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MHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OF CASSON FLUID OVER A MOVING POROUS PLATE WITH VISCOUS DISSIPTION AND THERMAL RADIATION EFFECTS

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Abstract: The motion of temperature dependent viscosity of steady MHD boundary layer flow of Casson fluid and heat transfer over a porous moving plate with viscous dissipation in presence of porous medium and thermal radiation is considered. The viscosity of Casson fluid is assumed to be vary as a linear function of temperature. By using suitable transformation, the governing partial differential equations corresponding to momentum and energy equations are converted into non linear coupled ordinary differential equations and solved by fourth order Runge-Kutta Gill method with shooting technique. A parametric study is performed to illustrate the influence of Prandtl number, Eckert number, Permeability parameter, Magnetic parameter, non-Newtonian Casson parameter, Velocity ratio parameter, Temperature ratio parameter, Radiation parameter, Variable plastic dynamic viscosity parameter and suction parameter on the fluid velocity and temperature profiles within the boundary layer. The flow controlling parameters are found to have a profound effect on the resulting flow profiles.

Keywords: Casson fluid, variable viscosity, viscous dissipation, thermal radiation, heat transfer, boundary layer flow, porous medium, moving plate

Introduction

Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear. The examples of Casson fluid are of the type jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. Human blood can also be treated as Casson fluid. Due to the presence of several substances like, protein, fibrinogen, and globulin in aqueous base plasma, human red blood cells can form a chainlike structure, known as aggregates or rouleaux. If the rouleaux behave like a plastic solid, then there exists a yield stress that can be identified with the constant yield stress in Casson's fluid described by Fung [10].This rheological model was introduced originally by Casson [8] in his research on a flow equation for pigment oilsuspensions of printing ink. Bird et al. [7] investigated the rheology and flow of plastic fluid model which exhibits shear thinning characteristics, yield stress and high shear viscosity. Venkatesan et al. [37] stated that blood shows Newtonian fluid's character when it flows through larger diameter arteries at high shear rates, but it exhibits a remarkable non-Newtonian behaviour when it flows through small diameter arteries at low shear rates.

Fredrickson [9] investigated the steady flow of a Casson fluid in a tube. Mustafa *et al*. [28] studied the unsteady boundary layer flow and heat transfer of a Casson fluid over a moving flat plate with a parallel free stream using the Homotopy Analysis Method (HAM). Boundry layer behaviour on continuous solid surfaces and boundary layer equations are studied by Sakiadis [33] and Abu-Sitta[1]. Howarth [17] has discussed on the solution of the laminar boundary layer equations. Rao et al. [31] reported that the Casson fluid model is reduced to a Newtonian fluid at a very high wall shear stress, i.e. when the wall stress is much greater than yield stress. Hayat et al. [14] investigated Soret and Dufour effects on magnetohydrodynamic (MHD) flow of Casson fluid. In all of the above mentioned studies, fluid viscosity and fluid thermal conductivity was assumed to be constant within the boundary layer.

However, it is known that the physical properties of the fluid may change significantly when expose to temperature. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affects the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall is also affected greatly. In industrial systems, fluids can be subjected to extreme conditions such as high temperature, pressure, high shear rates and external heating (Ambient Temperature) and each of these factors can lead to high temperature being generated within the fluid. According to Anyakoha [4], Batchelor [6] and Meyers et al. [24] and other researchers in fluid dynamics, it is a well-known fact that the properties which are most sensitive to temperature rise are viscosity and thermal conductivity. Mukhopadhyay [25] adopted Batchelor's model of temperature dependent fluid viscosity when he studied the effect of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge assuming constant thermal conductivity. Salem and Fathy [34] investigated the effects of variable properties on MHD heat and mass transfer flow near a stagnation point towards a stretching sheet in a porous medium with thermal radiation and adopted the model of Prasad et al. [30] for temperature dependent viscosity and thermal conductivity and also incorporated the stagnation point velocity into the momentum equation.

The radiative effects have important applications in physics and engineering processes. The radiations due to heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends, to some extent to the heat controlling factors (Mukhopadhyay et al. [26]). High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids (Mukhopadhyay and Gorla [27]). The effect of radiation on heat transfer problems have studied by Hossain and Takhar [16], Takhar et al. [36], Hossain et al. [15], Hayat et al. [11,12] and Hayat and Qasim [13]. Many excellent theoretical models have been developed for radiative-convection flows and radiative-conductive transport. These generally utilize an algebraic approximation to the non-linear integro-differential equation for radiative heat transfer. Abdelhafez [2] studied the heat transfer from flat surface moving in parallel free stream.

Hussaini et al. [18] also discussed the boundary layer on a flat plate which has a constant velocity opposite in direction to that of them uniform mainstream. Some important studies in this topic are made by Ishak et al. [20–23] and Weidman et al. [38]. Ishak [19] combined these two problems by using the composite velocity introduced by Afzal et al. [3].

Casson fluid flow over a vertical porous surface with chemical reation in the presence of magnetic field has been studied by Arthur et.al.[5] the boundary layer flow of Casson fluid accompanied by heat transfer towards an exponentially porous stretching sheet in presence of thermal radiation is presented by Pramanik [29].

The purpose of the present study is to unravel the behavior of velocity and temperature profiles of Casson fluid flow with variable viscosity and viscous dissipation in the presence of thermal radiation and magnetic field. It is hoped that the results obtained will not only provide useful information for industrial applications, but also serve as a complement to the previous study.

