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BIANCHI TYPE I COSMOLOGICAL MODEL FOR DUST DISTRIBUTION WITH VARIABLE G AND LAMBDA

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Abstract: Bianchi Type I Cosmological Model for Dust Distribution with Variable G and Lambda is investigated by assuming average scale factor R as R^3= ABC = exp3Ht. G and Λ have been considered in such a way that energy is conserved and no assumption is made on energy density ρ. Λ is cosmological constant. The metric potentials A, B and C are considered as functions of cosmic time t. The physical and kinematical significance of the model are also discussed.

Keywords: Bianchi type-I space-time, Dust fluid, Hubble parameter, Variable G and Λ.

1. Introduction

Einstein’s field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics.

The isotropy of the present day universe makes the Bianchi Type-I model a prime consideration for studying the possible effects of an anisotropy in the early universe on modern-day data observations. Solutions to the field equations may also be generated by a law of variation of scale factor which was proposed by Pavon[19]. The behavior of the cosmological scale factor R (t) in the solution of Einstein's field equations with Robertson-Walker metric has been the subject of several studies. In earlier literature cosmological models with cosmological term is proportional to scale factor have been studied by Hoyle, et al[13], Olson,et al. [18], Beesham [7], Maia, et al. [17], Torres, et al. [25], Chen and Wu [10], Bali et al. [3,4]. The idea of variable gravitational constant G in the framework of general relativity was first proposed by Dirac [12]. Lau [16] worked in the framework of general relativity and proposed modification linking the variation of G. A number of authors investigated Bianchi’s models, using this approach given by (Abdel-Rahman [1], Berman [8], Kalligas et al. [14], Abdussattar and Vishwakarma [2], Vishwakarma [26], Pradhan et al. [21], Singh and Tiwari [23]), Bali et al. [5,6]. Borges and Carneiro [9].

In this paper, we have investigated Bianchi Type I dust fluid cosmological model with variable G and Λ assuming the condition on average scale factor R as $R^3=ABC = e^{3H_0 t}$. An obvious one is equation of state $p=\gamma\rho, 0\le\gamma\le 1$ is general condition for barotropic equation of state, p being isotropic pressure and ρ the matter density. This includes dust filled universe p = 0 (Friedman model) for γ = 0. We also assume that the shear (σ) is proportionate to expansion (θ). The motivation for assuming σ/θ = constant is explained by the Thorne [24]. The observations of the velocity – redshift relation for extra galactic sources suggest that the Hubble expansion of the universe is isotropic today to within 30% (Kantowski and Kristian [15]). More precisely, the redshift studies place the limit $\sigma/H\le$ 0:30 where σ is the shear which is the eigen value of shear tensor σ_{ij} and H is a Hubble constant. Collins et al. [11] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous hyper surface satisfies the condition σ/θ = constant, θ the expansion in the model. The condition $\sigma \propto \theta$ for the metric (1) leads to $A=(BC)^n$ where n is the constant. we consider G and Λ in such a way that energy is conserved and no assumption is made on energy density ρ. Various physical aspects of the model are also discussed.

2. Metric and Field Equations

We have considered the Bianchi Type I metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{1}$$

where A, B, C are functions of t-alone.

The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij} + \Lambda g_{ij} \tag{2}$$

Where G is the gravitational constant and Λ the cosmological constant, G and Λ are time-dependent.

The energy-momentum tensor T_{ij} for perfect fluid distribution is taken as

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} \tag{3}$$

Where p is the isotropic pressure , ρ the matter density. We assume the coordinates to be comoving so that

$v^1 = v^2 = v^3 = 0$ and $v^4 = 1$, Thus $v_4 = -1$.

The Einstein’s field equation (2) for the metric (1) leads to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda \tag{5}$$

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-8\pi Gp+\Lambda \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-8\pi Gp+\Lambda \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}=8\pi G\rho+\Lambda \tag{8}$$

where a dot denotes differentiation with respect to ‘t’.

An additional equation for time changes of G and Λ is obtained by Bianchi identities

$$\left(R_{ij}-\frac{1}{2}Rg_{ij}\right)_{;j}=0 \tag{9}$$

This leads to

$$\left(8\pi GT_i^{\ j}-\Lambda g_i^{\ j}\right)_{;j}=0 \tag{10}$$

Thus, we have $8\pi T_i^{\ j}G_{;j}+8\pi GT_{i\ ;j}^{\ j}-g_i^{\ j}\Lambda_{;j}-\Lambda g_{i\ ;j}^{\ j}=0$

This yield

$$8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]+8\pi\dot{G}\rho+\dot{\Lambda}=0 \tag{11}$$

The conservation of energy $T_{i\ ;j}^{\ j}=0$ splits equation (11) into two equations

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{12}$$

And $8\pi\dot{G}\rho+\dot{\Lambda}=0$ (13)

The Hubble parameter (H) is given by

$$H=\frac{\theta}{3}=\frac{\dot{R}}{R} \tag{14}$$

The expansion (θ) and shear (σ) are given by

$$\theta=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{15}$$

$$\sigma=\frac{k}{\sqrt{3}R^3} \tag{16}$$

The generalized deceleration parameter q is defined as

$$q=-1-\frac{\dot{H}}{H^2}=-\frac{\ddot{R}/R}{\dot{R}^2/R^2} \tag{17}$$

Where R is the average scale factor of Bianchi type-I universe for which $R^3=ABC$ (18)

3. Solution of field equations

Now assuming that the model is filled with dust of perfect fluid i.e. $\gamma = 0$.

