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HALL AND SORET EFFECTS ON MHD CONVECTION FLOW OF SECOND ORDER FLUID THROUGH POROUS MEDIUM IN A VERTICAL CHANNEL WITH THERMAL RADIATION

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Abstract: An analysis of MHD convective flow of a second grade, incompressible and electrically conducting fluid in an infinite vertical channel is investigated. A magnetic field of uniform strength is applied along the axis perpendicular to the planes of the plates. The magnetic field is strong enough so that the Hall currents have to be taken into account. The temperature difference of the channel plates is high to cause heat radiation. A closed form solutions of the governing equations are obtained for the velocity and the temperature fields. The effects of different parameters involved in the expressions for the velocity, temperature and concentration profiles are studied with the help of figures. The amplitudes and the phase angles of the wall shear stress, Nusselt number and Sherwood number are also discussed with the help of graphs.

Key Words: hall current, viscoelastic, MHD convection, radiation, soret.

1. Introduction

Free convective flows in a porous medium have received much attention in recent time due to its wide applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. Moreover, considerable interest has been shown in radiation interaction with convection for heat transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing-emitting fluids. The effects of transversely applied magnetic field on the flow of an electrically conducting viscous fluid have been discussed widely owing to their astrophysics, geophysical and engineering applications. Hakiem [1] investigated hydromagnetic oscillatory flow on free convection radiation through porous medium with constant suction velocity. Raptis and Perdikis [2] Unsteady flow through a highly porous medium in the presence of radiation. Makinde and Mhone [3] Heat transfer to MHD oscillatory flow in a channel filled with porous medium. Alagoa et al. [4] studied radiative and free convective effects of a MHD flow through a porous medium between infinite parallel plates with time-dependent suction.

In majority of hydromagnetic flows the Hall current effect is neglected. But when the strength of the magnetic field is strong, one cannot neglect the effects of Hall current. The influence of electromagnetic force is noticeable and causes anisotropic electrical conductivity in the plasma. This anisotropy in the electrical conductivity in the plasma produces a current called the Hall current. Reddy and Bathaiah [5] studied the Hall effects on MHD Couette flow through a porous straight channel. Jana and Dutta [6] analyzed Hall effects on MHD Couette flow in a rotating system. Attia [7] studied Hall current effects on velocity and temperature fields of an unsteady Hartmann flow. Hossain and Rashid [8] presented the analysis on Hall current effect on hydromagnetic free convection flow along a porous flat plate with mass transfer. Singh and Pathak [9] presented an analysis of an oscillatory rotating MHD Poiseuille flow with injection/suction and Hall currents. Singh and Kumar [10] investigated of Hall current and rotation effects together on free convection MHD flow in a porous channel. Hall currents and surface temperature oscillation effects on natural convection magnetohydrodynamic heat-generating flow have been considered by Takhar and Ram [11].

Recently, researchers in engineering and scientific fields have shown great interest in the study of a non-Newtonian fluid due to its importance in industrial processes. Many authors have examined the flow, heat and mass transfer in non-Newtonian fluids of different types. Sarpakaya [12] discussed the steady flow of a uniformly conducting non-Newtonian incompressible fluid between two parallel plates. The fluid considered is under the influence of a constant pressure gradient. Chaudhary and Jain [13] analyzed the Hall effect on an MHD mixed convection flow of a viscoelastic fluid past an infinite vertical porous plate with mass transfer and radiation. Chaudhary and Jha [14] discussed heat and mass transfer in an elastico-viscous fluid past an impulsively started infinite vertical plate with the Hall effect. Saxena and Dubey [15] studied MHD free convection heat and mass transfer flow of viscoelastic fluid embedded in a porous medium of variable permeability with radiation effect and heat source in slip flow regime.

The thermal-diffusion effects, which are caused by the temperature gradient (called the Soret effect) is an interesting macroscopically physical phenomenon in fluid mechanics. Usually, in heat and mass transfer problems the variation of density with temperature gives rise to the considered buoyancy effect under natural convection and hence the temperature will influence the diffusion species. Alam et al. [16] studied the Soret Dufour effect on a steady MHD combined free –forced convection and mass transfer flow past a semi-infinite vertical plate. Pal and Mondal [17, 18] examined the effect of Soret and Dufour on an MHD non-Darcy unsteady mixed convection heat and mass transfer over a stretching sheet. Seethamahalakshmi et al. [19] discussed MHD free convective mass transfer flow past an infinite vertical porous plate with variable suction and soret effect.

Singh and Kumar [20] analyzed fluctuating heat and mass transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous plate in slip- flow regime. Kumar and Singh [21] studied soret and Hall effects on oscillatory MHD free convective flow of radiating fluid in a rotating vertical porous channel filled with porous medium. Singh et al. [22] analyzed heat and mass transfer in an unsteady MHD free convective flow through a porous medium bounded by vertical porous channel in the presence of radiative heat and Hall current. Recently, Jha et al. [23] investigated the influence of soret effect on MHD mixed convection flow of viscoelastic fluid past a vertical surface with Hall effect. Coleman and Noll [24] studied an approximation theorem for functional with application in continuum mechanics. Meyer [25] studied heat transfer rates by MHD techniques. Markovitz and Coleman [26] studied incompressible second order fluids, Garg [27] discussed the combined effect of hall current and thermal radiations on unsteady flow of an electrically conducting flow past an impulsively started infinite vertical porous plate with variable temperature. Garg also [28] analyzed MHD flow due to moving vertical porous plate with variable temperature in the presence of thermal radiations. Recently Garg et al [29] investigated mixed convective MHD flow through porous medium in hot vertical channel with spanwise cosinusoidal temperature and thermal radiations. Very recently Garg et al. [30, 31] studied unsteady free convective flow through porous medium in vertical channel under different boundary conditions with thermal radiations.

The aim of the paper is to study the effects of Hall and soret effects on MHD convective flow through porous medium in a vertical channel when the temperature and concentration at either of the plates of the channel varies periodically with time. The channel is filled with the highly porous medium. A magnetic field of uniform strength is applied perpendicular to the planes of the plates. Exact solutions of the partial differential equations are obtained and the effects of different flow parameters on the flow and temperature are discussed with the aid of graphs.