Mathematical formulation

A steady two dimensional laminar flow of a viscous incompressible electrically conducting Casson fluid over a moving porous flat plate under the influence of viscous dissipation and thermal radiation in the presence of uniform transverse managnatic field is considered for a theoretical study. The motion of incompressible fluid is induced because of the plate moving with constant velocity U_w in the same or opposite direction to the free steam U_∞. It is assumed that a uniform magnetic field of strength B_0 is applied in the perpendicular direction towards the flow. Electrical conductivity of fluid is assumed to be small so that induced magnetic field can be neglected in comparison to applied magnetic field.

Under the above assumptions, taking into the account thermal radiation and viscous dissipation term, the governing equations for boundary layer flows and heat transfer are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\mu}{\rho k}(u-U_\infty)+\frac{\sigma_e B_0^2}{\rho}(u-U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where u and v are the components of velocity respectively in the x and y directions, μ is the coefficient of fluid viscosity ρ is the fluid density, k is the porosity, T is the temperature, κ is the thermal conductivity of the fluid, q_r is the radiative heat flux and C_p is the specific heat at constant pressure.

From the definition of viscosity $\left(\tau = \mu(\partial u/\partial y|_{y=o}\right)$, according to Mukhopadhyay[25] it is assumed that the rheological equation of an isotropic and incompressible flow of a Casson fluid can be written as

$$\tau_{ij}=\left(\mu_b+\frac{P_y}{\sqrt{2\pi}}\right)2e_{ij} \quad \text{when} \quad \pi>\pi_c$$

$$\tau_{ij}=\left(\mu_b+\frac{P_y}{\sqrt{2\pi_c}}\right)2e_{ij}$$

$$\text{when } \pi<\pi_c \tag{4}$$

P_y is known as yield stress of the fluid, mathematically expressed as

$$P_y=\frac{\mu_b\sqrt{2\pi}}{\beta} \tag{5}$$

μ_b is known as plastic dynamic viscosity of the non-Newtonian fluid, π is the product of the component of deformation rate with itself i.e. $\pi = e_{ij}e_{ij}$, where e_{ij} is the $(i, j)^{th}$ component of the deformation rate and π_c is the critical value of based on the non-Newtonian model. In a case of Casson fluid (non-Newtonian) flow, where $\pi > \pi_c$, it is possible to say that

$$\mu=\mu_b+\frac{P_y}{\sqrt{2\pi}} \tag{6}$$

Substituting (5) into (6), the kinematics viscosity of Casson fluid is now depending on plastic dynamic viscosity μ_b, density ρ and Casson parameter β

$$\mu = \mu_b \left(1 + \frac{1}{\beta}\right) \tag{7}$$

Rosseland approximation requires that the media is optically dense media and radiation travels only a short distance before being scattered or absorbed. Since we are considering a situation in which the radiation of heat within optically thick Casson fluid exists before the heat is scattered, radiative heat transfer is taken into account. Rosseland equation which is a simplified model of Radiative Transfer Equation (RTE) is adopted to account for this effect. When material has a great extinction coefficient, it can be treated as optically thick. q_r is the radiative heat flux and is defined using the Rosseland approximation Raptis[32] and Sparrow et al.[35] as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{8}$$

where σ^* is the Stefan–Boltzmann constant and k^* is known as the absorption coefficient.

It is assumed that plastic dynamic viscosity of Casson fluid varies as a linear function of temperature. This assumption is valid since it is know that the physical properties of the fluid may change significantly with temperature. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affects the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. Hence modified governing equations are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_b(T)}{\rho}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho}\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial y}\frac{\partial \mu_b(T)}{\partial T}\frac{\partial T}{\partial y} - \frac{\mu_b}{\rho k}\left(1+\frac{1}{\beta}\right)(u-U_\infty) + \frac{\sigma_e B_0^2}{\rho}(u-U_\infty) \tag{9}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu_b(T)}{\rho C_p}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{\rho C_p}\frac{4\sigma^*}{3k^*}\frac{\partial^2 T^4}{\partial y^2} \tag{10}$$

The appropriate boundary conditions for the problem are given by

$$\begin{aligned} y = 0: &\quad u = U_w, \quad v = V_w, \quad T = T_w \\ y \to \infty: &\quad u = U_\infty, \; T = T_\infty \end{aligned} \tag{11}$$

Here V_w is the velocity of suction $(V_w < 0)$ or injection $(V_w > 0)$, T_w is the wall temperature, T_∞ is the free stream temperature assumed to be constant with $T_w > T_\infty$.

