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SPATIALLY HOMOGENEOUS BIANCHI TYPE I COSMOLOGICAL MODEL FOR DUST DISTRIBUTION WITH VARIABLE BULK VISCOSITY

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Abstract: Spatially homogeneous Bianchi Type I Cosmological model for dust distribution with variable bulk viscosity is investigated. The model represents anisotropic space-time in general but for large values of time, it isotropizes. The bulk viscosity prevents the matter density to vanish. The model represents decelerating and accelerating phases of universe. The spatial volume increases with time and the model has Point Type singularity.

Key Words: Spatially, homogeneous, Bianchi I, Anisotropic, dust distribution, bulk viscosity.

1. Introduction

The anisotropic space-times provide a systematic way to investigate cosmological models more general than Friedmann-Robertson-Walker (FRW) models. But FRW models are unstable near the singularity as pointed out by (Partridge and Wilkinson [13]). Therefore, spatially homogeneous and anisotropic Bianchi space-time (I – IX) are undertaken to study the universe in its early stages of evolution. Among these, Bianchi Type I space-time is the simplest one and is anisotropic generalization of zero curvature of FRW models.

The matter distribution is satisfactorily described by perfect fluid distribution due to large scale distribution of galaxies in our universe. However, a realistic treatment of the problem requires the consideration of material distribution other than perfect fluid. Bianchi Type I cosmological models for viscous fluid distribution have been investigated by Belinski and Khalatnikov [5]. They have pointed out that Bianchi Type I models with viscous fluid approach isotropic steady state asymptotically. Bali [1] investigated viscous fluid cosmological model with magneto hydrodynamic matter source using the ansatz $\zeta\theta$ = constant (Brevik et al. [8]). Barrow [7] investigated radiation dominated model with constant coefficient of bulk viscosity. The effect of viscosity on cosmological models is also investigated by Maartens and Mendez [11], Beesham [9], Saha [17], Bali and Singh [2,4], Bali et al. [3], Ram and Verma [15], Brevik and Gron [6].

In this paper we have investigated spatially homogeneous anisotropic Bianchi type I cosmological model for dust distribution with variable bulk viscosity. To get the deterministic model of universe, we have assumed the matter density $\rho = 3H^2, \zeta \alpha \rho^{1/2}$ as considered by Barrow [7] and Gron [10] and conservation equation $T^{j}_{i;j} = 0$ is taken into account where H is Hubble parameter and ζ the coefficient of bulk viscosity.

2. Metric and Field Equations

We consider spatially homogeneous Bianchi Type I space-time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \tag{1}$$

where A, B, C are metric potentials and are functions of cosmic time t-alone.

The Einstein field equation is taken into the form

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -T_{ij} \tag{2}$$

(in geometrized unit $8\pi G=1$, $c = 1$)

with

$$T_{ij} = (\rho + p) v_i v_j + p\, g_{ij} - \zeta\theta (g_{ij} + v_i v_j) \tag{3}$$

where ρ is the matter density, p the isotropic pressure, ζ the coefficient of bulk viscosity, θ the expansion in the model, g_{ij} the metric tensor, v^i the flow vector satisfying

$$g_{ij} v^i v^j = -1 \tag{4}$$

We also assume the coordinate axes to be comoving so that

$$v^1 = 0 = v^2 = v^3. \text{Thus } v^4 = 1.$$

The Einstein field equation (2) with (3) and (4) for the space-time (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -p + \zeta\theta \tag{5}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -p + \zeta\theta \tag{6}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -p + \zeta\theta \tag{7}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} = \rho \tag{8}$$

3. Solution of Field Equations

Equation (5),(6),(7) lead to

$$\frac{A_{44}}{A}-\frac{B_{44}}{B}+\frac{C_4}{C}\left(\frac{A_4}{A}-\frac{B_4}{B}\right) \tag{9}$$

and

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{A_4}{A}\left(\frac{B_4}{B}-\frac{C_4}{C}\right) \tag{10}$$

We also assume that the average scale factor R satisfies

$$R^3=ABC=e^{3Ht} \tag{11}$$

where H is Hubble constant.

Thus

$$\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}=3H \tag{12}$$

Equations (9), (10) and (12) lead to

$$\frac{\left(\frac{A_4}{A}-\frac{B_4}{B}\right)_4}{\left(\frac{A_4}{A}-\frac{B_4}{B}\right)}=-3H \tag{13}$$

and

$$\frac{\left(\frac{B_4}{B}-\frac{C_4}{C}\right)_4}{\left(\frac{B_4}{B}-\frac{C_4}{C}\right)}=-3H \tag{14}$$

The conservation equation $T^j_{i;j}=0$ for dust distribution leads to

$$\rho_4+\rho\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)-\zeta\theta\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right) \tag{15}$$

To get the deterministic solution, we assume that $\rho = 3H^2, \zeta = a\rho^{1/2}$ as considered by Barrow [7] and Gron [10]. Using these conditions in equation (15), we have

$$6H\,H_4 + 9H^3\left(1 - \frac{1}{\sqrt{3}}\right) = 0 \tag{16}$$

where $\zeta = \frac{1}{3}\rho^{1/2}$ (a=1/3 for simplicity).

