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BIANCHI TYPE III INFLATIONARY UNIVERSE WITH FLAT POTENTIAL AND CONSTANT DECELERATION PARAMETER IN GENERAL RELATIVITY

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Abstract: Bianchi Type III inflationary model with flat potential and massless scalar field is investigated. To get an inflationary universe, we have considered a flat region in which the potential V(φ) is constant and decelerating parameter is also constant. We find that the spatial volume increases with time representing inflationary scenario. The Hubble parameter decreases with time. The Higgs field evolves slowly but the universe expands. The models represent decelerating and accelerating phases of universe and isotropize at late time. These results match with recent astronomical observations. The models also have Point Type singularities.

Key Words: Bianchi Type III, inflationary, flat potential, deceleration parameter.

2010 Mathematics Subject Classification: 83CXX, 83FXX.

1. Introduction

The stage of exponential expansion of universe is termed as Inflation. The inflationary scenario is the satisfactory solution to some of conceptual issues in cosmology and is not understood by the standard Big Bang cosmology. The inflationary scenario explains several mysteries of modern cosmology like homogeneity, the isotropy, flatness of observed universe and primordial magnetic monopole problem. Guth [7] introduced the concept of inflation while investigating the problem of why we do not see magnetic monopole today. Guth [7] has also suggested that rapid expansion is due to false vacuum energy and after inflation, the universe is filled with bubbles. This inflationary scenario is also confirmed by Cosmic Microwave Background (CMB) observations (Bassett et al. [6]). The notion of inflation is one of the best mechanism at early stage of evolution of universe to explain the flat, homogeneous and isotropic nature to present day universe. In inflationary models, the universe undergoes a phase transition characterized by the evolution of Higgs field (ϕ). The inflation will take place if the potential $V(\phi)$ has flat region and in this region, the ϕ field evolves slowly but the universe expands in an exponential way due to the vacuum field energy as suggested by Stein-Schabes [13]. The flat part of the potential is naturally associated with a vacuum energy which can be identified as an effective cosmological constant (Λ) and it makes the universe to enter an inflationary period. Various authors viz. Linde [9], Abrecht and Steinhardt [2], Abbott and Wise [1], Mijic et al. [10], La and Steinhardt [8]. Rothman and Ellis [11] have pointed out that we can have the solution of isotropy problem if we work with anisotropic metrics and these can be isotropized in a very general circumstances. Following Rothman and Ellis [11], Bali and Jain [3], Bali [4], Bali and Swati [5] investigated inflationary cosmological models using Bianchi Type I and VIII space-times in which the potential $V(\phi)$ is considered as constant. Singh and Kumar [12] investigated Bianchi Type II inflationary models with constant deceleration parameter in general relativity considering flat region in which potential $V(\phi)$ is constant.

In this paper, we have investigated inflationary cosmological model in context of Bianchi Type III space-time in which potential $V(\phi)$ is constant and deceleration parameter are also assumed as constant. We find that Higgs field evolves slowly for one model but the universe expands. For another model, the Higgs field is initially large but decreases with time. The spatial volume increases with time representing inflationary scenario. The model represents decelerating and accelerating phases of universe which match with recent astronomical observations.

2. Metric and Field Equations

We consider Bianchi Type III line-element in orthogonal form as

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2\alpha x}B^2dy^2 + C^2dz^2 \tag{1}$$

where A, B, C are metric potentials and are function of t-alone.