To get the deterministic model, we have assumed the dust fluid condition i.e.

$$p = 0 \tag{19}$$

Using equations (14) and (15) we get,

$$\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=3H \tag{20}$$

Which implies that $R^3=ABC=e^{3Ht}$ (21)

Now as $\sigma \propto \theta$ which leads to $A = (BC)^n$ (22)

Differentiating (22) w.r.to t, we get $\frac{\dot{A}}{A}=n\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)$

From the equations (20) and (22) we get,

$$A=ae^{\frac{3nHt}{n+1}} \tag{23}$$

And $\frac{\dot{A}}{A}=\frac{3nH}{n+1}$, $\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=\frac{3H}{n+1}$ (24)

Using equations (5), (6) and (24) we get,

$$\frac{\ddot{C}}{C}-\frac{\dot{C}^2}{C^2}+3H\frac{\dot{C}}{C}=\frac{9nH^2}{n+1} \tag{25}$$

On solving (25), we get

$$C=b\exp\left[\frac{3nHt}{n+1}-\frac{ae^{-3Ht}}{3H}\right] \tag{26}$$

And from the equations (24) and (26) we get,

$$B=m\exp\left[\frac{3(1-n)Ht}{n+1}+\frac{ae^{-3Ht}}{3H}\right] \tag{27}$$

Using equations (12), (19) and (20) we get,

$$\rho=\alpha e^{-3Ht} \tag{28}$$

Solving equations (6) , (8) and (24) we get,

$$G=\frac{e^{3Ht}}{8\pi\alpha}\left[\frac{9H^2(n-1)(1-2n)}{(n+1)^2}+\frac{6Ha(1-2n)e^{-3Ht}}{n+1}-2a^2e^{-6Ht}\right] \tag{29}$$

From the equations (4),(6) and (31) we get,

$$\Lambda=\frac{9H^2}{(n+1)^2}\left(-n+n^2+1\right)+\frac{3Ha}{n+1}(1-2n)e^{-3Ht}+a^2e^{-6Ht} \tag{30}$$

Now considering $e^{3Ht}=T$. The values of metric potentials A, B and C, matter density ρ, Gravitational and Cosmological constants G and Λ are given as

$$A=aT^{\frac{n}{n+1}}$$

$$B=mT^{\frac{(1-n)}{(1+n)}}\,\mathrm{e}^{\frac{a}{3HT}}$$

$$C=bT^{\frac{n}{(1+n)}}\,\mathrm{e}^{-\frac{a}{3HT}}$$

$$\rho=\frac{\alpha}{T}$$

$$G = \frac{T}{8\pi\alpha}\left[\frac{9H^2(n-1)(1-2n)}{(n+1)^2} + \frac{6Ha(1-2n)}{(n+1)T} - \frac{2a^2}{T^2}\right]$$

$$\Lambda = \left[\frac{9H^2}{(n+1)^2}\left(n^2-n+1\right) + \frac{3Ha}{(n+1)T}(1-2n) + \frac{a^2}{T^2}\right]$$

The metric (1) yields as

$$ds^2 = -\frac{dT^2}{9H^2T^2} + a^2T^{\frac{2a}{n+1}}dx^2 + m^2T^{\frac{2(1-n)}{(1+n)}}e^{\frac{2a}{3HT}}dy^2 + b^2T^{\frac{2n}{(1+n)}}e^{\frac{2a}{3HT}}dz^2 \tag{31}$$

Special case: Here when we take $n = \frac{1}{2}$, then we get

$$A = aT^{\frac{1}{3}}, B = mT^{\frac{1}{3}}e^{\frac{a}{3HT}}, C = bT^{\frac{1}{3}}e^{-\frac{a}{3HT}}, \rho = \frac{\alpha}{T}, G = -\frac{a^2}{4\pi\alpha T}, \ \Lambda = 3H^2 + \frac{a^2}{T^2} \tag{32}$$

For the model (1), The Spatial volume is R^3= ABC = lT, l = abm, being a constant.

$$\frac{\dot{R}}{R} = \frac{H}{l} \ and \ \frac{\ddot{R}}{R} = \frac{H^2}{l^2} \tag{33}$$

The generalized deceleration parameter q is defined as

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} - 1 \tag{34}$$

Which leads to de-Sitter universe for n = ½.

Also the Volume expansion θ and shear σ are defined as θ = $v^i_{;i}$ and $\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$, with σ_{ij} being the shear tensor. Here Volume expansion θ = $3\frac{\dot{R}}{R}$ = 3H = $\frac{1}{T}$ and $\sigma = \frac{k}{\sqrt{3}R^3}$ =

$$\frac{\kappa}{l\sqrt{3}T} \tag{35}$$

4. Conclusion

The matter density ρ, shear σ, gravitational constant G and cosmological constant Λ are initially very large. As t increases, the scale factor increases and the matter density $\rho \to 0$, also as t increases, gravitational constant G, cosmological constant Λ and shear σ decreases. When n=1/2, $\Lambda\,\alpha\,\frac{1}{t^2}$ which matches with the result as obtained by Beesham

[7]. Universe exhibits an accelerating phase as q < 0 for the model. Since $\frac{\sigma}{\theta} \neq 0$ in general, therefore anisotropy is maintained. Thus the model represents shearing, non-rotating and accelerating model of the universe. Initially the model starts with a big-bang at T = 0 and the expansion θ decreases with time T, after that, it represents accelerating universe which matches with the result as explained by Riess et al. [22] and Perlmutter et al. [20]. The decelerating expansion at the initial epoch provides apparent provision for the formation of large structure of universe. Finally, the solutions presented in the paper are new and useful or better understanding of the evolution of the universe in Bianchi type-I dust fluid space-time with variable G and Λ.

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