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ON CR-STRUCTURE AND F(2ν+5,1)-STRUCTURE SATISFYING F^{2V+5}+F=0

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Abstract: The purpose of this paper is to show a relationship between CR-structures and F (2ν + 5, 1)-structure satisfying $F^{2\nu+5} + F = 0$

Keywords: CR-submanifolds, CR-structure, Nijenhuis Tensor.

1. Introduction

CR-submanifolds of a kahlerian manifold have been defined by Bejancu [1], and are now being studied by various authors viz. Blair and Chen [2], Yano and Kon [9]. The theory of F-structure was developed by Yano [10], Yano and Ishihara [11], Das [3], Goldberg [6], and (k+1) structure was developed by Das and Nivas [4].

Let F be a non-zero tensor field of the type (1, 1) and of class C ∞on an

n-dimensional manifold M (Nikkie [7]).

$$F(2\nu+5) + F = 0, \tag{1}$$

The rank of (F) = r =constant.

Let us define the operators on M as follows [7]

$$l = -F\,2\nu+4\,,$$
$$m = I + F\,2\nu+4. \tag{2}$$

Where I denotes the identity operator. We state the following two theorems[7]

Theorem 1.1. Let M be an F (2ν + 5, 1)-structure manifold satisfying(1), then

$$l + m = I$$
$$l^2 = l\,, m^2 = m \tag{3}$$
$$\text{And } lm = ml = 0$$

Thus for (1, 1) tensor field F(≠ 0) satisfying (1), there exist complementary distributions Dl and Dm corresponding to the projection operators l and m respectively. Then, dim Dl = r and dim Dm = (n − r).

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Theorem 1.2. We have,

a- $lF = Fl\ , mF = Fm = 0$

b- $F^{2\nu+4}m = 0$ (4)

c- $F^{2\nu+4}l = -1$

Thus F ν +2 acts on Dl as an almost complex structure and on Dm as a null operator.

2. Nijenhuis Tensor

The Nijenhuis tensor N (X, Y) of F satisfying (1) in M is expressed as follows for every vector field X, Y on M .

$$N(X, Y) = [F X, F Y] - F [F X, Y] - F [X, F Y] + F\ 2\ [X, Y] \quad (5)$$

Definition 2.1. If X , Y are two vector fields in M , then their Lie bracket [X, Y] is defined by

$$[X, Y] = X Y - Y X \quad (6)$$

3. CR-Structure

Let M be a differentiable manifold and TcM be its complexified tangent bundle. A CR-structure on M is a complex subbundle H of TcM such that HP ∩ H¯ p = 0 and H is involutive i.e. for complex vector fields X and Y in H, [X, Y] is in H. In this case, we say M is a CR-manifold. [5,8]

Let F -structure given by equation (1) be an integrable structure of rank r = 2$\underline{m}$ on M. We define complex sub bundle H of TcM by

HP = {X − $\sqrt{-1}$F X, X ∈ χ(Dl)}, where χ(Dl) is the F (Dm) module

of all differentiable sections of Dl then Re(H) = Dl and HP ∩ H¯ p = 0,

where H¯ p denotes the complex conjugate of HP .

Theorem 3.1. If P and Q are two elements of H then the following relations holds

$$[P, Q] = [X, Y] - [FX, FY] - \sqrt{-1}([X, FY] + [FX, Y]) \quad (7)$$

Proof. Let us define $P = X - \sqrt{-1}FX$ and Q=$Y - \sqrt{-1}FY$, then by direct calculation and on simplifying, we obtain

$$[P, Q] = \left[X - \sqrt{-1}FX, Y - \sqrt{-1}FY\right] = [X, Y] - [FX, FY] - \sqrt{-1}([X, FY] + [FX, Y])$$

Theorem 3.2. If F (2ν + 5, 1)-structure satisfying equation (1) is integrable then

we have $$-F\ 2\nu + 3\ ([F X, F Y] + F\ 2\ [X, Y]) = l([F X, Y] + [X, F Y]). \quad (8)$$

Proof. From equation (5), we have

N (X, Y) = [F X, F Y] − F [F X, Y] − F [X, F Y] + F 2[X, Y].

Since N (X, Y) = 0, we obtain

$$[F X, F Y] + F 2 [X, Y] = F ([F X, Y] + [X, F Y]. \quad (9)$$

Operating (3.3) by (−F 2ν +3), we get

(−F 2ν +3)([F X, F Y] + F 2[X, Y]) = (−F 2ν +4)([F X, Y] + [X, F Y])

In view of equation (2) in the above equation, we obtain (8), which proves the theorem.

Theorem 3.3. The following identities hold

$$mN (X, Y) = m[F X, F Y]. \quad (10)$$

$$mN (F 2\nu +3 X, Y)$$
$$= m[F 2\nu +4 X, F Y]. \quad (11)$$

Proof. The proof of equations (10) and (11) follows easily by virtue of theorems 1.1, 1.2 and equation (5).

