation (2) for metric (1) leads to

$$\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}}{R}\frac{\dot{A}}{A} = 8\pi G\left[\rho - \frac{1}{2} f\, \dot{C}^2\right] \tag{5}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{2\dot{R}}{R}\frac{\dot{A}}{A} + \frac{\ddot{A}}{A} = 8\pi G\left[\frac{1}{2} f\dot{C}^2 - p\right] \tag{6}$$

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = 8\pi G\left[\frac{1}{2} f\dot{C}^2 - p\right] \tag{7}$$

where p being the isotropic pressure, ρ the matter density, G the gravitational constant.

3. Solution of Field Equations

The conservation equation

$$[G\, T_i^j]_{;j} = 0 \tag{8}$$

leads to

$$\dot{G}\left(\rho - \frac{1}{2} f\, \dot{C}^2\right) + 8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + 3\rho\frac{\dot{R}}{R} + \rho\frac{\dot{A}}{A} - 3f\dot{C}^2\frac{\dot{R}}{R} - f\dot{C}^2\frac{\dot{A}}{A} + 3p\frac{\dot{R}}{R} + p\frac{\dot{A}}{A}\right] = 0 \tag{9}$$

which yields $\dot{C} = 1$ when used in source equation.

Using $\dot{C} = 1$ in equation (5), we get

$$8\pi\rho G = \frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}}{R}\frac{\dot{A}}{A} + 4\pi G f \tag{10}$$

After using barotropic fluid condition $p = \gamma\rho\,(0 \le \gamma \le 1)$, p is the isotropic pressure and ρ the matter density and $\dot{C} = 1$ in equation (7), we have

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = 4\pi Gf - 8\pi G\gamma\rho \tag{11}$$

Using the value of $8\pi G\gamma\rho$ from equations (10) in (11), we have

$$\frac{3\ddot{R}}{R} + \frac{3\dot{R}^2}{R^2} = 4\pi Gf - \gamma\left[\frac{3\dot{R}^2}{R^2} + \frac{3\dot{R}}{R}\frac{\dot{A}}{A} + 4\pi Gf\right] \tag{12}$$

To get the deterministic solution is terms of cosmic time t, we assume

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R} \tag{13}$$

Equation (12) leads to

$$\frac{\ddot{R}}{R} + (1+2\gamma)\frac{\dot{R}^2}{R^2} = \frac{4\pi Gf}{3}(1-\gamma) \tag{14}$$

For the variation of G, we assume

$$G = R^n \tag{15}$$

where n is a constant and R the scale factor.

Equations (14) and (15) lead to

$$\ddot{R} + (1+2\gamma)\frac{\dot{R}^2}{R} = \frac{4\pi f}{3}(1-\gamma)R^{n+1} \tag{16}$$

To get the solution of equation (16), we assume $\dot{R} = F(R).$ This leads to $\ddot{R} = FF'$ with $F' = \frac{dF}{dR}.$ Thus equation (16) leads to

$$\frac{dF^2}{dR} + \frac{2(1+2\gamma)}{R}F^2 = \frac{8\pi f(1-\gamma)}{3}R^{n+1} \tag{17}$$

which leads to

$$F^2 = \frac{R^{n+2}}{n+4\gamma+4} + \frac{L}{R^{4\gamma+2}} \tag{18}$$

where $K = \dfrac{8\pi f(1-\gamma)}{3}$ and L is constant of integration.

From equation (18), we have

$$\frac{R^{1+2\gamma}dR}{\sqrt{R^{n+4\gamma+4} + \dfrac{L(n+4\gamma+4)}{K}}} = \sqrt{\frac{K}{n+4\gamma+4}}\, dt \tag{19}$$

To obtain the value of R in terms of cosmic time t, we assume that

$$n = -(2\gamma+2) \tag{20}$$

Using condition (20) in (19), we have

$$\frac{R^{1+2\gamma}dR}{\sqrt{R^{2+2\gamma} + \dfrac{L(2\gamma+2)}{K}}} = \sqrt{\frac{K}{2\gamma+2}}\, dt \tag{21}$$

Equation (21) leads to

$$R^{2\gamma+2} = [(at+b)^2 - m] \tag{22}$$

Thus, $R = [(at+b)^2 - m]$ (23)

where

$$a = \sqrt{\frac{K(\gamma+1)}{2}} \tag{24}$$

$$m = \frac{L(2\gamma+2)}{K} \tag{25}$$

and b is a constant of integration.