The following relations are introduced for u and v respectively as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{12}$$

Where ψ(x, y) is the stream function. Introduce similarity variables as

$$\psi = \sqrt{U\upsilon x}\, f(\eta) \quad \text{and} \quad \eta = y\sqrt{\frac{U}{\upsilon x}}, \tag{13}$$

dimensionless temperature, temperature dependent viscosity respectively as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \mu_b(T) = \mu_b^* \left[a + b(T_w - T) \right] \tag{14}$$

where μ_b^* is the constant value of the coefficient of viscosity far from the plate, a and b are constant. The case when a = 1 is considered only. The equation of continuity (1) is identically satisfied and the momentum and energy equation are reduced as:

$$\left(1+\frac{1}{\beta}\right)\left[1+A(1-\theta)\right] f''' - A\left(1+\frac{1}{\beta}\right)\theta' f'' + \frac{1}{2} f f'' - K\left(1+\frac{1}{\beta}\right)\left[1+A(1-\theta)\right](f'-\varepsilon) + M(f'-\varepsilon) = 0 \tag{15}$$

$$\theta''\left[1+\frac{4}{3N}\left\{1+(\theta_r-1)\theta\right\}^3\right] + \frac{4}{N}(\theta_r-1)\left[1+(\theta_r-1)\theta\right]^2\theta'^2 + \frac{1}{2}\Pr f\theta' + \Pr Ec\left(1+\frac{1}{\beta}\right)\left[1+A(1-\theta)\right] f''^2 = 0 \tag{16}$$

Together with the boundary conditions

$$\eta = 0: \quad f(\eta) = S \quad f'(\eta) = 1-\varepsilon \quad \theta = 1$$

$$\eta \to \infty: \quad f'(\eta) = \varepsilon$$

$$\theta = 0 \tag{17}$$

Where the non-dimensional parameters

$A = b\left(T_w - T\right)$ (Variable plastic dynamic viscosity parameter)

$\beta = \frac{\mu_b\sqrt{2\pi}}{P_y}$ (Non-Newtonian Casson parameter)

$N = \frac{\kappa k^*}{4\sigma^* T_\infty^3}$ (Radiation parameter)

$\Pr = \frac{\mu C_p}{\kappa}$ (Prandtl number)

$Ec = \frac{U^2}{C_p(T_w - T_\infty)}$ (Eckert number)

$\varepsilon = \frac{U_\infty}{U}$ (Velocity ratio parameter)

$K = \frac{\upsilon x}{kU}$ (Permeability parameter)

$M = \frac{\sigma_e B_0^2 x}{\rho U}$ (Magnetic parameter)

$\theta_r = \frac{T_w}{T_\infty}$ (Temperature ratio parameter)

$R_{ex} = \frac{U_w x}{\upsilon}$ (Local Reynold number)

$S = V_0$ (Suction/Injecton parameter)

The physical quantities of interest are the local skin friction coefficient c_f and heat transfer rate i.e. the Nussult number Nu_x are:

$$c_f = \frac{\tau_w}{\rho U_\infty^2} \tag{18}$$

where the surface shear stress τ_w is defined as

$$\tau_w = \left(\mu_b + \frac{P_y}{\sqrt{2\pi}} \right) \frac{\partial u}{\partial y}\bigg|_{y=0} \tag{19}$$

and the heat transfer between the surface and the fluid conventionally expressed is dimensionless as a local Nussult number is given by

$$Nu_x = \frac{x}{\kappa\left(T_w - T_\infty\right)} q_w \tag{20}$$

Where the surface heat flux q_w is defined as

$$q_w = -\kappa \left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{21}$$

Using equation(7) and the similarity variables given by (13) and (14), we obtain

$$\sqrt{R_{ex}}C_f = \left(1+\frac{1}{\beta}\right)f''(0) \qquad \frac{Nu_x}{\sqrt{R_{ex}}} = -\theta'(0) \tag{22}$$

Numerical Solution:

The set of non-linear coupled differential equations (15) and (16) together with the boundary conditions (17) are solved numerically by using Runge Kutta Gill method along with shooting techniques using prescribed parameters. The quadratic interpolation is based on locally approximating the nonlinear functions by a quadratic function and the root of the quadratic function is taken as an improved approximate to the root of nonlinear functions. The procedure is applied repetitively to converge. Runge Kutta Gill method is selected because it reduces (minimize) round off error and this method of integrating systems of first order do not require preceding function values to be known . Order analysis, Consistency analysis and Stability analysis shows that Runge Kutta Gill is also of order four, stable and consistent. The constants are selected to reduce the amount of storage required in the solution of a large number of simultaneous first order differential equation, in addition; the Runge Kutta Gill variant method probably most often used in machine integration because of the storage savings. The Boundary Value Problem (BVP) cannot be solved on an infinite interval, and it would be impractical to solve it on a very large finite interval. Therefore, we imposed the infinite boundary condition at a finite point. Following method of superposition is adopted to reduce the dimensionless governing equations, (15) and (16) together with boundary conditions (17) to system of first order nonlinear ordinary differential equations. Let

$$f = f_1, \quad f' = f_2, \quad f'' = f_3, \quad \theta = f_4, \text{ and } \quad \theta' = f_5 \tag{23}$$

$$f_3' = \frac{Af_3 f_5}{\left[1+A(1-f_4)\right]} - \frac{1}{2}\frac{f_1 f_3}{\left(1+\frac{1}{\beta}\right)\left[1+A(1-f_4)\right]} + K(f_2-\varepsilon) - \frac{M(f_2-\varepsilon)}{\left(1+\frac{1}{\beta}\right)\left[1+A(1-f_4)\right]} \tag{24}$$

$$f_5' = -\left[\frac{3N}{3N+4\{1+(\theta_r-1)f_4\}^3}\right]\left[\frac{4}{N}(\theta_r-1)\{1+(\theta_r-1)f_4\}^2 f_5^2 + \frac{1}{2}\Pr f_1 f_5 + \Pr Ec\left(1+\frac{1}{\beta}\right)\{1+A(1-f_4)\} f_3^2\right]$$

And (25)

$$f_1(0) = S, \quad f_2(0) = 1-\varepsilon \quad f_4(0) = 1$$

$$f_2(\infty) = \varepsilon, \quad f_4(\infty) = 0 \tag{26}$$

According to the method of shooting, equation (17) is used to obtain $f''(0)$ and $\theta'(0)$. To integrate the corresponding Intial Value Problem (IVP) (24) to (26), $f''(0)$ and $\theta'(0)$ are required but no such values exist after the non-dimensionalization of the boundary conditions. Suitable guess values are chosen and then integration is carried out.