2. Basic Equations

The equations governing the unsteady MHD convective flow of a viscoelastic (second order), incompressible and electrically conducting fluid in a vertical channel filled with porous medium in the presence of magnetic field are:

Equation of Continuity:

$$div\,\vec{V} = 0 \tag{1}$$

Momentum Equation:

$$\rho\left[\frac{\partial\vec{V}}{\partial t^*} + \left(\vec{V}\cdot\nabla\right)\vec{V}\right] = \vec{F} + \vec{J}\times\vec{B} - \frac{\mu}{K^*}\vec{V} + \nabla.\exists, \tag{2}$$

Energy Equation:

$$\rho C_p\left[\frac{\partial T^*}{\partial t^*} + \left(\vec{V}.\nabla\right)T^*\right] = k\nabla^2 T^* - \nabla q\ , \tag{3}$$

Species Concentration Equation:

$$\frac{\partial C^*}{\partial t^*} + (\vec{V}.\nabla)C^* = D\nabla^2 C^* + D_T\nabla^2 T^* - K_r(C^* - C_1). \tag{4}$$

Kirchhoff's First Law:

$$div\,\vec{j} = 0\ , \tag{5}$$

General Ohm's Law:

$$\vec{j} + \frac{\omega_e\tau_e}{B_0}(\vec{j} \times \vec{B}) = \sigma\left[\vec{E} + \vec{V} \times \vec{B} + \frac{1}{e\eta_e}\nabla p_e\right], \tag{6}$$

Gauss's Law of Magnetism:

$$div\,\vec{B} = 0\ , \tag{7}$$

On the right hand side of (2) the first term $\vec{F} = g\beta(T^* - T_1) + g\beta^*(C^* - C_2)$ accounts for the force due to buoyancy, the second term is the Lorentz force due to magnetic field and the third term is the resistance due to porous matrix. In the last term $\exists$ is the Cauchy stress tensor and the constitutive equation derived by Coleman and Noll [24] for an incompressible homogeneous fluid of second order is

$$\exists = -p^*I + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_1^2. \tag{8}$$

Here $-p^*I$ is the interdeterminate part of the stress due to constraint of incompressibility,μ_1, μ_2 and μ_3 are the material constants describing viscosity, elasticity and cross-viscosity respectively. The kinematic A_1 and A_2 are the Rivelen Ericson constants defined as

$$A_1 = (\nabla\vec{V}) + (\nabla\vec{V})^T,\ A_2 = \frac{dA_1}{dt} + (\nabla\vec{V})^T A_1 + A_1(\nabla\vec{V}),$$

where ∇ denotes the gradient operator and d/dt the material time derivative. According to Markovitz and Coleman [26] the material constantsμ_1, μ_3 are taken as positive and μ_2 as negative. $\vec{B}$ is the magnetic induction vector, $\vec{j}$ is the current density and all other quantities have their usual meanings and have been defined in the text time to time.

The last term in equation (3) stands for heat due to radiation and is given by

$$\frac{\partial q^*}{\partial z^*} = 4a^*\sigma^*(T^{*4} - T_1^4), \tag{9}$$

for an optically thin gray gas. Here a^* is the mean absorption coefficient and σ^* is Stefan-Boltzmann constant. Assume the temperature differences within the flow sufficiently small, T^{*4} may be expressed as a linear function of the temperature. This is accomplished by expanding T^{*4} in a Taylor series about T_1 and neglecting higher order terms, thus

$$T^{*4} \cong 4T_1^{*3}T^* - 3T_1^4. \tag{10}$$

Substituting (10) into (9) and simplifying, we obtain

$$\frac{\partial q^*}{\partial z^*} = 16a^*\sigma^*T_1^3(T^* - T_1). \tag{11}$$

The substitution of equation (11) into the energy equation (3) for the heat due to radiation, we get

$$\frac{\partial T^*}{\partial t^*} + (\vec{V}.\nabla)T^* = \frac{k}{\rho C_p}\nabla^2 T^* - \frac{16a^*\sigma^* T_1^3}{\rho C_p}(T^* - T_1). \tag{12}$$

In equation (4) the last term stands for the chemical reaction and the second last term is the thermal-diffusion caused by the temperature gradient (called Soret effect).

3. Formulation of the problem

Consider an unsteady MHD free convective flow of an electrically conducting, viscoelastic (second order), incompressible fluid through a porous medium bounded between two insulated infinite vertical plates in the presence of Hall current and thermal radiation. The plates are at a distance 'd' apart. We introduce a Cartesian coordinate system with X^*-axis oriented vertically upward along the centreline of the channel. The Z^*-axis taken perpendicular to the planes of the plates lying in $z^* = \pm\frac{d}{2}$ planes and a strong magnetic field is applied along this axis. The non-uniform temperature at the plate $z^* = +\frac{d}{2}$ and the non-uniform species concentration at the plate $z^* = -\frac{d}{2}$ is assumed to be varying periodically with time. The temperature difference between the plates is high enough to induce the heat due to radiation.

The diagram of the physical problem is shown below.

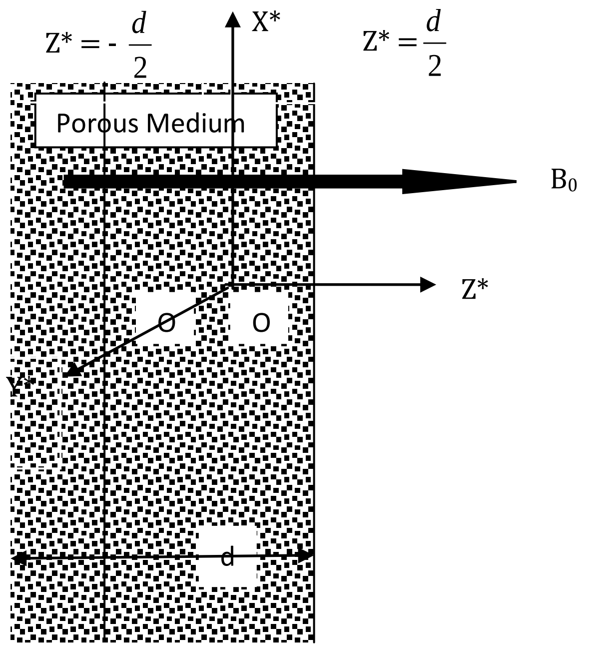

Figure 1. Schematic diagram of the physical problem.

Since the plates of the channel are of infinite extent, all the physical quantities depend only on z^* and t^* only for this problem of fully developed laminar flow. Let *(u*, v*, w*)* be the components of velocity in the directions *(x*, y*, z*)* respectively. Since the plates are non-porous, therefore equation of continuity (1) on integration gives $w^* = 0$. A strong transverse magnetic field of uniform strength B_0 is applied along the Z^*-axis. So equation (7) for the magnetic field $\vec{B} = \left(B_x^*, B_y^*, B_z^*\right)$ gives $B_z^* = B_0(constant)$.