We assume the co-ordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1$$

The Einstein's field equations (in geometrized units $8\pi G = c = 1$) in the case of massless scalar field ϕ with potential $V(\phi)$ as given by Stein-Schabes [13] are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{2}$$

with

$$T_{ij} = \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right]g_{ij} \tag{3}$$

The conservation relation leads to

$$\frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}\partial^{\mu}\phi)=-\frac{dV}{d\phi} \tag{4}$$

The field equations (2) for the line-element (1) lead to non-linear differential equations

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=-\frac{1}{2}\dot{\phi}^2+V(\phi) \tag{5}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=-\frac{1}{2}\dot{\phi}^2+V(\phi) \tag{6}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{\alpha^2}{A^2}=-\frac{1}{2}\dot{\phi}^2+V(\phi) \tag{7}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{\alpha^2}{A^2}=\frac{1}{2}\dot{\phi}^2+V(\phi) \tag{8}$$

$$\frac{B_4}{B}-\frac{A_4}{A}=0 \tag{9}$$

Equation (9) leads to

$$A = aB \tag{10}$$

where a is constant of integration.

The equation (4) for scalar field (ϕ) leads to

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=\frac{dV}{d\phi} \tag{11}$$

3. Solution of Field Equations

We are interested in inflationary solution so flat region is considered. Thus $V(\phi)$ is constant = K.

Now equation (11) leads to

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=0 \tag{12}$$

where suffix '4' indicates ordinary partial derivative with respect to t.

From equation (12), we have

$$\phi_4 = \frac{\ell}{ABC} = \frac{\ell}{aB^2C} \text{ Using equation(10)} \tag{13}$$

where ℓ is constant of integration.

The scale factor R for line-element (1) is given by

$$R^3 = ABC = a\,B^2C \tag{14}$$

using equation (10). To find the deterministic model of the universe, we assume that deceleration parameter (q) is constant i.e.

$$q = -\frac{R_{44}/R}{R_4^2/R^2} = \beta \text{ (constant)} \tag{15}$$

Two cases arise:

Case (i) $\beta > 0$

Case (ii) $\beta < 0$

Case (i): $\beta > 0$ i.e, $\beta = b > 0$

From equation (15) , we have

$$\frac{R_{44}}{R_4} + b\frac{R_4}{R} = 0 \tag{16}$$

Which leads to

$$R = (\gamma t + \delta)^{\frac{1}{b+1}} \tag{17}$$

Where $\gamma=\ell(b+1)$, ℓ is constant of integration.

To get the deterministic value of B and C, we assume that shear (σ) is proportional to expansion (θ). This leads to

$$B = C^n \tag{18}$$

Also

$$R^3 = (\gamma t + \delta)^{\frac{3}{b+1}} \tag{19}$$

Which leads to

$$aB^2C = (\gamma t + \delta)^{\frac{3}{b+1}} \tag{20}$$

From equation (18) and (20), we have

$$C^{2n+1} = \frac{1}{a}(\gamma t + \delta)^{\frac{3}{b+1}} = \frac{1}{a}T^{\frac{3}{b+1}} \tag{21}$$

Equation (18) and (21) leads to

$$B = C^n = \frac{1}{a^{\frac{n}{2n+1}}} T^{\frac{3n}{(b+1)(2n+1)}} \tag{22}$$

And

$$A = aB = a^{\frac{(n+1)}{(2n+1)}} T^{\frac{3n}{(b+1)(2n+1)}} \tag{23}$$

Whereγt+δ=T

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = \frac{-1}{\gamma^2} dT^2 + T^{\frac{6n}{(b+1)(2n+1)}} \left\{ dX^2 + e^{-\frac{2\alpha X}{a^{\frac{(n+1)}{(2n+1)}}}} dY^2 \right\} + T^{\frac{6}{(b+1)(2n+1)}} dZ^2 \tag{24}$$

Where

$$a^{\frac{n+1}{2n+1}} x = X$$

$$\frac{y}{a^{\frac{n}{2n+1}}} = Y$$

$$\frac{z}{a^{\frac{1}{2n+1}}} = Z$$

Case (ii): If β < 0 then β=-b, b > 0.

Equation (15) leads to

$$\frac{R_{44}}{R_4} - b\frac{R_4}{R} = 0 \tag{25}$$

From equation (25), we have

$$R = (\lambda t + N)^{\frac{1}{1-b}} \tag{26}$$

Where λ = M(1-b), M and N being constants of integration and b < 1.