The calculated values for and are compared with that of boundary condition (17). Interpolation is employed and better estimate for $f''(0)$ and $\theta'(0)$ are obtained, IVP are solved using Runge Kutta Gill method. To improve the solutions, quadratic interpolation method which is superior (i. e. faster convergence rate) more than linear interpolation. Hence, guess values were chosen wisely. The above procedure is repeated until results up to the desired degree of accuracy 0.000001 is obtained.

Results and Discussion

In order to bring out the sailent features of the flow and heat transfer characteristics, the computation has been carried out using the method describe in the previous section for various values of temperature dependent plastic dynamic variable viscosity parameter A, non-Newtonian Casson parameter β, Velocity ratio parameter ε, Temperature ratio parameter θ_r, Permeability parameter K, Magnetic parameter M, Radiation parameter N, Suction/Injection parameter S, Prandtl number Pr and Eckert number Ec. For the illustration of the results, numerical values are portrayed in Fig.1 to Fig.19.

Now to study the effect of velocity ratio parameter ε, two cases are considered. In the first case we have assumed that both the plate and the free stream move in the same direction so that $0 < \varepsilon < 1$. The case when $\varepsilon = 0$ is corresponds to a plate moving in an otherwise quiescent fluid $(U_\infty = 0)$, $\varepsilon = 1$ corresponds to flow over a stationary plate $(U_w = 0)$ and $\varepsilon = \frac{1}{2}$ is equivalent to $U_w = U_\infty$ so that the plate and the free- stream move with the same speed. When $0 < \varepsilon < \frac{1}{2}$, the plate moves faster than the free-stream while the case when $\frac{1}{2} < \varepsilon < 1$ corresponds to the free-stream moving faster than the plate. When $\varepsilon < 0$ and $\varepsilon > 1$, the plate and fluid move in opposite directions. The influence of velocity ratio parameter ε on velocity and temperature profile are presented in Fig.1 and Fig.2. Fig.1 describes that with the increasing velocity ratio parameter ε, fluid velocity decreases near the plate but increases after a fixed point $(\eta \approx 1)$. Fig.2 exhibits that temperature decreases with increasing value of velocity ratio parameter.

Now the effect of suction parameter on velocity and temperature profile is illustrated for two cases: one, when the plates and the free stream move in the same direction and the other when the plate and the free stream move in opposite direction. From the both velocity profile Fig.3 and Fig.4, it is clear that due to increasing suction, fluid velocity increases in both cases. In other words momentum boundary layer thickness decreases with increasing value of suction parameter S due to sucking fluid through the porous wall. This effect is much more when the plate and the fluid move in opposite direction. Fig.5 shows that increase in suction parameter results decrease in temperature when the plate and the fluid move in same direction and thermal boundary layer thickness decrease. While the plate and the fluid move in opposite direction, temperature decreases with increase in suction parameter but it increases after a fixed point $(\eta \approx 5)$ as shown by Fig.6.

Fig.7 demonstrates effect of permeability parameter K, on velocity profile. As permeability parameter increases velocity decreases. Temperature as well as thermal boundary layer thickness increases with increasing value of permeability parameter as shown in Fig.8. It is very clear from Fig.9 and Fig.10, that the effect of magnetic parameter M is to decrease both velocity and temperature. This is due to fact that the tranverse magnetic field induces a Lorentz force which leads to provide resistance to the fluid flow.

The effect of variable plastic dynamic viscosity parameter A, on velocity and temperature profile is presented by Fig.11 and Fig.12. Velocity increases with increasing value of variable plastic dynamic viscosity parameter but it decrease after a fixed point $(\eta = 2.5)$. With increasing value of variable plastic dynamic viscosity parameter temperature and thermal boundary layer thickness increases.

Fig.13 describes that temperature decrease with increasing value of radiation parameter N. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature decreases. Fig.14 depicts the effect of radiation parameter on temperature gradient profile. The magnitude of temperature gradient decreases with increasing radiation parameter near the plate but it increases after a fixed point $(\eta = 2)$.

In Fig.15, it is observed that an increase in Prandtl number Pr, results in decrease of temperature and thermal boundary layer thickness. The reason is that the smaller value of Pr are equivalent to increasing the thermal conductivity and therefore heat is able to diffuse away from the plate more rapidly for higher value of Pr. The effect of Eckert number Ec, is on temperature to increase which is presented by Fig.16. There is no effect of Prandtl number Pr and Eckert number Ec on velocity profile.

The effect of non-Newtonian Casson parameter β on velocity and temperature profile is depicted by Fig.17 and Fig.18. Both velocity and temperature decreases with increasing value of non-Newtonian Casson parameter. The nature of fluid tends towards to the nature of Newtonian fluid as $\beta \rightarrow \infty$. Fig.19 depicts the temperature distribution for various values of temperature ratio parameter θ_r in a case when the plate and fluid move in same direction. It is clearly observed from this figure that fluid temperature increases as θ_r increases. θ_r > 1 when $T_w > T_\infty$ and this is the usual case that is considered in this problem. θ_r < 1 represents the case when $T_w < T_\infty$ and this is possible if the porous plate is kept on ice pad.