If $\left(j_x^*, j_y^*, j_z^*\right)$ are the components of electric current density $\vec{j}$. The equation of conservation of electric charge (5) gives $j_z^* = constant$. For non-conducting plates $j_z^* = 0$ at the plates and hence zero everywhere in the fluid. Under the usual assumptions that the electron pressure (for a weakly ionized gas), the thermoelectric pressure, ion slip and the external electric field arising due to polarization of charges is negligible. It is assumed that no applied and polarization voltage exists. This corresponds to the case where no energy is being added or extracted from the fluid by electrical means (Meyer [26]) i.e. electrical field $\bar{E} = 0$. Therefore, the generalized Ohm's law (5) takes the form:

$$\vec{j} + \frac{\omega_e \tau_e}{B_0}\left(\vec{j} \times \vec{B}\right) = \sigma\left(\vec{V} \times \vec{B}\right).$$

After using equation $j_z^* = 0$, equation (13) in component form becomes

$$j_x^* + \omega_e \tau_e\, j_y^* = \sigma B_0 v^*, \tag{14}$$

$$j_y^* - \omega_e \tau_e\, j_x^* = -\sigma B_0 u^*. \tag{15}$$

Solving (14) and (15) for j_x^* and j_y^*, we get

$$j_x^* = \frac{\sigma B_0}{(1+H^2)}(Hu^* + v^*) \tag{16}$$

and $$j_y^* = \frac{\sigma B_0}{(1+H^2)}(Hv^* - u^*) \tag{17}$$

where $H = \omega_e \tau_e$ is the Hall parameter.

Using the velocity and the magnetic field distribution as stated above the MHD convective flow through porous medium is governed by the following Cartesian equations:

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \frac{\partial^2 u^*}{\partial z^{*2}} + \vartheta_2 \frac{\partial^3 u^*}{\partial z^{*2}\partial t^*} + \frac{\sigma B_0^2(Hv^* - u^*)}{\rho(1+H^2)} - \frac{\vartheta_1 u^*}{K^*} + g\beta(T^* - T_1) + g\beta^*(C^* - C_2), \tag{18}$$

$$\frac{\partial v^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \vartheta_1 \frac{\partial^2 v^*}{\partial z^{*2}} + \vartheta_2 \frac{\partial^3 v^*}{\partial z^{*2}\partial t^*} - \frac{\sigma B_0^2(Hu^* + v^*)}{\rho(1+H^2)} - \frac{\vartheta_1 v^*}{K^*}, \tag{19}$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}, \tag{20}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial z^{*2}} - \frac{16 a^* \sigma^* T_1^3}{\rho C_p}(T^* - T_1), \tag{21}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial z^{*2}} + D_T\frac{\partial^2 T^*}{\partial z^{*2}} - K_r(C^* - C_2), \tag{22}$$

where ρ is the density, ϑ_1 is the kinematic viscosity, ϑ_2 is the viscoelasticity, p^* is the pressure, t^* is the time, σ is the electric conductivity, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration, k is the thermal conductivity, c_p is the specific heat at constant pressure, D is the molecular diffusivity, D_T is the coefficient of thermal diffusion and K_r is the chemical reaction. Equation (20) shows the constancy of the hydrodynamic pressure along the axis of rotation.

The boundary conditions for the problem are

$$z^* = -\frac{d}{2}: \quad u^* = v^* = 0, T^* = T_1, C^* = C_2 + (C_1 - C_2)\cos\omega^* t^*, \tag{23}$$

$$z^* = \frac{d}{2}: \quad u^* = v^* = 0,\ T^* = T_1 + (T_2 - T_1)\cos\omega^* t^*, C^* = C_2, \tag{24}$$

where ω^* is the frequency of oscillations.

Introducing the following non-dimensional quantities

$$x,y,z = \frac{x^*,y^*,z^*}{d}, t = \frac{t^*U}{d}, \omega = \frac{\omega^* d}{U},\ u,v = \frac{u^*,v^*}{U}, \theta = \frac{T^*-T_1}{T_2-T_1}, C = \frac{C^*-C_2}{C_1-C_2}, p = \frac{p^*}{\rho U^2}, \tag{25}$$

into equations (18),(19), (21) and (22), we get

$$Re\frac{\partial u}{\partial t} = -Re\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial z^2} + \gamma Re\frac{\partial^3 u}{\partial z^2 \partial t} + \frac{M^2(Hv-u)}{(1+H^2)} - K^{-1}u + Gr\,\theta + Gm\,C, \tag{26}$$

$$Re\frac{\partial v}{\partial t} = -Re\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial z^2} + \gamma Re\frac{\partial^3 v}{\partial z^2 \partial t} - \frac{M^2(Hu+v)}{(1+H^2)} - K^{-1}v, \tag{27}$$

$$RePr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - N^2\theta, \tag{28}$$

$$ReS_c\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial z^2} + S_c\,S_0\frac{\partial^2 \theta}{\partial z^2} - K_r S_c\,C, \tag{29}$$

where U is the mean axial velocity, ‘*’ represents the dimensional quantities,

$\gamma = \frac{v_2}{d^2}$ is the visco-elastic parameter,

$Re = \frac{Ud}{\vartheta_1}$ is the Reynolds number,

$Gr = \frac{g\beta\, d^2(T_2-T_1)}{\vartheta_1 U}$ is the Grashof number,

$Gm = \frac{g\beta^* d^2(C_1-C_2)}{\vartheta_1 U}$ is the modified Grashof number,

$M = B_0 d\sqrt{\frac{\sigma}{\rho\vartheta_1}}$ is the Hartmann number,

$H = \omega_e\tau_e$ is the Hall parameter,

$K = \frac{K^*}{d^2}$ is the permeability of the porous medium,

$Pr = \frac{\rho \vartheta_1 c_p}{k}$ is the Prandtl number,

$N = 4d\sqrt{\frac{a^* \sigma^* T_1^3}{k}}$ is the radiation parameter,

$S_c = \frac{\vartheta_1}{D}$ is the Schmidt number,

$S_o = \frac{D_T (T_2 - T_1)}{\vartheta_1 (C_1 - C_2)}$ is the soret parameter,

$K_r = \frac{K_r^* d^2}{\vartheta_1}$ is the chemical reaction parameter.

The boundary conditions in the dimensionless form become

$$z = -\tfrac{1}{2}: \quad u = v = 0, \;\; \theta = 0, C = cos\, \omega t\, , \tag{30}$$

$$z = \tfrac{1}{2}: \quad u = v = 0, \;\; \theta = \cos \omega t\, , C = 0. \tag{31}$$

We shall assume now that the fluid flows under the influence of pressure gradient varying periodically with time in the X*-axis only is of the form

$$-\frac{\partial p}{\partial x} = A\, cos\omega t \;\text{ and }\; -\frac{\partial p}{\partial y} = 0\, , \tag{32}$$

where A is a constant.