Equation (14) and (26) leads to

$$aB^2C = (\lambda t + N)^{\frac{3}{1-b}} \tag{27}$$

Using (18) and (27), we have

$$C^{2n+1} = \frac{1}{a}(\lambda t + N)^{\frac{3}{1-b}} = \frac{1}{a}\tau^{\frac{3}{1-b}} \tag{28}$$

Now

$$B = C^n = \frac{1}{a^{n/2n+1}} \tau^{\frac{3n}{(1-b)(2n+1)}} \tag{29}$$

And

$$A = aB = a^{\frac{n+1}{2n+1}}\tau^{\frac{3n}{(1-b)(2n+1)}} \tag{30}$$

Where λt+N = λτ

After suitable transformation of coordinates, the metric (1) leads to

$$ds^2 = -d\tau^2 + \tau^{\frac{6n}{(1-b)(2n+1)}}\left\{dX^2 + e^{-\frac{2\alpha X}{a^{n+1}/2n+1}}dY^2\right\} + \tau^{\frac{6}{(1-b)(2n+1)}}dZ^2 \tag{31}$$

Where $a^{\frac{n+1}{2n+1}}x = X, \frac{1}{a^{\frac{n}{2n+1}}}y = Y, \frac{1}{a^{\frac{1}{2n+1}}}z = Z$

4. Physical and Geometrical Aspects

The rate of Higgs fields (φ), the spatial volume (R^3), the expansion (θ), the shear(σ), the Hubble parameter(H) for the model (24) are given by

$$\phi_4 = \frac{l}{aC^{2n+1}} = \frac{l}{T^{\frac{3}{b+1}}} \tag{32}$$

Which leads to

$$\phi = \frac{l(b+1)}{\gamma(b-2)}T^{\frac{b-2}{b+1}} + m \qquad , b \neq 2 \tag{33}$$

Where m is constant of integration and b > 0.

The Higgs field evolves slowly but the universe expands.

$$R^3 = T^{\frac{3}{b+1}} \tag{34}$$

$$H = \frac{\gamma}{(1+b)T} \tag{35}$$

$$\theta = 3H = \frac{3\gamma}{(1+b)T} \tag{36}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{(n-1)C_4}{\sqrt{3}\,C}$$

$$= \frac{\sqrt{3}(n-1)\gamma}{(1+b)(2n+1)T} \tag{37}$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(2n+1)} \tag{38}$$

The above mentioned quantities for the model (31) are given by

$$\phi_4 = \frac{l}{aC^{2n+1}} = \frac{l}{\tau^{\frac{3}{1-b}}} \tag{39}$$

Which leads to

$$\phi = -\frac{l(1-b)}{\lambda(b+2)}\tau^{-\frac{(b+2)}{(1-b)}} + L \tag{40}$$

Where L is constant of integration and b < 1.

$$R^3 = \tau^{\frac{3}{1-b}} \;; \qquad b < 1 \tag{41}$$

$$H = \frac{\lambda}{(1-b)\tau} \tag{42}$$

$$\theta = 3H = \frac{3\lambda}{(1-b)\tau} \tag{43}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{(n-1)C_4}{\sqrt{3}C}$$

$$= \frac{\sqrt{3}(n-1)\lambda}{(1-b)(2n+1)\tau} \tag{44}$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(2n+1)} \tag{45}$$

5. Conclusion

The spatial volume increases with time representing inflationary scenario in the models (24) and (31). These models represent decelerating and accelerating phases of universe if β > 0 and β< 0 respectively. The Higgs field for the model (24) evolves slowly but the universe expands. However, for the model (31), the Higgs field is initially large but decreases with time. The Hubble parameter is initially large but decreases with time for both the models. The models in general represent anisotropic space time but these models isotropic at late time. The models are isotropic for n=1 also. The models have Point Type singularity at T=0 and τ=0 respectively.

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