Equation (16) leads to

$$H = \frac{2}{\alpha t + \beta} \tag{17}$$

where

$$\alpha = \sqrt{3}\left(\sqrt{3} - 1\right) \tag{18}$$

Thus, equation (13), (14) and (17) leads to

$$\frac{A_4}{A} - \frac{B_4}{B} = \beta_1(\alpha t + \beta)^{-6/\alpha} \tag{19}$$

$$\frac{B_4}{B} - \frac{C_4}{C} = \beta_2(\alpha t + \beta)^{-6/\alpha} \tag{20}$$

where β_1 and β_2 are constants. From equations (12), (19) and (20), we have

$$A = \beta_3(\alpha t + \beta)^{2/\alpha} \exp\left[\frac{2\beta_1 + \beta_2}{3(\alpha - 6)}(\alpha t + \beta)^{\frac{\alpha - 6}{\alpha}}\right] \tag{21}$$

$$B = \beta_4(\alpha t + \beta)^{2/\alpha} \exp\left[\frac{\beta_2 - \beta_1}{3(\alpha - 6)}(\alpha t + \beta)^{\frac{\alpha - 6}{\alpha}}\right] \tag{22}$$

$$C = \beta_5(\alpha t + \beta)^{2/\alpha} \exp\left[\frac{2\beta_2 + \beta_1}{3(\alpha - 6)}(\alpha t + \beta)^{\frac{\alpha - 6}{\alpha}}\right] \tag{23}$$

where $\beta_3, \beta_4, \beta_5$ are constants and $1 < \alpha < 6$.

After suitable transformation of coordinates, the space-time (1) leads to the form

$$ds^2 = -\frac{dT^2}{\alpha^2} + T^{4/\alpha} \exp\left[2\left\{\frac{2\beta_1+\beta_2}{3(\alpha-6)} T^{\frac{\alpha-6}{\alpha}}\right\}\right] dX^2$$

$$+ T^{4/\alpha} \exp\left[2\left\{\frac{\beta_2-\beta_1}{3(\alpha-6)} T^{\frac{\alpha-6}{\alpha}}\right\}\right] dY^2$$

$$+ T^{4/\alpha} \exp\left[2\left\{\frac{2\beta_2+\beta_1}{3(\alpha-6)} T^{\frac{\alpha-6}{\alpha}}\right\}\right] dZ^2 \tag{24}$$

where

$$\alpha t + \beta = T \tag{25}$$

4. Physical and Geometrical Aspects

The Hubble parameter (H), the matter density (ρ), the coefficient of bulk viscosity (ζ), the expansion (θ), the shear (σ), the spatial volume (R^3), the deceleration parameter (q) for the model (24) are given by

$$H = \frac{2}{T} \tag{26}$$

$$\rho = \frac{12}{T^2} \tag{27}$$

$$\zeta = \frac{\rho^{1/2}}{3} = \frac{2}{\sqrt{3}\,T} \tag{28}$$

$$\theta = \frac{6}{T} \tag{29}$$

$$\sigma = \frac{\beta_1^2+\beta_2^2+\beta_1\beta_2}{\sqrt{3}(6-\alpha)T^{6/\alpha}} \tag{30}$$

$$R^3 = ABC = \ell\, T^{6/\alpha} \tag{31}$$

$$q = -\frac{\alpha(2\alpha-1)}{2} < 0 \quad \text{as } \alpha > 1 \tag{32}$$

5. Discussion and Conclusion

The model starts with a big-bang at T = 0 and the expansion decreases with time. Since $\frac{\sigma}{\theta} \neq 0,$ hence anisotropy is maintained. However, for large values of time, the model isotropizes. The spatial volume increase with time. The deceleration parameter q < 0 indicates that the model represents accelerating universe in presence of bulk viscosity.

The matter density (ρ) is initially large but decreases with time. The bulk viscosity prevents the matter density to vanish. The expansion decreases with time. Later on, it represents accelerating universe which matches with the results as obtained by Riess et al. [16] and Perlmutter et al. [14]. The decelerating expansion at the initial stage provides obvious provision for the formulation of large scale structure of universe. The model as Point Type singularity at T = 0 (MacCallum [12]).

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