4. Solution of the problem

Now combine equations (26) and (27) into single equation by introducing a complex function F = u + iv, we get

$$Re\frac{\partial F}{\partial t} = A\, cos\, \omega t + \frac{\partial^2 F}{\partial z^2} + \gamma Re \frac{\partial^3 F}{\partial z^2 \partial t} - \left(\frac{M^2(1+iH)}{(1+H^2)} + K^{-1}\right) F + Gr\, \theta + Gm\, C. \tag{33}$$

In order to solve the problem it is convenient to adopt complex notations and assume the solution of the problem as

$$F(z,t) = F_0(z) e^{i\omega t}, \theta(z,t) = \theta_0(z) e^{i\omega t}, C(z,t) = C_0(z) e^{i\omega t}, -\frac{\partial y}{\partial x} = A\, cos\omega t = Ae^{i\omega t}, \tag{34}$$

with corresponding boundary conditions as

$$z = -\tfrac{1}{2}: \quad F = 0, \;\; \theta = 0, C = cos\, \omega t, \tag{35}$$

$$z = \tfrac{1}{2}: \quad F = 0, \;\; \theta = cos\, \omega t, C = 0. \tag{36}$$

The boundary conditions (35) and (36) in complex notations can also be written as

$$z = -\tfrac{1}{2}: \quad F = 0, \;\; \theta = 0, C = e^{i\omega t}, \tag{37}$$

$$z = \frac{1}{2}: \quad F = 0, \ \ \theta = e^{i\omega t}, C = 0. \tag{38}$$

Substituting expressions (34) in equations (33), (28) and (29), we get

$$a^2 \frac{d^2F_0}{dz^2} - m^2F_0 = -ARe - Gr\,\theta_0 - Gm\,C_0\ , \tag{39}$$

$$\frac{d^2\theta_0}{dz^2} - n^2\theta_0 = 0, \tag{40}$$

$$\frac{d^2C_0}{dz^2} - l^2\theta_0 = -S_cS_0\frac{d^2\theta_0}{dz^2}\ , \tag{41}$$

where $a = \sqrt{1 + i\omega\gamma Re}$, $l = \sqrt{Sc(K_r + i\omega Re)}$, $m = \sqrt{\frac{M^2(1+iH)}{(1+H^2)} + K^{-1} + i\omega Re}$, $n = \sqrt{N^2 + i\omega RePr}$.

The transformed boundary conditions reduce to

$$z = -\frac{1}{2}: \quad F_0 = 0, \ \ \theta_0 = 0, C_0 = 1, \tag{42}$$

$$z = \frac{1}{2}: \quad F_0 = 0, \ \ \theta_0 = 1, C_0 = 0. \tag{43}$$

The ordinary differential equations (39), (40) and (41) are solved under the boundary conditions (42) and (43) for the velocity, temperature and species concentration fields. The solution of the problem is obtained as

$$F(z,t) = \left[\begin{gathered} \frac{ARe}{m^2}\left(1 - \frac{\cosh\frac{m}{a}z}{\cosh\frac{m}{2a}}\right) + \frac{Gr}{(a^2n^2-m^2)}\left\{\frac{\sinh\frac{m}{a}\left(\frac{1}{2}+z\right)}{\sinh\frac{m}{a}} - \frac{\sinh n\left(\frac{1}{2}+z\right)}{\sinh n}\right\} \\ + \frac{Gm}{(a^2l^2-m^2)}\left\{\frac{\sinh\frac{m}{a}\left(\frac{1}{2}-z\right)}{\sinh\frac{m}{a}} - \frac{\sinh l\left(\frac{1}{2}-z\right)}{\sinh l}\right\} \\ -\frac{GmS_cS_on^2}{(n^2-l^2)}\left\{\frac{\sinh l\left(\frac{1}{2}+z\right)}{(a^2l^2-m^2)\sinh l} - \frac{\sinh n\left(\frac{1}{2}+z\right)}{(a^2n^2-m^2)\sinh n} - \frac{a^2(n^2-l^2)\sinh\frac{m}{a}\left(\frac{1}{2}+z\right)}{(a^2l^2-m^2)(a^2n^2-m^2)\sinh\frac{m}{a}}\right\}\end{gathered}\right] e^{i\omega t}. \tag{44}$$

$$\theta(z,t) = \frac{\sinh n\left(\frac{1}{2}+z\right)}{\sinh n}e^{i\omega t}\ , \tag{45}$$

$$C(z,t) = \left[\frac{\sinh l\left(\frac{1}{2}-z\right)}{\sinh l} + \frac{S_cS_on^2}{(n^2-l^2)}\left\{\frac{\sinh l\left(\frac{1}{2}+z\right)}{\sinh l} - \frac{\sinh n\left(\frac{1}{2}+z\right)}{\sinh n}\right\}\right]e^{i\omega t}. \tag{46}$$

The primary velocity is given by the real part of complex function F(z,t).

From the velocity field given by equation (44), we can obtain the skin-friction τ_L at the left plate in terms of its amplitude and phase angle as

$$\tau_L = \left(\frac{\partial F}{\partial z}\right)_{z=-\frac{1}{2}} = |F|\ \cos(\omega t + \varphi)\ , \tag{47}$$

with $F_r + i\,F_i = \frac{ARe}{ma}\tanh\frac{m}{2a} + \frac{Gr}{(a^2n^2-m^2)}\left\{\frac{\frac{m}{a}}{\sinh\frac{m}{a}} - \frac{n}{\sinh n}\right\} - \frac{Gm}{(a^2l^2-m^2)}\left\{\frac{m}{a}\coth\frac{m}{a} - l\coth l\right\}$

$$-\frac{GmS_cS_on^2}{(n^2-l^2)}\left\{\frac{l}{(a^2l^2-m^2)\,sinh\,l}-\frac{n}{(a^2n^2-m^2)\,sinh\,n}-\frac{am\,(n^2-l^2)}{(a^2l^2-m^2)(a^2n^2-m^2)\,sinh\,\frac{m}{a}}\right\}. \quad (48)$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle $\varphi = tan^{-1}\frac{F_i}{F_r}$. (49)

From the temperature field given in equation (45) the heat transfer coefficient Nu (Nusselt number) in terms of its amplitude and the phase angle can be obtained as

$$Nu = \left(\frac{\partial\theta}{\partial z}\right)_{z=-\frac{1}{2}} = |H|\cos(\omega t + \psi), \quad (50)$$

where $H_r + i\,H_i = \frac{n}{\sinh\,n}$. (51)

The amplitude $|H|$ and the phase angle ψ of the heat transfer coefficient Nu (Nusselt number) are given by

$$|H| = \sqrt{H_r^2 + H_i^2} \text{ and } \psi = \tan^{-1}\left(\frac{H_i}{H_r}\right) \text{ respectively.} \quad (52)$$

Similarly, The amplitude and the phase angle of the Sherwood number at the left plate (z=-0.5) can be obtained from equation (46) for the species concentration as

$$Sh = \left(\frac{\partial C}{\partial z}\right)_{z=-\frac{1}{2}} = |C|\cos(\omega t + \zeta), \quad (53)$$

where $C_r + iC_i = -l\coth l + \frac{S_cS_on^2}{(n^2-l^2)}\left(\frac{l}{\sinh\,l} - \frac{n}{\sinh\,n}\right)$. (54)

The amplitude $|C|$ and the phase angle ζ of Sherwood number (Sh) are given by

$$|C| = \sqrt{C_r^2 + C_i^2} \text{ and } \zeta = \tan^{-1}\left(\frac{C_i}{C_r}\right) \text{ respectively.}$$

5. Results and Discussion

The problem of unsteady MHD convective flow of viscoelastic fluid through the porous medium bounded by two infinite vertical plates is analyzed. The temperature and the species concentration both vary periodically in time. The Hall current have been taken into account due to the transverse application of the magnetic field. Numerical calculations have been carried out to study the effects of viscoelastic parameter γ, Reynolds number Re, Grashof number Gr, modified Grashof number Gm, Hartmann number M, Hall parameter H, porous medium permeability parameter K, Prandtl number Pr, radiation parameter N, Schmidt number Sc, soret parameter S_o, chemical reaction parameter K_r and the frequency of oscillations ω on the velocity, temperature, species concentration fields and the shear stress, Nusselt number and Sherwood number in terms of their amplitude and the phase angle.