Conclusions

The numerical solutions for steady MHD boundary layer flow and heat transfer for a Casson fluid over a porous moving plate with variable viscosity in presence of viscous dissipation and thermal radiation are analyzed. The main findings of this investigation may be summarized as follows:

1) Momentum boundary layer thickness increases with increasing non-Newtonian Casson parameter while thermal boundary layer thickness decreases in this case.

2) Temperature increases with the increase in Eckert number and temperature ratio parameter while decreases with the increase in Prandtl number and velocity ratio parameter.

3) Velocity decreases near the plate with increasing velocity ratio parameter but it increases after a fixed point while it increases with increasing variable plastic dynamic viscosity parameter but it decreases after a fixed point.

4) Temperature decreases with increasing radiation parameter while increases with increasing variable plastic dynamic viscosity parameter throughout the boundary flow.

5) Momentum boundary layer thickness decreases with increasing suction parameter in both cases: the plate and the fluid move in same or opposite direction.

6) Temperature decreases with increasing suction when the plate and the fluid move in same direction while it decreases with the increase in suction but it increases after fixed point in opposite case.

7) The effect of magnetic parameter is to decrease both velocity and temperature while permeability parameter is to decrease velocity and increase temperature.

8) Temperature gradient magnitude decrease with the increase in radiation parameter near the plate but it increases after a fixed point.

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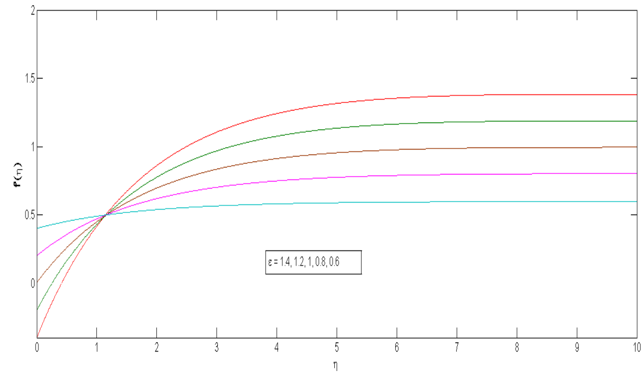

Fig.1 Velocity profile $f'(\eta)$ for several values of ε with K=M=0.1, S=A=N=β=1, Ec=0.3, Pr=0.7 and θ_r =1.1.

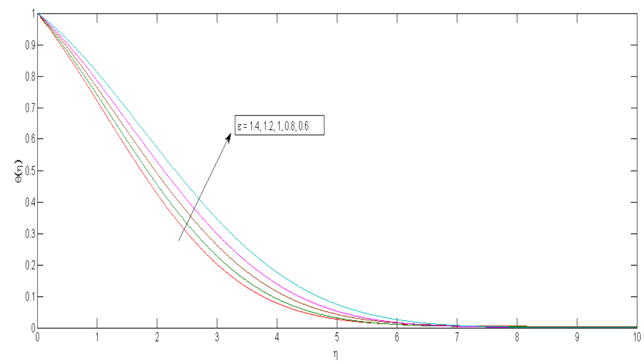

Fig.2 Temperature profile $\theta(\eta)$ for various values of ε with K=M=0.1, S=A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

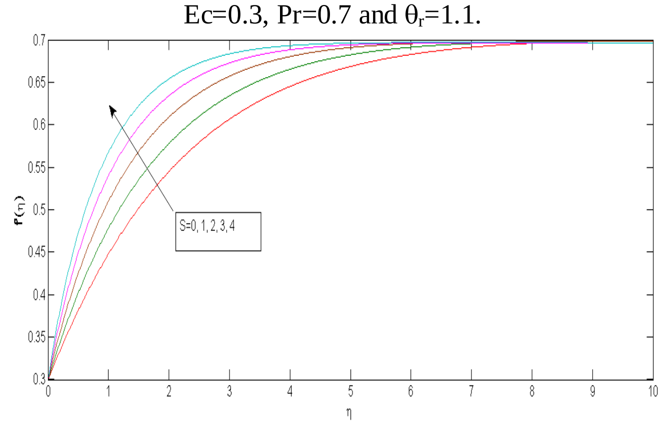

Fig.3 Velocity profile $f'(\eta)$ for several values of S with K=M=0.1, ε = 0.7, A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1 when the plate and the fluid move in same direction.

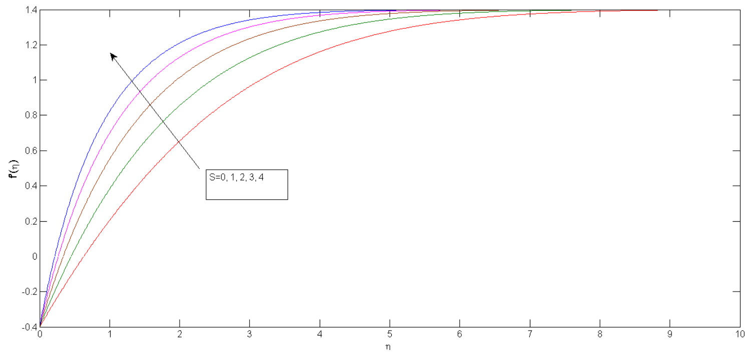

Fig.4 Velocity profile *f'(η)* for several values of S with K=M=0.1, ε = 1.4, A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1 when the plate and the fluid move in opposite direction.