Theorem 3.4. For any two vector fields X and Y, the following conditions are equivalent

a. mN (X, Y) = 0,

b. m[F X, F Y] = 0,

c. mN (F 2ν +3 X, Y) = 0, (12)

d. m[F 2ν +4 X, F Y] =0,

e. m[F 2ν +4 lX, F Y] = 0.

Proof. In consequence of equations (1), (2), (5) and theorems 1.2, 3.3, the above identities can be proved to be equivalent.

Theorem 3.5. If F ν +2 acts on Dl as an almost complex structure, then

$$m[F \nu +2\ lX, F Y] = m[\sqrt{-1}X, F Y] = 0. \quad (13)$$

Proof. In view of equations (4) we see that F ν +2 acts on D_1 as an almost complex structure then equation (12) follows in an obvious manner. To show that m[F ν +2 lX, F Y] = 0, we use the definition 2.1 and in view of equation (4), the result follows directly.

Theorem 3.6. For X, Y ∈ χ(Dl), we have

$$l([X, F Y] + [F X, Y]) = [X, F Y] + [F X, Y] \quad (14)$$

Proof. Since [X, F Y] and [F X, Y] ∈ χ(Dl), on making use of (4) and definition 2.1 we obtain the result.

Theorem 3.7. The integrable F (2ν + 5, 1)-structure satisfying (1)

on M defines a CR-structure H on it such that ReH = Dl .

Proof. In view of the fact that [X, F Y] and [F X, Y] $\in \chi$ (Dl) and on using equations (7), (8) and theorem 3.6, we have [P, Q] $\in \chi$(Dl). Then F (2ν + 5, 1)-structure satisfying (1) on M defines a CR-structure.

Definition 3.8. Let $\widetilde{K}$ be the complementary distribution of $R_e(H)$ to TM. We define a morphism of vector bundles $F: TM \to TM$ given by $F(X) = 0 \ \forall \ X \in \chi F(\widetilde{K})$ such that

$F(X) = \frac{1}{2}\sqrt{-1}(P - \overline{P})$ where $P = X + \sqrt{-1}Y$, $Y \in X(H_P)$ and $\bar{P}$ is complex conjugate of P.

Corollary3.9.[3] If $P = X + \sqrt{-1}Y \ and \ \bar{P} = X - \sqrt{-1}Y$ belong to H_P and

$F(X) = \frac{1}{2}\sqrt{-1}(P - \overline{P})$, $F(Y) = \frac{1}{2}\sqrt{-1}(P + \overline{P})$

And$F(-Y) = -\frac{1}{2}(P + \overline{P})$ then $F(X) = -Y$, $F^2(X) = -X$ and $F(-Y) = -X$

Theorem 3.10. If M has a CR-structure H, then we have F 2ν +5 +F =0 and consequently F (2ν + 5, 1)-structure satisfying (1) is defined on M such that the distributions Dl and Dm coincide with Re(H) and Ke respectively.

Proof .Suppose M has a CR-structure. Then in view of definition 3.8

and corollary 3.9, we have

$$F (X) = -Y. \tag{15}$$

Operating (15) by F 2K we get

$$F\ 2\nu\ (F (X) = F\ 2\nu\ (-Y) \tag{16}$$

We can write the right hand side of (16) as follows

$$F\ 2\nu +1\ (X) = F\ 2\nu -1\ (F\ (-Y) \tag{17}$$

On making use of corollary 3.9, the above equation becomes

$$F\ 2\nu +1\ (X) = F\ 2\nu -1\ (-X)$$

$$= -F\ 2\nu -1\ (X)\ , \tag{18}$$

which can be written as

$$F\ 2\nu +1\ (X) = -F\ 2\nu -2(F\ (X\ ))$$

$$= -F\ 2\nu -2\ (-Y)$$

$$= F\ 2\nu -2(Y) \tag{19}$$

We continue simplifying in this manner and obtain

$$F\ 2\nu +1(X) = -F(X) \tag{20}$$

i.e

$$F\ 2\nu +1(X) + F(X) = 0. \tag{21}$$

Similarly we have $F^{2\nu+3}(X) = F^{2\nu+1}(-X)$

$$= -F^{2\nu+1}(X) \tag{22}$$

i.e $F^{2\nu+3}(X) = -F^{2\nu}(F(X)$

$$= -F^{2\nu}(-Y) \tag{23}$$

$$= F^{2\nu}(Y)$$

We continue simplifying in this manner and obtain

$$F^{2\nu+3}(X) = -F(X) \tag{24}$$

$$F^{2\nu+3}(X) + F(X) = 0. \tag{25}$$

Again, we continue simplifying in this manner and obtain,

$$F^{2\nu+5}(X) + F(X) = 0. \tag{26}$$

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