The variations of primary velocity profiles under the influence of different parameters against z i.e., over the channel width are shown graphically in Fig. 2. The values of various parameters listed in Table 1 represent different curves in Fig. 2. The velocity profiles remain parabolic in the channel. In order to study the effect of each of the parameter every curve is compared with the dotted curve I (---). Comparison of curves III, IV, V, VII, VIII and XII with the dotted curve I (---) reveals that the primary velocity increases with the increase of Reynolds number Re, Grashof number Gr, modified Grashof number Gm, Hall parameter H, permeability of porous the medium parameter K and soret parameter S_o. The increase of Reynolds number which is the ratio of inertial to viscous forces implies that the velocity increases. The buoyancy force due to thermal diffusion and molecular diffusion enhance the flow velocity. At the same time as expected physically also that the resistance posed by the porous matrix reduces with increasing permeability of the porous medium which consequently leads to the gain in the velocity.

Similarly, the comparison of curves II, VI, IX, X, XI, XIII and XIV with the dotted curve I (---) shows that the primary velocity decreases with increasing viscoelastic parameter γ, Hartmann number M, Prandtl number Pr, radiation parameter N, Schmidt number Sc, Chemical reaction parameter K_r and the frequency of oscillation ω. The decrease of velocity due M is because of the reason that effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. The decrease with the increase of Prandtl number Pr is due to the fact that increasing Prandtl number means (Prandtl number being the ratio of the viscous to the thermal diffusion) the dominance of the viscous over the thermal diffusion. Thus, the fluid flow is resisted because of this predominance property of the viscous fluid which leads to the decrease in velocity. To be more realistic the two values of the Prandtl number chosen represent air (Pr=0.7) and water (Pr=7) the most commonly found fluids on earth. As expected the flow velocity is less in water (Pr=7) than in air (Pr=0.7).

The amplitude $|F|$ of the skin-friction τ_L on the left plate (y = -0.5) is plotted in Figure 3 against ω the frequency of oscillations. The values of various parameters listed in Table 2 represent different curves in this Figure. It is quite evident that the amplitude of the skin friction goes on decreasing with increasing frequency of oscillations ω. In order to study the effect of each parameter every curve is compared with the dotted curve I (---). Comparison of curves IV, V, VII, VIII and XII with the dotted curve I (---) shows that the amplitude increases with the increase of Grashof number Gr, modified Grashof number Gm, Hall parameter H, of porous medium permeability K and soret parameter S_o. Similarly, comparing curves II, VI, IX, X, XI and XIII with the dotted curve I (---) show that the amplitude $|F|$ decreases with increasing viscoelastic parameter γ, Hartmann number M, Prandtl number Pr, radiation parameter N, Schmidt number Sc and chemical reaction parameter Kr. It is interesting to note that with the increase of Reynolds number the amplitude increases for $\omega<15$ but decreases for $\omega>15$.

The effects of the variations of different flow parameters on the phase angle φ of the skin-friction τ_L are illustrated in Fig. 4. The values of various parameters listed in Table 2 represent different curves in Fig. 4. It is obvious from this figure that there is always a phase lag because the values of φ plotted against ω are negative throughout. This lag in the phase goes on increasing further as the frequency of oscillations ω increases. Every curve in the figure is compared with the basic dotted curve I (---) to know the effect of each of the flow parameter. By the comparison of curves V, X, XII and XIII with the dotted curve I (---) we find that the lag in phase angle decreases with the increase of modified Grashof number Gm, radiation parameter N, soret parameter S_o and reaction parameter K_r. However, by the increase of viscoelastic parameter γ, Reynolds number Re, Grashof number Gr, Hartmann number M, Hall parameter H, porous medium permeability parameter K, Prandtl number Pr and Schmidt number Sc the lag in phase angle increases as is observed by the comparison of curves II, III, IV, VIII, IX and XI.

The temperature profiles are shown in Fig. 5. It is quite clear from this figure that the temperature decreases with the increase of each of the parameters involved i.e., Reynolds number, Prandtl number Pr, radiation parameter N and the frequency of oscillations ω. Similarly, Fig. 6. shows that the species concentration increases with the increase of Reynolds number Re, soret parameter S_o and the frequency of oscillations ω but decreases with chemical reaction parameter K_r. The variation in species concentration is insignificant due to the increase of Schmidt number S_c. A careful study of the curves reveals that the species decreases in left half but increases in right half of the channel. The amplitude $|H|$ and the phase angle of the Nusselt number are presented in Figs. 7. and 8. respectively. Fig. 7. reveals that $|H|$ decreases sharply with the increase of Reynolds number and the Prandtl number as frequency ω increases. However, the amplitude remains almost constant for increased radiation parameter N as ω increases. The phase angle ψ of rate of heat transfer shown in Fig. 8. oscillates between phase lag and phase lead for increased Reynolds number Re and the Prandtl number Pr as frequency of oscillations ω increases. Over the values of ω chosen a phase lag is noticed for small Reynolds number and increased radiation parameter N. The amplitude $|C|$ and phase angle ζ of the Sherwood number are shown in Figs. 9. and 10. Fig. 9. shows that the amplitude $|C|$ increases with the increase of Reynolds number Re, Schmidt number Sc and reaction parameter K_r but decreases with the increase of soret number S_o. It is also very clear from Fig. 10. that the phase angle ζ increases with the increase of Reynolds number Re and Schmidt number Sc but decreases with the increase of reaction parameter K_r. A phase lead is observed for increased Re, S_c and K_r but for increased soret number S_o there is a phase lag.