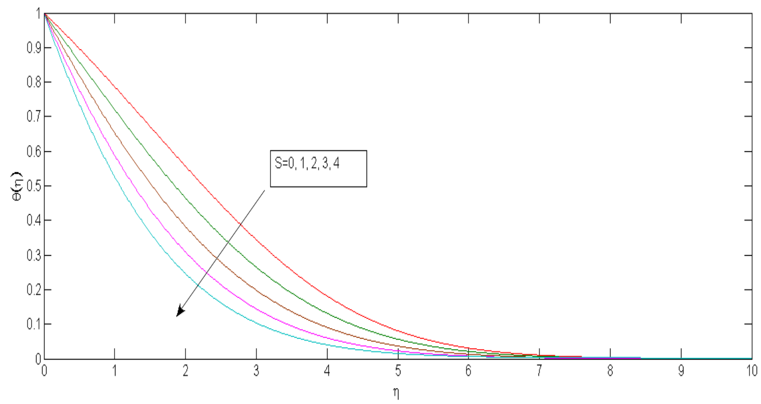

Fig.5 Temperature profile *θ(η)* for various values of S with K=M=0.1, ε = 0.7, A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1 when the plate and the fluid move in same direction.

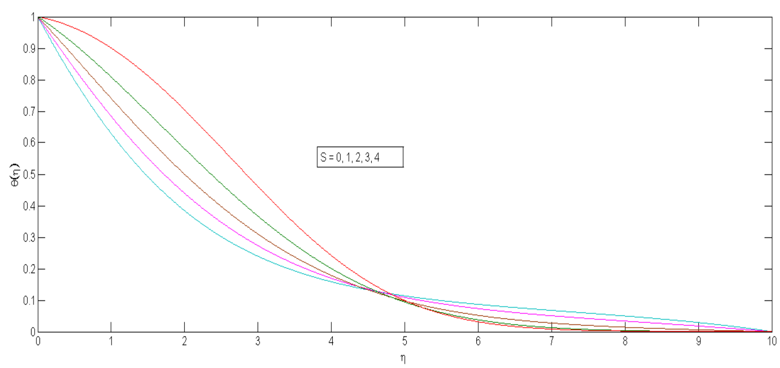

Fig.6 Temperature profile *θ(η)* for various values of S with K=M=0.1, ε = 1.4, A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1 when the plate and the fluid move in opposite direction.

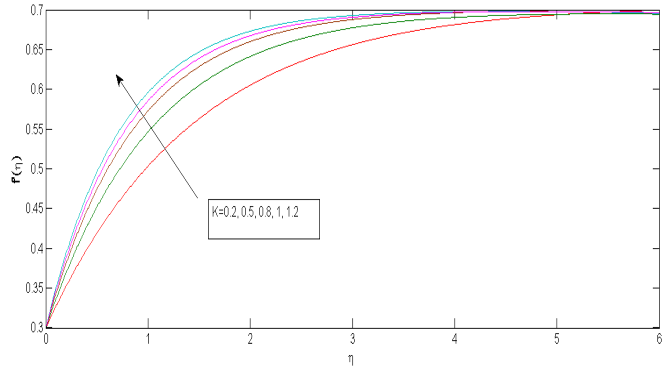

Fig.7 Velocity profile $f'(\eta)$ for several values of K with M=0.1, ε = 0.7, S=A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

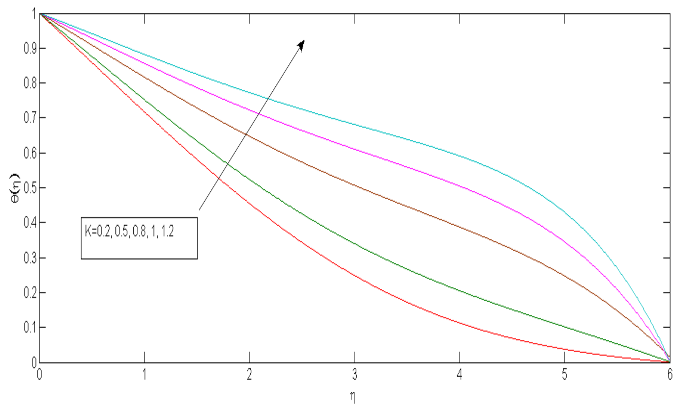

Fig.8 Temperature profile $\theta(\eta)$ for various values of K with M=0.1, ε = 0.7, S=A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

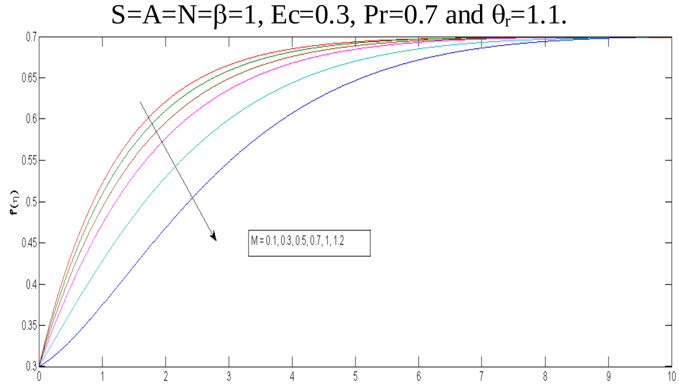

Fig.9 Velocity profile $f'(\eta)$for several values of M with K=0.3, ε = 0.7, S=A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