Thus, we have

$$G = R^n = R^{-(2\gamma+2)} = [(at+b)^2 - m]^{-1} \tag{26}$$

From equations (10), (13),(22) and (26), we have

$$8\pi\rho = \frac{6a^2(at+b)^2}{(\gamma+1)^2[(at+b)^2 - m]} + 4\pi f \tag{27}$$

After using the value of R given by (22), the metric (1) leads to

$$ds^2 = dt^2 - [(at+b)^2 - m]^{\frac{2}{(\gamma+1)}}[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2] - [(at+b)^2 - m]^{\frac{2}{(\gamma+1)}} dy^2 \quad (28)$$

Now equation (9) leads to

$$\rho\dot{G} - \frac{1}{2}\dot{G}f\dot{C}^2 + G\dot{\rho} - fG\dot{C}\ddot{C} + 4G\rho\frac{\dot{R}}{R}(1+\gamma) - 4Gf\dot{C}^2\frac{\dot{R}}{R} = 0 \quad (29)$$

Using equations (22), (26) and (27) in equation (29), we get

$$\dot{C}^2\, t^{\frac{2(\gamma-3)}{(\gamma+1)}} = \int \left[\frac{-24}{8\pi f(\gamma+1)^2} - 2\right] t^{\frac{\gamma-7}{\gamma+1}} dt \quad (30)$$

where we have set the constants a = 1, b = 0, m = 0 to get deterministic value of C. Now (30) leads to

$$\dot{C}^2 = \left[\frac{-24}{8\pi f(\gamma+1)^2} - 2\right]\left(\frac{\gamma+1}{2\gamma-6}\right) \quad (31)$$

where we have used $a = 1, a = \sqrt{\frac{K(\gamma+1)}{2}}$ and $K = \frac{8\pi f(1-\gamma)}{3}$ in equation (31), we have

$$\dot{C}^2 = 1 \quad (32)$$

which leads to

$$C = t \quad (33)$$

The metric (1) for a = 1, b = 0 leads to

$$ds^2 = dt^2 - t^{2/(\gamma+1)}[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2] - t^{2/(\gamma+1)} dy^2 \quad (34)$$

4. Physical Aspects

The homogeneous mass density ρ, gravitational constant G, scale factor $\bar{a}(t)$ and deceleration parameter (q) for the model (28) are given by

$$8\pi\rho = \frac{6a^2(at+b)^2}{(\gamma+1)^2[(at+b)^2 - m]} + 4\pi f \quad (35)$$

$$G = [(at+b)^2 - m]^{-1} \quad (36)$$

$$\bar{a}(t) = [(at+b)^2 - m]^{\frac{1}{(\gamma+1)}} \quad (37)$$

$$q = \gamma + m\frac{\gamma+1}{2} > 0 \tag{38}$$

Setting a = 1, b = 0, m = 0, we have

$$8\pi\rho = \frac{6}{(\gamma+1)^2} + 4\pi f \tag{39}$$

$$G = \frac{1}{t^2} \tag{40}$$

$$\bar{a}(t) = t^{\frac{2}{\gamma+1}} \tag{41}$$

$$q = \gamma \tag{42}$$

Particular Cases:

(i) For γ = 0 (Dust distribution), equations (39), (40), (41) and (42) leads to

$$8\pi\rho = 6 + 4\pi f \tag{43}$$

$$G = \frac{1}{t^2} \tag{44}$$

$$\bar{a}(t) = t^2 \tag{45}$$

$$q = 0 \tag{46}$$

(ii) For γ = 1 (Stiff fluid distribution)

$$8\pi\rho = \frac{3}{2} + 4\pi f \tag{47}$$

$$\bar{a}(t) = t \tag{48}$$

$$q = 1 \tag{49}$$

(iii) For γ = 1/3 (Radiation dominated)

$$8\pi\rho = \frac{27}{8} + 4\pi f \tag{50}$$

$$\bar{a}(t) = \frac{3}{2} + 4\pi f \tag{51}$$

$$q = \frac{1}{3} \tag{52}$$

5. Conclusion

In presence of creation field (C), the matter density is positive and in particular cases, it is uniform. The scale factor increases with time which shows that the model represents expanding universe. These results agree with Hoyle-Narlikar theory (1964). The constancy of density is due to the presence of an appropriate creation of C-field with negative energy field. The creation field increases with time which is in agreement with Hoyle and Narlikar theory. The gravitational constant (G) decreases with time and for large t, it tends to zero. The mass density is constant even though scale factor R and A approach infinity which is consistent with steady state theory where matter creation via the creation field maintains the balance. For dust model i.e. for $\gamma = 0$, our model reduces to the model obtained by Bali and Meghna (2013). In the model (34), the density parameter $\rho_c = \frac{3H^2}{8\pi G}$ is constant and is less than 1 which shows that universe is open and will continue to expand for ever. The deceleration parameter $q > 0$ indicates that the model (34) represents decelerating universe in all the cases. The deceleration parameter $q > 0$ shows that the model (34) represents decelerating universe. The decelerating expansion provides obvious provision for the formation of large structure in the universe. The formation of structures in the universe is better supported by decelerating expansion. Thus the model is asymptotically relevant as mentioned by Bali[16].

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