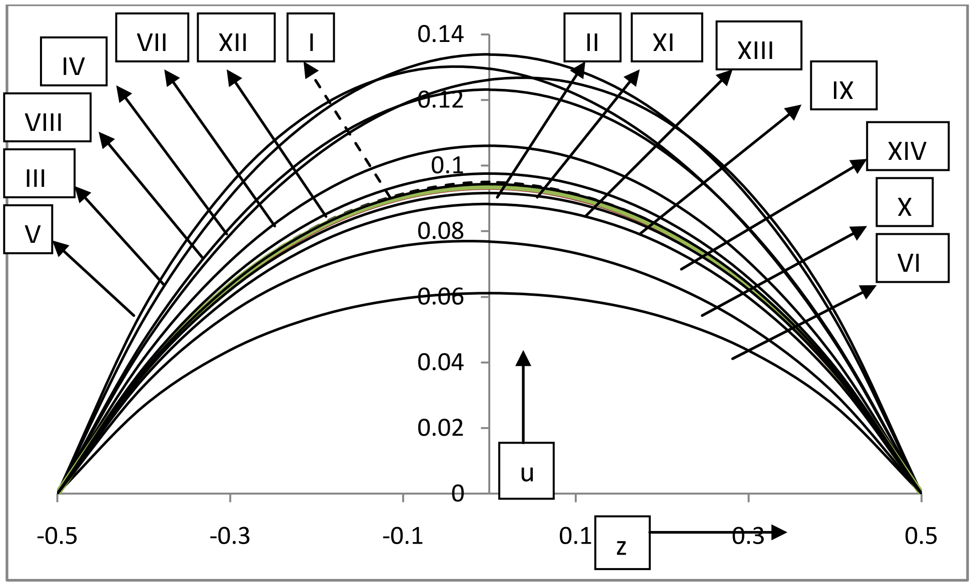

Fig. 2. Velocity profiles.

Table 1. Sets of parameter values plotted in Fig. 2. for A=2 and t=0.

γ	Re	Gr	Gm	M	H	K	Pr	N	Sc	So	Kr	ω	Curve
0.2	0.2	1	1	2	1	0.2	0.7	1	0.22	1	1	1	I(---)
0.5	0.2	1	1	2	1	0.2	0.7	1	0.22	1	1	1	II
0.2	0.5	1	1	2	1	0.2	0.7	1	0.22	1	1	1	III
0.2	0.2	2	1	2	1	0.2	0.7	1	0.22	1	1	1	IV
0.2	0.2	1	2	2	1	0.2	0.7	1	0.22	1	1	1	V
0.2	0.2	1	1	4	1	0.2	0.7	1	0.22	1	1	1	VI
0.2	0.2	1	1	2	3	0.2	0.7	1	0.22	1	1	1	VII
0.2	0.2	1	1	2	1	1.0	0.7	1	0.22	1	1	1	VIII
0.2	0.2	1	1	2	1	0.2	7.0	1	0.22	1	1	1	IX
0.2	0.2	1	1	2	1	0.2	0.7	5	0.22	1	1	1	X
0.2	0.2	1	1	2	1	0.2	0.7	1	0.94	1	1	1	XI
0.2	0.2	1	1	2	1	0.2	0.7	1	0.22	5	1	1	XII
0.2	0.2	1	1	2	1	0.2	0.7	1	0.22	1	5	1	XIII
0.2	0.2	1	1	2	1	0.2	0.7	1	0.22	1	1	5	XIV

Table 2. Sets of parameter values plotted in Figs. 3. and 4. for A=2.

γ	Re	Gr	Gm	M	H	K	Pr	N	Sc	So	Kr	Curve
0.2	0.2	1	1	2	1	0.2	0.7	1	0.22	1	1	I(---)
0.3	0.2	1	1	2	1	0.2	0.7	1	0.22	1	1	II
0.2	0.5	1	1	2	1	0.2	0.7	1	0.22	1	1	III
0.2	0.2	2	1	2	1	0.2	0.7	1	0.22	1	1	IV
0.2	0.2	1	2	2	1	0.2	0.7	1	0.22	1	1	V
0.2	0.2	1	1	4	1	0.2	0.7	1	0.22	1	1	VI
0.2	0.2	1	1	2	3	0.2	0.7	1	0.22	1	1	VII
0.2	0.2	1	1	2	1	1.0	0.7	1	0.22	1	1	VIII
0.2	0.2	1	1	2	1	0.2	7.0	1	0.22	1	1	IX
0.2	0.2	1	1	2	1	0.2	0.7	5	0.22	1	1	X
0.2	0.2	1	1	2	1	0.2	0.7	1	0.94	1	1	XI
0.2	0.2	1	1	2	1	0.2	0.7	1	0.22	5	1	XII
0.2	0.2	1	1	2	1	0.2	0.7	1	0.22	1	5	XIII

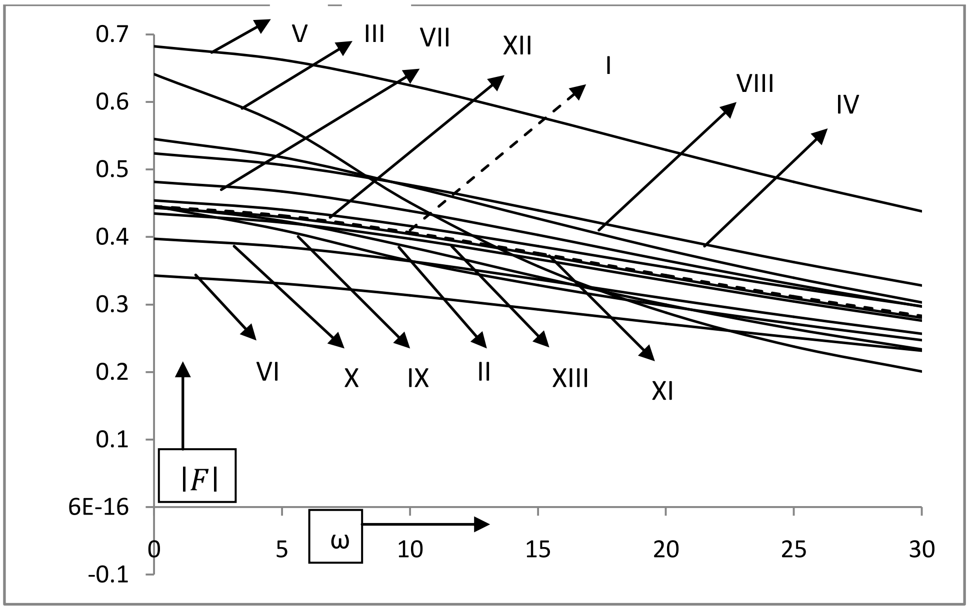

Fig. 3. Amplitude of shear stress.