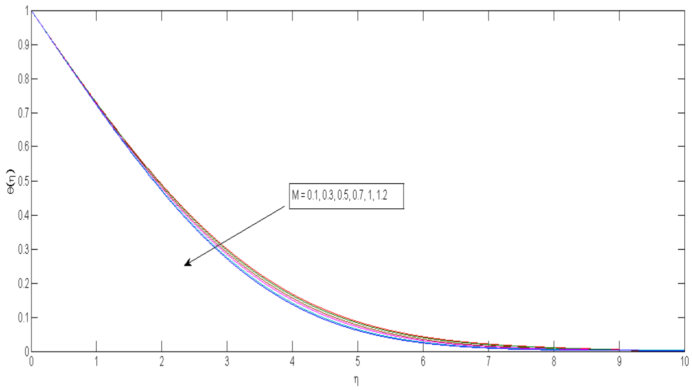

Fig.10 Temperature profile $\theta(\eta)$ for various values of M with K=0.3, ε = 0.7, S=A=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

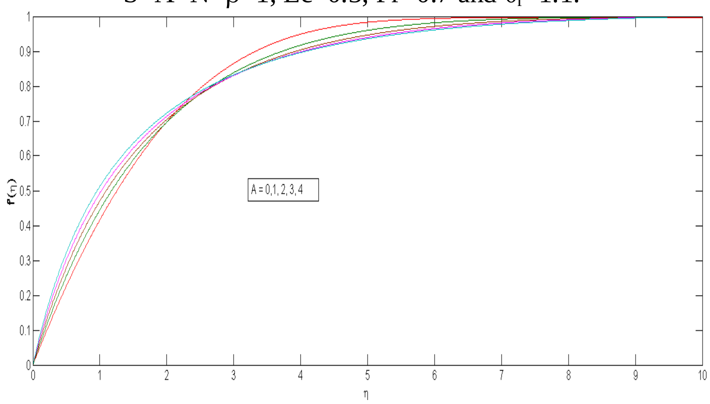

Fig.11 Velocity profile $f'(\eta)$ for several values of A with K=M=0.1, ε = 1, S=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

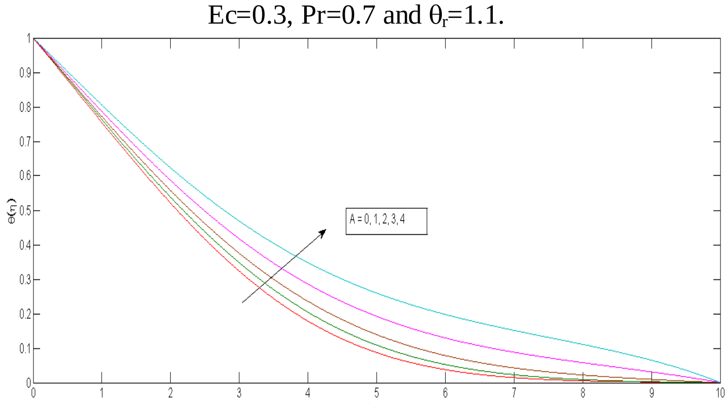

Fig.12 Temperature profile $\theta(\eta)$ for various values of A with K=M=0.1, ε = 1, S=N=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

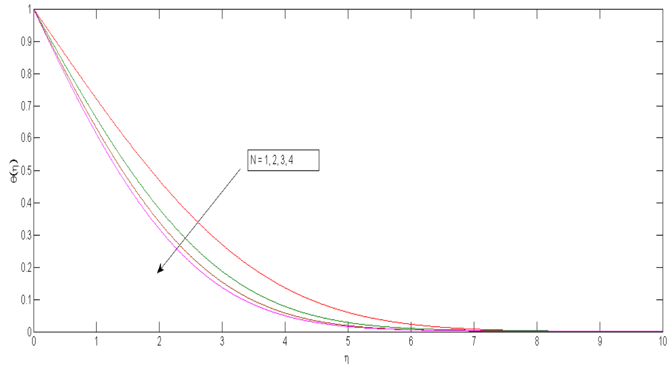

Fig.13 Temperature profile $\theta(\eta)$ for various values of N with K=M=0.1, ε = 0.7, S=A=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

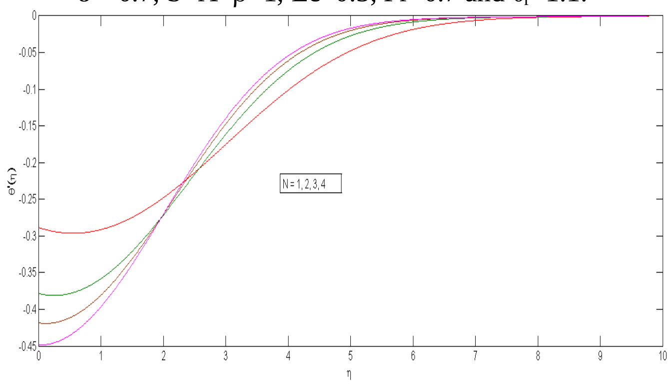

Fig.14 Temperature gradient profile $\theta'(\eta)$ for various values of N with K=M=0.1, ε = 0.7, S=A=β=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