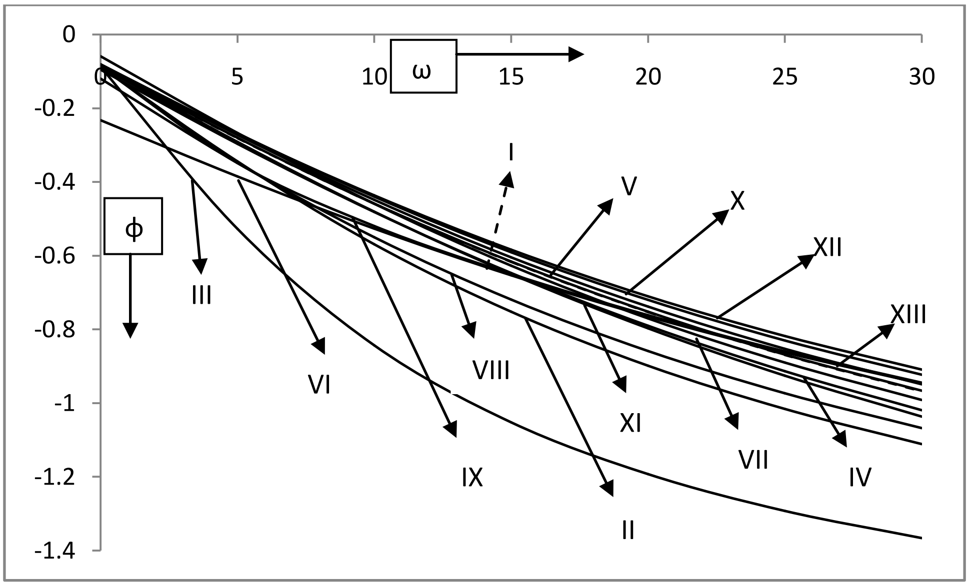

Fig. 4. Phase angle of shear stress.

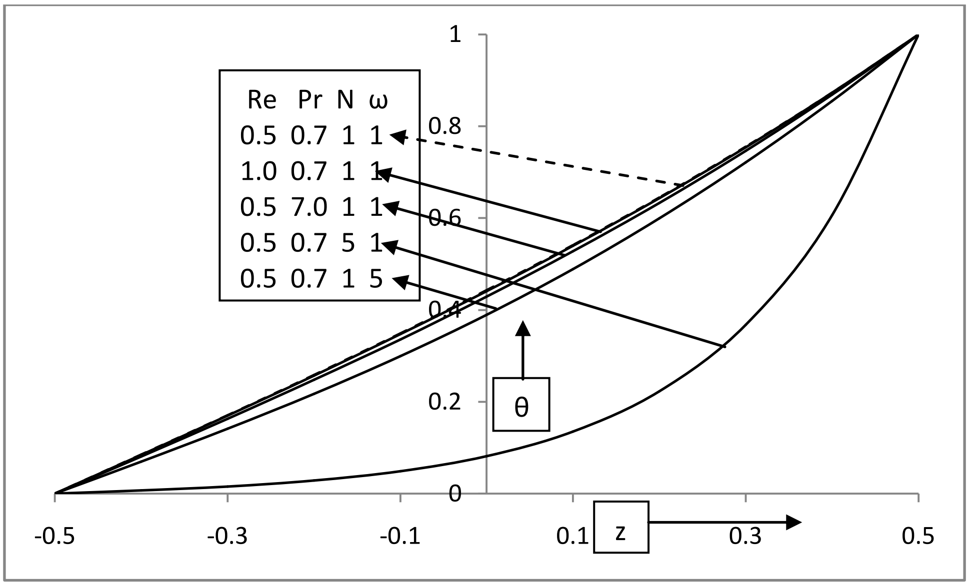

Fig. 5. Temperature profile for t=0.

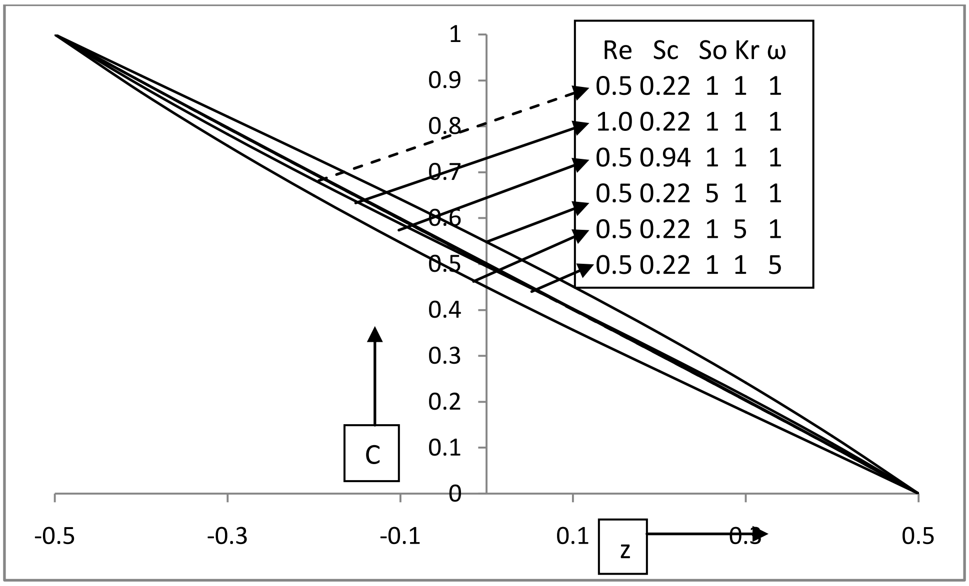

Fig. 6. Species concentration for t=0.

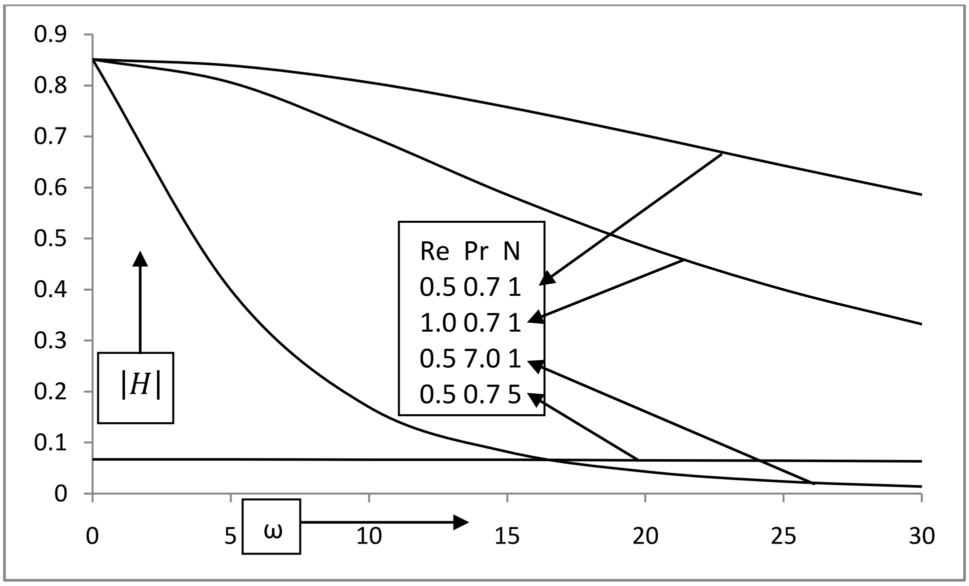

Fig. 7. Amplitude of Nusselt number.

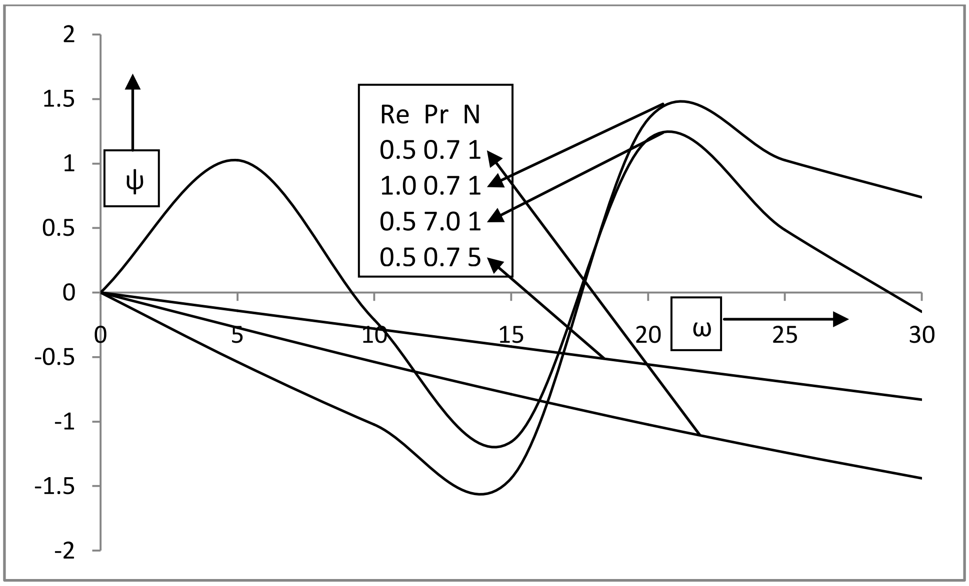

Fig. 8. Phase angle of Nusselt number.

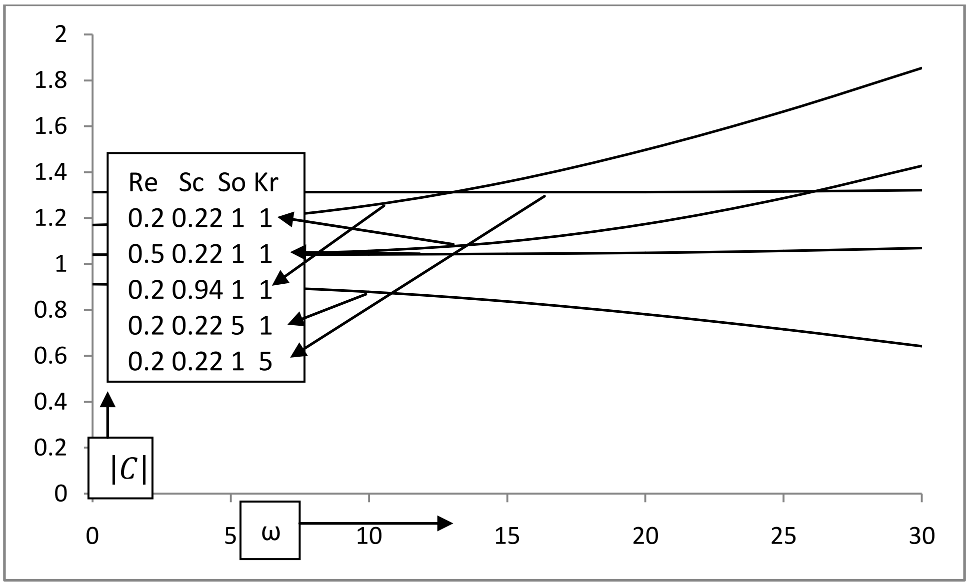

Fig. 9. Amplitude of Sherwood number.

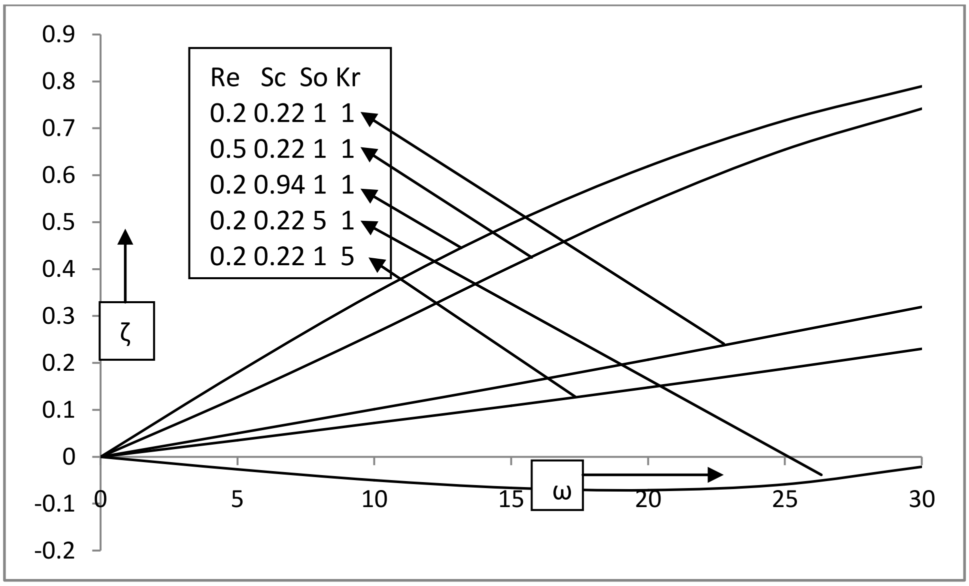

Fig. 10. Phase angle of Sherwood number.

6. Conclusions

An exact solution of heat and mass transfer oscillatory MHD convection flow through the porous medium in a vertical channel is analyzed. Following conclusions of the study are made:

- The velocity increases with the increase of Reynolds number, Grashof number, modified Grashof number, Hall parameter, porous medium permeability and soret parameter.
- The velocity decreases with increasing viscoelastic parameter γ, Hartmann number M, Prandtl number Pr, radiation parameter N, Schmidt number Sc, Chemical reaction parameter K_r and the frequency of oscillation ω.
- The amplitude increases with the increase of Grashof number Gr, modified Grashof number Gm, Hall parameter H, of porous medium permeability K and soret parameter S_o.
- The amplitude $|F|$ decreases with increasing viscoelasticity γ, Hartmann number M, Prandtl number Pr, radiation parameter N, Schmidt number Sc and chemical reaction parameter Kr.
- The increasing frequency of oscillations ω leads to the decrease of the amplitude of skin friction while to an increase in the phase lag.
- The temperature and the concentration both decrease with the increase of the parameters involved.
- The amplitude of Nu decreases with Prandtl number Pr or radiation N.
- The amplitude $|C|$ increases with increase of Re, Sc and K_r but decreases with increasing S_o.
- The phase angle ψ of Nu oscillates between phase lag and phase lead for increased Reynolds number Re and the Prandtl number Pr as frequency of oscillations ω increases.
- A phase lead is observed for increased Re, S_c and K_r but for increased soret number S_o there is a phase lag.

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