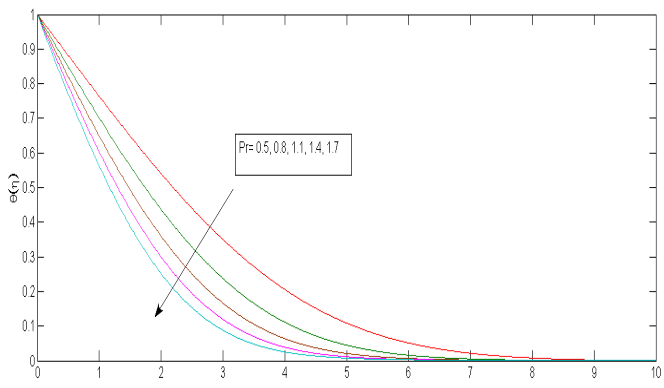

Fig.15 Temperature profile $\theta(\eta)$ for various values of Pr with K=M=0.1, ε = 0.7, S=A=N=β=1, Ec=0.3 and θ_r=1.1.

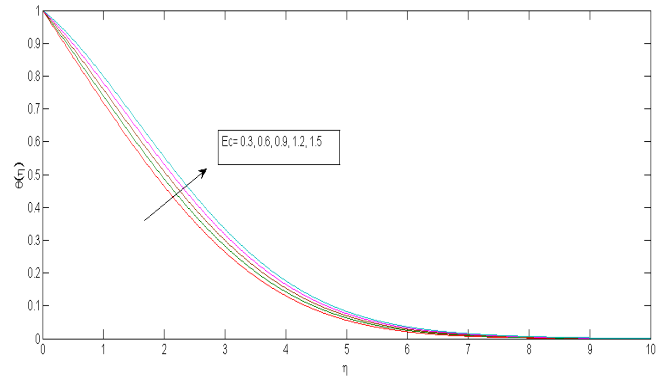

Fig.16 Temperature profile $\theta(\eta)$ for various values of Ec with K=M=0.1, ε = 0.7, S=A=N=β=1, Pr=0.7 and θ_r=1.1.

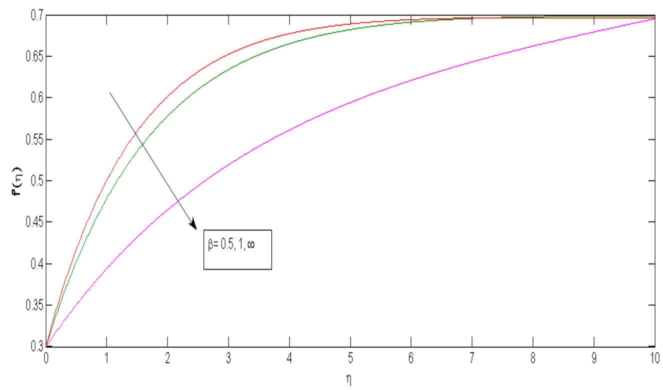

Fig.17 Velocity profile $f'(\eta)$ for several values of β with K=M=0.1, ε = 1, S=N=A=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

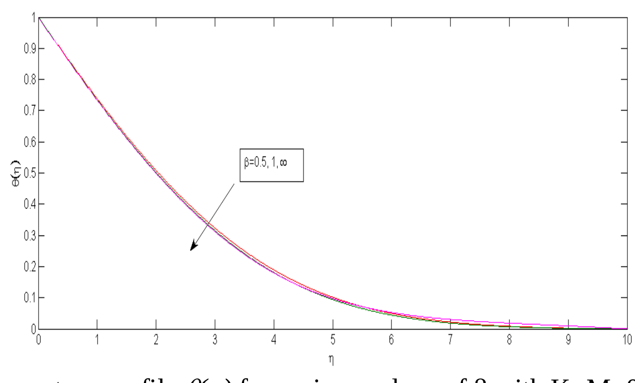

Fig.18 Temperature profile $\theta(\eta)$ for various values of β with K=M=0.1, ε = 0.7, S=A=N=1, Ec=0.3, Pr=0.7 and θ_r=1.1.

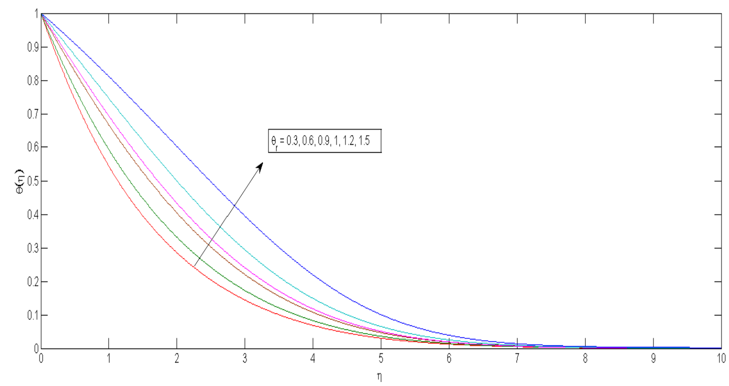

Fig.19 Temperature profile $\theta(\eta)$ for various values of θ_r with K=M=0.1, $\varepsilon = 0.7$, S=A=N=β=1, Ec=0.3 and Pr=0.7.